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The main difficulty which Demosthenes presents to the student lies in the close logical connection of his arguments; and most commentaries consist largely of translation or paraphase. Paraphase is dangerous, as it may lead a novice to a belief that he quite understands a piece of Latin or Greek, when he is some way from doing so. I have, therefore, taken the bull by the horns, and have given a continuous rendering, as close as I could decently make it. Its aim is purely commentatorial—to save its weight in notes. It is intended to show what Demosthenes said, but not how well he said it. And, I may say, I believe that every lecturer and tutor in Oxford will admit that an undergraduate, or sixth-form boy, cannot get full value out of reading the De Corona without such help.

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LONDON: SIMPKIN, MARSHALL, & CO.

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PREFACE.

It may perhaps appear somewhat rash to add another to the many text books on Arithmetic already in use. It is believed, however, that the present work will be found to contain some features sufficiently distinctive and valuable to warrant its appearance.

The chief aim of the work is to place prominently before the student the fact that the principle of 'cause and effect' is as applicable to Arithmetic as to other sciences, and that by working on this plan he may obtain his results in about half the time required by other methods. In each division of the subject this is shown by a typical example worked out in full, followed by ample exercises, with examination-papers at regular intervals. Complex examples of fractions and decimals are introduced later, so that the pupil may reach as early as possible the more interesting part of his work, the arithmetic of ordinary life. The so-called 'inverse rules' are learnt at the same time as the 'direct rules,' and thus the difficulties of proportion wholly disappear.

Tables and rules are placed at the beginning of the volume, and will be found useful for reference.

One thousand examples, carefully selected from papers set by the Universities, Civil Service and Army Examination Boards, and other Educational bodies, have been placed at the end of the volume.

The Answers are published in a separate form.

In order to insure accuracy I have tested most of the results with my pupils; but, of course, in a work of this kind, consisting of more than 7,000 examples, some errors must still remain. I should, therefore, be truly grateful to anyone who will kindly inform me or the publisher of such imperfection when detected.

FREDERICK SPARKS.

THE COLLEGE, PLYMOUTH:
November 1884.

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VARIOUS TABLES.

1. NOTATION TABLE.

1.	I.	11.	XI.	30.	XXX.	400.	CD.
2.	II.	12.	XII.	40.	XL.	500.	D or IↃ.
3.	III.	13.	XIII.	50.	L.	600.	DC.
4.	IV.	14.	XIV.	60.	LX.	700.	DCC.
5.	V.	15.	XV.	70.	LXX.	800.	DCCC.
6.	VI.	16.	XVI.	80.	LXXX.	900.	CM.
7.	VII.	17.	XVII.	90.	XC.	1000.	M or CIↃ.
8.	VIII.	18.	XVIII.	100.	C.	2000.	MM.
9.	IX.	19.	XIX.	200.	CC.	5000.	V or IↃↃ.
10.	X.	20.	XX.	300.	CCC.	10000.	CCIↃↃ.

The first column gives the Arabic or common arithmetical figures; the second, the Roman numerals (still in occasional use, chiefly for inscriptions), consists of seven letters, viz. I, one; V, five; X, ten; L, fifty; C, one hundred; D (originally IↃ), five hundred; and M (originally CIↃ), one thousand. Other numbers are expressed by a combination of the above symbols, viz.:—(1) When a symbol is followed by one of equal or less value, the whole expression indicated is the sum of the values of the symbols, thus: III stands for 3; VII for 7; LXXXV for 85; CCCXXXIII for 333. (2) When a symbol is preceded by one of less value it indicates that the latter is to be subtracted from the former, thus: IV stands for 4; XL for 40; XCIX or IC for 99; CM for 900. (3) Every Ↄ annexed to IↃ increases its value tenfold, thus: IↃↃ is 5,000; IↃↃↃ is 50,000; and so on. (4) Every C prefixed and Ↄ annexed to CIↃ increases the value tenfold, thus: CCIↃↃ is 10,000; CCCIↃↃↃ is 100,000; and so on. (5) A horizontal line placed over a symbol increases its value a thousandfold, thus: V̄ is 5000; C̄ is 100,000; D̄ is 500,000; and so on. Therefore either M.DCCCLXXXIV, or CIↃ.IↃCCCLXXXIIII, or Ī.DCCCLXXXIV represents 1884.

2. NUMERATION TABLE.

Units	1	Hundreds.
Tens	21	
Hundreds	321	
Thousands	4,321	Thousands.
Tens of Thousands	54,321	
Hundreds of Thousands	654,321	
Millions	7,654,321	Millions.
Tens of Millions	87,654,321	
Hundreds of Millions	987,654,321	

3. MULTIPLICATION TABLE CONDENSED IN SQUARES.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

4. SQUARES, CUBES, SQUARE AND CUBE ROOTS.

No.	Square	Cube	Square Root	Cube Root
1	1	1	1	1
2	4	8	1·414213	1·259921
3	9	27	1·732050	1·442249
4	16	64	2·	1·587401
5	25	125	2·236068	1·709976
6	36	216	2·449489	1·817121
7	49	343	2·645751	1·912931
8	64	512	2·828427	2·
9	81	729	3·	2·080083
10	100	1000	3·162277	2·154434
11	121	1331	3·316624	2·223980
12	144	1728	3·464101	2·289428
13	169	2197	3·605551	2·351335
14	196	2744	3·741657	2·410142
15	225	3375	3·872983	2·466212
16	256	4096	4·	2·519842
17	289	4913	4·123105	2·571281
18	324	5832	4·242640	2·620741
19	361	6859	4·358898	2·668402
20	400	8000	4·472136	2·714417

5. MONEY TABLE.

All coinage of money for general circulation is done at a large Government Office in London, called *The Mint.* From that place issue the gold coinage of sovereigns and half-sovereigns; the silver coinage of crowns, half-crowns, florins, shillings, sixpences, fourpences, and threepences, and the copper coinage of pence, half-pence, and farthings. The standard for gold coin is 22 carats of fine gold and 2 carats of copper melted together. From a pound Troy of standard gold, there are coined $46\frac{29}{40}$ sovereigns, or £46 14*s.* 6*d.* The standard for silver is 11 oz. 2 dwts. (or $\frac{37}{40}$) of fine silver, and 18 dwts. (or $\frac{3}{40}$) of alloy, which is now coined into 66*s.*, instead of 62*s.* as formerly. The standard for bronze coinage is 95 parts of copper, 4 of tin, and 1 of zinc. From a pound Avoirdupois of bronze are coined 48 pence, or 80 half-pence, or 160 farthings. The half-penny measures exactly an inch in diameter. Bronze coins can also be used as convenient letter weights, viz., 3 pence, or 5 half-pence, or 10 farthings = 1 ounce. A pound Avoirdupois of copper used to be coined into 24 pence, or 48 half-pence, or 96 farthings. Their respective values are as under, viz.:—

Copper.

2 farthings......... = 1 halfpenny.
2 halfpence ...... = 1 penny.
12 pence .... = 1 shilling.
24 pence .... = 1 florin.
30 pence .... = 1 half-crown.
60 pence .... = 1 crown.

[See also page 80.]

Silver.

4 threepences ...... = 1 shilling.
3 fourpences ...... = 1 shilling.
2 sixpences .... = 1 shilling.
2 shillings .... = 1 florin.
2 shillings and sixpence ... = 1 half-crown.
2 half-crowns ...... = 1 crown.

Gold.

2 crowns .... = 1 half-sovereign.
2 half-sovereigns = 1 sovereign.
20 shillings ...... = 1 sovereign.

Symbols.

£ *s.* *d.* are the symbols for pounds, shillings, and pence, and are abbreviations of the Latin words, *libra*, *solidus*, and *denarius.*

The Bank of England also issues paper money called *bank notes*, which are of the respective values of *five pounds* to *one thousand pounds.* These are at any time exchangeable for gold by the law of the country. Another kind of paper money is issued by the Post Office, called *Postal Orders,* of the respective values of one shilling to twenty shillings, which are at any time exchangeable for coin, but subject to a small deduction if not cashed within limited periods. Copper coinage is not a legal tender for more than 12*d.*, nor silver coinage for more than 40*s.*; bank notes are a legal tender for any amount *above* £5. Postal Orders are not legal tender. Gold coinage is the standard of this country.

OTHER ENGLISH COINS (*not now in common use*).

A groat	= fourpence	Silver.	A half-guinea	= 10*s.* 6*d.*	Gold.
A tester	= sixpence		A guinea	= £1 1*s.*	
A noble	= 6*s.* 8*d.*	Gold.	A carolus	= £1 3*s.*	
An angel	= 10*s.*		A jacobus	= £1 5*s.*	
A mark	= 13*s.* 4*d.*		A moidore	= £1 7*s.*	

DECIMAL COINAGE SUGGESTED IN PARLIAMENTARY REPORT, 1853.

(The mil .... = 0·24 penny).
10 mils .... = 1 cent (2·4 pence).
10 cents .... = 1 florin (24 pence).
10 florins .... = 1 pound (240 pence).

6. TROY WEIGHT.

4 grains (*grs.*) .... = 1 carat.
6 carats, or 24 grs. ...... = 1 pennyweight (*dwt.*)
20 pennyweights .... = 1 ounce (*oz.*)
12 ounces .... = 1 pound (*lb.*)
25 pounds .... = 1 quarter (*qr.*)
100 pounds .... = 1 hundredweight (*cwt.*)
20 hundredweights ...... = 1 ton of gold or silver.

Gold, silver, jewels, amber, precious stones, electuaries, and all liquids are weighed by this weight, which is also used for determining specific gravities and in philosophical researches.

VARIATIONS.—Diamonds and pearls are weighed by the carat, which contains 4 *grs.*, but 5 diamond *grs.* are only = to 4 Troy *grs.* 150 diamond carats = the *oz.* Troy, ∴ a carat of diamonds or pearls = 3⅕ *grs.* Troy. In gold the carat is the 24th part of the pound, or 10 *dwts.* (4 *grs.* = 1 carat), ∴ the *carat* grain is = 2½ *dwts.* or 60 *grs.*, and the carat = 240 *grs.* This measurement is used solely for the purpose of determining the *fineness* of gold, the whole weight being supposed to be divided into 24 equal parts (carats). If the gold be pure, it is said to be 24 carats fine; if 22 parts be pure and 2 alloy, it is said to be 22 carats fine. Jewellers' gold is 18 carats fine.

The proportion of a pound Troy to a pound Avoirdupois is as 14 to 17, the former containing 5,760 grains, and the latter 7,000.

7. APOTHECARIES' MEASURES.

WEIGHT.		FLUID MEASURE.	
20 grains (*gr.*)	= 1 scruple (℈)	60 minims (♏)	= 1 drachm (*f* ʒ)
3 scruples	= 1 drachm (ʒ)	8 drachms	= 1 ounce (*f* ℥)
8 drachms	= 1 ounce (℥)	20 ounces	= 1 pint (*O*)
12 ounces	= 1 pound (*lb.*)	8 pints	= 1 gallon (*gall.*)

The grain, ounce, and pound are the same as in Troy weight; in fact, the only difference in the two measures of weight is in the subdivisions of the pound. Apothecaries compound their medicines by these weights, but buy and sell by Avoirdupois.

8. AVOIRDUPOIS WEIGHT.

16 drachms (*dr.*) = 1 ounce (*oz.*)
16 ounces ...... = 1 pound (*lb.*)
14 pounds ...... = 1 stone (*st.*)
28 pounds ...... = 1 quarter (*qr.*)
4 quarters ... = 1 hundred (*cwt.*)
20 hundred...... = 1 ton (*t.*)

VARIATIONS.

1 great pound of silk... = 24 oz.
1 stone of butchers' meat or fish............ = 8 lbs.
1 firkin of butter ...... = 56 lbs.
1 pack of wool .... = 240 lbs.
1 sack of flour .... = 280 lbs.
1 fodder of lead ...... = 19½ cwt.

By Avoirdupois weight, which derives its name from *avoirs* (goods or chattels) and *poids* (weight), are weighed all the larger and coarser sorts of goods. An Act of Parliament passed in 1878 declares that 'all articles sold by weight shall be by Avoirdupois weight, except gold, silver, platinum, diamonds, and other precious metals or stones, and drugs when sold by retail; and that such excepted articles and none others may be sold by Troy weight.'

9. COAL WEIGHT.

14 pounds .... = 1 stone.
28 ,, ... = 1 quarter-cwt.
56 ,, ... = 1 half cwt.
1 sack of 112 lbs. .... = 1 cwt.
1 large sack of 224 lbs.......... = 2 cwt.
20 cwt., or 10 large sacks...... = 1 ton.
53 cwt. .... = 1 Newcastle chaldron.
21 tons 4 cwt. .... = 1 barge or keel.
20 keels, or 424 tons........... = 1 shipload.
140 cwt., or 7 tons..... .... = 1 room.

By Act of Parliament passed in 1831 it is directed that all coals be sold by weight instead of measure; 10 sacks of 224 lbs. each to one ton.

10. HAY AND STRAW WEIGHT.

36 pounds ... = 1 truss of straw.
36 ,, ... = 1 ,, old hay.
60 ,, ... = 1 ,, new hay.
36 trusses...... = 1 load.
18 cwt.......... = 1 ,, of old hay.

19 cwt. 32 lbs. = 1 load of new hay.
11 ,, 64 ,, = 1 ,, straw.
1 cu. yd. of new hay weighs 6 stone.
1 ,, oldish ,, 8 ,,
1 ,, old ,, 9 ,,

Hay is considered as new for three months, and is called old on the 1st of September. To find the weight of hay contained in a stack, multiply the length by its breadth, and the result by its height, all in feet; divide the product by 27, which will give the number of cubic yards; again multiply by 6, 8, or 9, according to the age of the hay as above, and the product will be the weight in stones. In measuring the height, deduct two-thirds of the distance in feet fiom the eaves to the top.

11. CATTLE WEIGHT.

To calculate the weight of cattle, measure round the animal close behind the shoulder, then along the back from the fore part of the shoulder-blade to the bone at the tail. Multiply the square of the girth by five times the length, both expressed in feet. Divide the product by 21, and the result is the weight of the four quarters in stones of 14 lbs. In very fat cattle the weight is about a twentieth more than ascertained in this manner, while very lean ones weigh about a twentieth less. The quarters are little more than half the weight of the animal while living. The skin weighs about the eighteenth, and the tallow about the twelth of the whole.

12. MEASURES OF LENGTH.

3 barleycorns...... = 1 inch (*in.*)
12 inches .... = 1 foot (*ft.*)
3 feet .... = 1 yard (*yd.*)
5½ yards .... = 1 pole (*po.*)
4 po., or 100 links = 1 chain (*ch.*)
40 po., or 10 chains = 1 furlong (*fur.*)
8 furlongs .... = 1 mile (*m.*)
3 miles .... = 1 league (*lea.*)
60 geographical m. = 1 degree (*deg.*)
69½ miles .... = 1 degree.
360 degrees .... = circumf. of globe.

12 lines ...... = 1 inch.
3 inches ... = 1 palm.
4 inches ...... = 1 hand (for measuring the height of horses).
9 inches ...... = 1 span.
18 ,, ... = 1 cubit.
2½ feet ...... = 1 military pace.
5 ,, ... = 1 itinerary pace.
2 yards ...... = 1 fathom (*fth*).
120 fathoms, or 24 yds. ...... = 1 cable's length.
6076 feet ...... = 1 knot.

365,172 feet, or 69·1613 miles = 1 degree of the equator.
364,565 feet, or 69·046 miles = 1 degree of the meridian, or 60 knots

The old Scotch and Irish miles are $1\frac{1}{8}$ and $1\frac{3}{11}$ English.

Among mechanics the inch is usually divided into eighths, but in scientific calculations it is generally divided into decimals, or otherwise the foot is decimally divided.

13. CLOTH MEASURE.

2¼ inches .... = 1 nail.
4 nails, or 9 inches .... = 1 quarter.
3 quarters .... = 1 Flemish ell.
4 quarters, or 36 inches .... = 1 yard.
5 quarters, or 45 inches .... = 1 English ell.
6 quarters .... = 1 French ell.
37·06 inches .... = 1 Scotch ell.

14. MEASURES OF SURFACE.—SQUARE OR LAND MEASURES.

144 inches (*sq. in.*) .... = 1 square foot (*sq. ft.*)
9 feet .... = 1 square yard (*sq. yd.*)
30¼ yards .... = 1 rod or pole (*sq. po.*)
16 poles .... = 1 chain (*sq. ch.*)

40 poles .... = 1 rood (*ro.*)
4 roods .... = 1 acre (*a.*)
640 acres .... = 1 square mile (*sq. m.*)
100 acres .... = 1 hide of land.
A square of flooring .... = 100 square feet.
A rod of brickwork .... = 272¼ square feet.

(The work is supposed to be 14 inches thick, or rather more than a brick and a half.)

8 hides were formerly a knight's fee, and 40 hides a baron's fee.

15. MEASURES OF SOLIDITY.

1728 cubic inches......................... = 1 cubic foot.
27 cubic feet .... = 1 cubic yard.

---

40 cubic feet of rough timber............... = 1 load.
50 ,, squared or hewn timber = 1 load.
42 ,, timber .... = 1 ton of shipping.

16. DRY MEASURE.

4 gills .... = 1 pint.
2 pints .... = 1 quart.
2 quarts .... = 1 pottle.
4 quarts .... = 1 gallon.
2 gallons .... = 1 peck.
4 pecks............... = 1 bushel.
4 bushels .... = 1 coomb.

3 bushels .... = 1 sack.
8 bushels, or 2 coombs = 1 quarter.
4 quarters .... = 1 chaldron.
5 quarters .... = 1 load.
2 loads, or 80 bushels = 1 last.
12 barrels .... = 1 last of meal.

This measure is used for grain, and dry goods, as fruit, potatoes, roots, &c. Heaped measures have been abolished since 1835.

17. WINE AND SPIRIT MEASURE.

4 gills .... = 1 pint (*pt.*)
2 pints .... = 1 quart (*qt.*)
4 quarts .... = 1 gallon (*gal.*)
10 gallons........ .... = 1 anker (*ank.*)
18 gallons........................ = 1 runlet (*run.*)
31½ gallons........................ = half a hogshead.
42 gallons........ .... = 1 tierce (*tce.*)
63 gallons........................ = 1 hogshead (*hhd.*)
84 gallons........................ = 1 puncheon (*pun.*)
2 hogsheads, or 126 gallons = 1 pipe or butt.
8 octave, or 4 quarter-casks = 1 pipe or butt.
2 pipes, or 252 gallons ...... = 1 tun.

Some branches of trade reckon 36 gallons = 1 tierce, 54 gallons = 1 hogshead, 108 gallons or 2 hogsheads = 1 butt or pipe. This, however, causes no inconvenience, as all the larger quantities, such as hogsheads, puncheons, &c., are gauged and charged according to the actual contents. The wine and spirit trade being an international commerce, there are a number of miscellaneous measures in use for the larger quantities, of which the principle are :—

30 gallons .... = 1 aum of hock, Moselle, and other German wines.
46 ,, ... = 1 hogshead of claret.
55 to 60 ,, ... = 1 ,, brandy.
60 ,, ... = 1 double aum of hock, &c.
92 ,, ... = 1 pipe of Madeira or Cape.
93 ,, ... = 1 ,, Marsala or Bronti.
90 to 100 ,, ... = 1 puncheon of rum.
100 ,, ... = 1 pipe of Teneriffe.
105 ,, ... = 1 butt of Tent, Malaga, or Mountain.
108 ,, ... = 1 ,, sherry.
115 ,, ... = 1 pipe of port or Masden.
117 ,, ... = 1 ,, Lisbon or Bucellas.
110 to 120 ,, ... = 1 puncheon of brandy.
110 to 130 ,, ... = 1 ,, Scotch whisky.

18. ALE AND BEER MEASURE.

2 pints......... = 1 quart.
4 quarts ...... = 1 gallon.
9 gallons ...... = 1 firkin (*fir.*)
2 firkins ...... = 1 kilderkin (*kil.*)
2 kilderkins, or 36 gals. = 1 barrel.
1½ barrel, or 54 gals. = 1 hogshd.
3 barrels, or 108 gals. = 1 butt.
2 butts, or 216 gals.... = 1 tun.

NOTE 1.—In London the gill is usually called a 'quartern,' and in the north of England the gill is termed a 'noggin,' and a half-pint a 'gill.'

NOTE 2.—To reduce cubic inches to bushels, multiply by 5 and divide by 11091; if to gallons, multiply by 40 and divide by 11091.

STANDARDS

Are carefully constructed measures or weights of acknowledged authority, by which others are tested and adjusted, till recently in the custody of the clerk of the House of Commons, but by Act of Parliament passed in 1866 transferred to the custody of the Board of Trade. The imperial standard yard is the unit wherefrom all other measures, lineal, superficial, or solid, are computed and ascertained. The imperial standard Troy pound, containing 5760 grains (and 7000 such grains to the pound Avoirdupois) is the unit wherefrom all other weights are computed and ascertained. The imperial standard gallon, containing 10 pounds Avoirdupois weight of distilled water weighed in air at a temperature of 62° Fahrenheit's thermometer with the barometer being at 30 inches, which fills 277·274 cubic inches, is the unit wherefrom all other measures of capacity are computed and ascertained.

WEIGHT OF DISTILLED WATER UNDER THE ABOVE CONDITIONS.

A cubic inch .... = 252·458 grains.
A cubic foot .... = 62·3206 lbs. Avoirdupois.
1·73298 cubic inch .... = 1 oz.
277·274 (*i.e.* an imperial gallon) = 10 lbs. Avoirdupois.

The diameter of a cylinder containing a gallon at one inch in depth is 18·78933 inches.

19. MEASURES OF TIME.

60 seconds (*s.*) = 1 minute (*m.*)
60 minutes = 1 hour (*h.*)
24 hours = 1 day (*d.*)
7 days = 1 week (*w.*)
4 weeks = 1 lunar month (*mth.*)

13 lunar months 1 day, or 12 calendar months, or 365 days = 1 year (*yr.*)
366 days .... = 1 leap year.
100 years .... = 1 century.

QUARTERLY TERMS.

Lady Day, March 25.
Midsummer, June 24.
Michaelmas, September 29.
Christmas, December 25.

In Scotland.

Candlemas, February 2.
Whitsuntide, May 15.
Lammas, August 1.
Martinmas, November 11.

A Solar Day is the duration of a complete rotation of the earth round its axis with respect to the sun—that is, the time which elapses between two successive transits of the sun across the meridian of any place. Although the motion of the earth's rotation in space is uniform, this interval is subject to fluctuations from the movement of the earth in its orbit, which is elliptical, the velocity of which is subject to a gradual acceleration and retardation. Clocks are therefore adjusted to an average or Mean Solar Day.

For astronomical purposes the day is reckoned to commence at noon, and is counted throughout the twenty-four hours, but in all other matters the day commences at midnight, and is divided into two equal portions, called morning or ante-meridian (A.M.), and evening or post-meridian (P.M.).

A Solar Year is determined by a revolution of the earth in its orbit round the sun from the vernal equinox to the vernal equinox again, and has been found by astronomers to be 365·24218 mean solar days, or 365 days 5 hours 48 m. 47½ sec. Therefore the year, which contains 365 days, is about ¼th of a day less than the solar year. This difference would in the course of time be very considerable and cause great confusion, especially in agricultural matters, where it is necessary that the course of the seasons, which depend upon the situation of the sun, shall be correctly indicated. In order to adjust this difference, Julius Cæsar, guided by his astronomer Sosigenes, enacted that every fourth year should consist of 366 days, called Leap or Bissextile year, the additional day being intercalated in the month of February, which is then made 29 days. The intercalary day is placed in the ecclesiastical calendar between the 24th and 25th; in the civil calendar between the 28th and March 1st. This correction is called the *Julian correction*. The length of a mean Julian year is therefore 365¼ or 365·25 days, which was afterwards found to be in excess of the actual mean solar year before mentioned, containing only 365·24218 days, and it was ascertained that the vernal equinox, which at the Council of Nice in the year 325 was supposed to correspond to the 21st of March, had in 1582 retrograded to the 11th. To get rid of this accumulated error, and so restore the equinox to its supposed former place, Pope Gregory caused ten days to be omitted in the year 1582, by calling the 5th of October the 15th; and, as the error according to Aloysius Lilius amounted to about 3 days in 400 years, it was ordered that the intercalations on centenary years should henceforth be omitted, excepting those which are multiples of 400. Consequently 1877, 1878, 1879, 1881, 1882, 1883, 1800, 1900; 2100, 2200 are ordinary years, and 1880, 1884, 1888, 1600, 2000, 2400, 2800 are bissextile years.

Every period of 400 years now contains 97 bissextile years and 303 ordinary years, together 146,097 days, and therefore, taking the 400th part of this number, an average or mean Gregorian year is 365·24250 days, which is only ·00032 day in excess, and amounts to a day in 3125 years. The Julian error, ·00778 day in excess, amounts to a day in 129 years.

This important adjustment, usually called the Gregorian Calendar, or New Style, has since been adopted in almost all Christian countries. It came into force in England on the 2nd of September 1752, when the error amounted to eleven days. The Julian Calendar formerly in use is called the Old Style, and is still adhered to in Russia.

The year is divided into 12 calendar months, whose lengths may easily be remembered by the following lines:—

Thirty days hath September, (30)
April, June, and November; (30 × 3 = 90)
February hath twenty-eight alone, (28)
And all the rest have thirty-one, (31 × 7 = 217)
Except in leap year, at which time 365 (an ordinary year).
February's days are twenty-nine. 1
366 (a leap year).

20. ANGULAR MEASURES.

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle.
4 right angles, or 360° = 1 circle.

21. COMMERCIAL NUMBERS.

12 articles = 1 dozen.
13 ditto = 1 long dozen.
12 dozen = 1 gross.
20 articles = 1 score.
5 score = 1 common hundred.
6 score = 1 great hundred.
80 deals = 1 quarter.
4 quarters = 1 hundred.
24 sheets = 1 quire of paper.
24 sheets = 1 do. outsides.
25 ditto = 1 printer's do.
20 quires = 1 ream.
21½ ditto = 1 printer's do.
2 reams = 1 bundle.
10 ditto = 1 bale.
5 doz. skins of parchment = 1 roll.

90 words in Chancery, 80 ditto in Exchequer, or 72 in common law = 1 folio.

22. SIZES OF BOOKS.

Folio books are the largest; of which	4	pages or	2	leaves	=	1	sheet.
Quarto, or 4to.	8	,, or	4	,,	=	1	,,
Octavo, or 8vo.	16	,, or	8	,,	=	1	,,
Duodecimo, or 12mo.	24	,, or	12	,,	=	1	,,
Sedecimo, or 16mo.	32	,, or	16	,,	=	1	
Octodecimo, or 18mo.	36	,, or	18	,,	=	1	,,
Quatuorvicesimo, or 24mo.	48	,, or	24	,,	=	1	,,
Duotrigesimo, or 32mo.	64	,, or	32	,,	=	1	,,

23. PRACTICE TABLES.

The rule of Practice is so called from the great facility it affords in calculation, and from its general use among merchants and tradesmen. It is a short and concise method of computing the value of commodities by taking aliquot parts. An aliquot part of a number is such a part as, being taken a certain number of times, will produce the number exactly; thus, 6*s.* 8*d.* is the third, or aliquot part of a pound; 3*d.* is the fourth, or aliquot part of a shilling; and so on, as demonstrated in the annexed tables.

ALIQUOT PARTS.

OF A POUND.

s. *d.*
10 0 = one-half.
6 8 = one-third.
5 0 = one-fourth.
4 0 = one-fifth.
3 4 = one-sixth.
2 6 = one-eighth.

s. *d.*
2 0 = one-tenth.
1 8 = one-twelfth.
1 4 = one-fifteenth.
1 3 = one-sixteenth.
1 0 = one-twentieth.
0 8 = one-thirtieth.

s. *d.*
0 6 = one-fortieth.
0 5 = one-forty-eighth.
0 4 = one-sixtieth.
0 3 = one-eightieth.
0 2 = 1-120th.
0 1 = 1-240th.

OF A SHILLING.

d.
9 = three-fourths.
8 = two-thirds.
6 = one-half.
4 = one-third.

d.
3 = one-fourth.
2 = one-sixth.
1½ = one-eighth.
1 = one-twelfth.

d.
¾ = one-sixteenth.
½ = one-twenty-fourth.
¼ = one-forty-eighth.

OF A PENNY.

¾*d.* = three-fourths.
½*d.* = one-half.
¼*d.* = one-fourth.

OF A TON.

cwts. qrs.
10 0 = one-half.
5 0 = one-fourth.

cwts. qrs.
4 0 = one-fifth.
2 2 = one-eighth.

cwt. qrs.
2 0 = one-tenth.
1 0 = one-twentieth.

OF A CWT.

qrs. lbs.
2 0 = one-half.
1 0 = one-fourth.
0 16 = one-seventh.

lbs.
14 = one-eighth.
8 = one-fourteenth.

lbs.
7 = one-sixteenth.
4 = one-twenty-eighth.

OF A QUARTER.

14 lbs. = one-half.
7 ,, = one-fourth.

4 lbs. = one-seventh.
3½ ,, = one-eighth.

2 lbs. = one-fourteenth.
1¾ ,, = one-sixteenth.

OF A POUND WEIGHT.

8 oz. = one-half.
4 ,, = one-quarter.

2 oz. = one-eighth.
1 ,, = one-sixteenth.

OF A POUND, TROY WEIGHT.

6 oz. = one half-pound.
4, &c., as in the parts of a shilling.

FOR ALIQUOT PARTS OF

Long Measure	......see page 186	Troy Weight	.........see page 189
Cloth ,,	...... ,, 187	Apothecaries' Weight	,, 190
Square ,,	...... ,, 188	Wine & Spirit Measure	,, 191
Cubic ,,	...... ,, 188		

24. TABLE OF £1 AT COMPOUND INTEREST.

Years	2 per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·020000	1·030000	1·040000	1·050000	1·060000
2	1·040400	1·060900	1·081600	1·102500	1·123600
3	1·061208	1·092727	1·124864	1·157625	1·191016
4	1·082432	1·125509	1·169859	1·215506	1·262477
5	1·104081	1·159274	1·216653	1·276282	1·338226
6	1·126162	1·194052	1·265319	1·340096	1·418519
7	1·148686	1·229874	1·315932	1·407100	1·503630
8	1·171659	1·266770	1·368569	1·477455	1·593848
9	1·195093	1·304773	1·423312	1·551328	1·689479
10	1·218994	1·343916	1·480244	1·628895	1·790848
11	1·243374	1·384234	1·539454	1·710339	1·898299
12	1·268242	1·425761	1·601032	1·795856	2·012196
13	1·293607	1·468534	1·665074	1·885649	2·123928
14	1·319479	1·512590	1·731676	1·979932	2·260904
15	1·345868	1·557967	1·800944	2·078928	2·396558
20	1·485947	1·806111	2·191123	2·653298	3·207135
25	1·640606	2·093778	2·665836	3·386355	4·291871
30	1·811362	2·427262	3·243398	4·321942	5·743491
35	1·999890	2·813862	3·946089	5·516015	7·686087
40	2·208040	3·262038	4·801021	7·039989	10·285718

25. TABLE OF PRESENT VALUE OF £1.

Years	2 per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	·980392	·970874	·961538	·952381	·943396
2	·961169	·942596	·924556	·907029	·889996
3	·942322	·915142	·888996	·863838	·839619
4	·923845	·888487	·854804	·822702	·792094
5	·905731	·862609	·821927	·783526	·747258
6	·887971	·837484	·790315	·746215	·704961
7	·870560	·813092	·759918	·710681	·665057
8	·853490	·789409	·730690	·676839	·627412
9	·836755	·766417	·702587	·644609	·591898
10	·820348	·744094	·675564	·613913	·558395
11	·804263	·722421	·649581	·584679	·526788
12	·788493	·701380	·624597	·556837	·496969
13	·773033	·680951	·600574	·530321	·468839
14	·757875	·661118	·577475	·505068	·442301
15	·743015	·641862	·555265	·481017	·417265
20	·672971	·553676	·456387	·376889	·311805
25	·609531	·477606	·375117	·295303	·232999
30	·552071	·411987	·308319	·231377	·174110
35	·500028	·355383	·253415	·181290	·130105
40	·452890	·306557	·208289	·142046	·097222

DEFINITIONS, RULES, AND FORMULÆ.

To be Committed to Memory.

⁂ *The numbers in brackets at the end of the rules refer to the paragraphs in the text.*

(1) If the name is given with the number, the number is said to be *concrete*; if the number alone is given, it is said to be *abstract*.

(2) This sign (+) is called *Plus*, and means that the quantities between which it stands are to be added together. These quantities are called the *Addenda*, and the answer is called the *Sum*.

(3) This sign (−) is called *Minus*, and means that the quantity following it is to be subtracted or taken away from the quantity before it. The quantity from which we subtract is called the *Minuend*, the quantity to be subtracted is called the *Subtrahend*, and the remainder is called the *Difference*.

(4) This sign (×) is called *Multiplied by*, and means that the number of things before it is to be *repeated as many times* as there are units in the number following it. The number of things to be repeated is called the *Multiplicand*, the number of units is called the *Multiplier*, and the result is called the *Product*. The Multiplier and Multiplicand are also called *Factors* of the Product.

(5) This sign (÷) is called *Divided by*, and means, 1st. How many times is the quantity following the sign contained in the quantity preceding the sign; 2nd. Distribute the quantity before the sign into as many equal parts as are indicated by the *Number* after the sign. The quantity before the sign is the *Dividend*, the quantity after the sign is the *Divisor*, and the result is the *Quotient*.

(6) If one number can be divided by another number *without remainder*, the divisor is called a *Measure* of the dividend, and the dividend a *Multiple* of the divisor.

(7) A number which has no measures but itself and unity is called a *Prime Number*; other numbers are called *Composite Numbers*.

(8) Divisibility of numbers :—

(1) A number is a multiple of 2, if its last digit is even.
(2) ,, ,, ,, 3, if the sum of its digits is a multiple of 3.
(3) A number is a multiple of 4, if the 2 last digits is a multiple of 4.

(4) A number is a multiple of 5, if the last digit is 5 or 0.
(5) ,, ,, ,, 6, if it is a multiple of both 2 and 3.
(6) A number is a multiple of 8, if its last 3 digits is a multiple of 8.
(7) A number is a multiple of 9, if the sum of its digits is a multiple of 9.
(8) A number is a multiple of 10, if its last digit is 0.
(9) ,, ,, ,, 11 if the sum of its digits added up alternately, beginning at the units place, is a multiple of 11.

(9) To find the factors of a composite number, or to resolve a number into its prime factors, apply the above tests, and divide the number by that factor as many times as possible, until the final quotient is a prime number: the prime factors required will be these several factors and the last quotient. (27)

(10) A number which is a measure of each of two or more numbers is called their *common measure*, and the greatest of these is called their *greatest common measure*, or *highest common factor*, which are indicated by the letters G.C.M. or H.C.F.

(11) To find the H.C.F. of 2 numbers:—(i.) Resolve both numbers into their prime factors, and the product of all the prime factors which they have in common is the H.C.F. required; or (ii.) Divide the greater of the 2 numbers by the less, the divisor by the remainder, and repeat the process until the remainder is zero: the last divisor is the H.C.F. (31)

(12) To find the H.C.F. of more than 2 numbers: Subtract the 2 largest numbers, the 2 smallest numbers, and find the H.C.F. of the 2 remainders. If one of the numbers be odd, and their H.C.F. even, divide it by 2, and the quotient will give the answer. (32)

(13) A *common multiple* of 2 or more numbers is a number which contains each of them exactly; or a number of which each of them is a factor. The *least common multiple*, written L.C.M., of 2 or more numbers is the least number which contains each of them as factors.

(14) To find the L.C.M. of several numbers: set down the different numbers in order, omitting any that are factors of others; divide by the lowest factor common to 2 or more of them, placing the quotient or the number not divisible by the factor immediately underneath the original number; repeat the process with the numbers thus obtained until a line of numbers prime to one another is arrived at; then the continued product of the divisors and the numbers in this line will be the L.C.M. required. (35)

(15) To find the L.C.M. of two large numbers: find their H.C.F., divide one of the numbers by it, and multiply this quotient by the other.

(16) A *fraction* is a portion of a whole or unit, and consists of two parts called *terms*; the *denominator* showing into how many portions the unit is divided, the *numerator* showing how many of these portions are to be taken.

(17) If the numerator and denominator of a fraction be multiplied or divided by the same number, the value of the fraction is not altered. (37)

(18) A fraction is in its lowest terms when the numerator and denominator are prime to each other.

(19) To reduce a fraction to its lowest terms divide its terms by their H.C.F.

(20) To compare fractions, reduce them to a common numerator or denominator.

(21) Fractions less than unity are called *simple* or *proper* fractions; fractions greater than unity are called *improper* fractions; quantities consisting of integers and fractions are called *mixed numbers*. (43) Fractions connected by the sign 'of' are called *compound fractions*. (47)

(22) To reduce a mixed number to an improper fraction: multiply the integer or whole number by the denominator, and add in the numerator for the numerator of the fraction, retaining for the denominator the original denominator.

(23) To reduce an improper fraction to a mixed number: divide the numerator by the denominator, setting down the quotient as the whole number, the remainder as the numerator of the fractional part of the mixed number, the denominator being the denominator of the original fraction. (44)

(24) To add fractions: express each fraction as equivalent fractions, with their lowest common denominator; add the new numerators together, place their sum over the common denominator, and reduce if necessary to a mixed number. (45)

(25) To subtract a fraction from a greater fraction: express each fraction as equivalent fractions with their lowest common denominator, subtract the new numerators, and place the difference over the common denominator. (46)

(26) To multiply fractions: arrange them with the sign (×) between them; reduce mixed numbers, if any, to improper fractions; remove factors common to numerator and denominator; the numerator and denominator of the required fraction is the product of the several numerators and of the several denominators respectively. (47)

(27) To divide fractions: invert the divisor and proceed as in multiplication.

(28) To simplify a complex fractional expression: write down, one under the other in separate lines, the different fractions; reduce each to its simplest form; then write thier results as they occur in the expression, separated by the necessary signs.

(29) A fraction of which the denominator is some power of 10 is called a *decimal* fraction, or simply a *decimal.*

(30) The *order* of a decimal is determined by the number of digits after the point.

(31) To add decimals together: arrange them under one another so that the 'points' form a vertical line, and then proceed as in ordinary addition.

(32) To subtract one decimal from another: arrange them so that the points are under one another, and then proceed as in ordinary subtraction.

(33) To multiply decimals together: multiply the factors together, disregarding the point, and make the number of decimal places in the product equivalent to the sum of those in the factors.

(34) To divide one decimal by another: make the divisor a whole number, then the first figure of the quotient will be of the same *order* as is the first figure of the first remainder. (68)

(35) To reduce a vulgar fraction to a decimal: divide the numerator by the denominator, the number of decimal places in the result being the same as that of cyphers employed to complete the operation.

(36) *Recurring*, *circulating*, or *repeating* decimals are decimals in which the digits are repeated. A *pure* circulator is one in which there are no figures except the recurring period. A *mixed* circulator is one in which there are other figures besides the recurring period. (73)

(37) A vulgar fraction reduced to its lowest terms will produce a circulating decimal if its denominator contains any prime factor other than 2 or 5; and the number of circulating figures must be less than the number expressed by the denominator.

(38) To reduce a *pure* circulator to a vulgar fraction: place all the digits of the period in the numerator, and as many nines as there are recurring figures in the denominator, and reduce to lowest terms. (76)

(39) To reduce a *mixed* circulator to a vulgar fraction: place in the numerator the difference between the number formed by the figures up to the end of the first period and the number formed by those that precede the first period; and in the denominator place 9 as many times as there are figures in the period, affixing as many cyphers as there are figures between the decimal point and the first period. (77)

(40) To add or subtract circulating decimals: find the L.C.M. of the numbers of recurring figures in each of the different decimals; then, after making the decimals of the same order, carry on all the decimals farther as many places as the number represented by the L.C.M., and proceed as usual. (79)

(41) In multiplication and division of recurring decimals, reduce the decimals to vulgar fractions, and express the product or quotient as a decimal. (81)

(42) To find the G.C.M. of vulgar fractions: reduce them to a common denominator; then the G.C.M. required will be the fraction, with the G.C.M. of the new numerators as its numerator, and its denominator the L.C.M. of the denominators. (87)

(43) To find the L.C.M. of vulgar fractions, reduce them to a common denominator; then the L.C.M. required will be the fraction, with the L.C.M. of the numerators as its numerator, and the common denominator as its denominator. (87)

(44) To find the G.C.M. or L.C.M. of decimals: make the order of the decimals the same; find the G.C.M. or L.C.M. of these numbers, disregarding the decimal points; and mark off from the result as many places as this order consists of. (88)

(45) When a number is multiplied by itself the product is called the square or the second power of that number.

(46) To find the least factor which will make a number a perfect square: resolve the number into its elementary factors, and introduce factors which will ensure each elementary factor being repeated an even number of times.

(47) The square root of a number is that number which multiplied by itself produces the given number. (91)

(48) To find the square root of a number:—(i) Resolve it, when possible, into the elementary factors, and the square root required will be the continued product of these factors repeated half as many times as they are in the square number.

(ii) Mark off the number into periods by putting a dot on every alternate digit, beginning at the units place. Find the highest square number in the first period on the left; the corresponding root will be the first figure of the answer. Subtract the square number from the first period, and annex to the remainder the two figures of the next period. Double the root already found, and prefix it as a divisor to the number last mentioned. Place the quotient both as the second figure of the root and also as an additional figure in the divisor. Multiply the divisor so completed by this last figure, subtract, and to the remainder annex the two figures of the next period; and continue the process until all the periods in the dividend have been brought down. The quotient will be the square root required. (94)

(49) To find the square root of a terminating decimal: proceed as above, and mark off in the root half the number of decimal places as in the square.

(50) To find the square root of a non-terminating decimal: unless otherwise directed, make the order of the decimal 10; proceed as above; then the square root will have 5 places of decimals. 5 more places may be obtained by cutting off, one by one, the figures in the divisor and proceeding by division. (97)

(51) To find the square root of a fraction:—(i) Find the square root of the numerator and of the denominator; (ii) Express the fraction as a decimal, and use the last two rules; or, (iii) Multiply the numerator and denominator by some factor which will make the denominator a square number, and then apply (i). (100)

(52) To find the square root of a circulating decimal: use rule 50; or reduce to a vulgar fraction, and use rule 51.

(53) When a number is multiplied by itself twice, the continued product is called the cube of the number.

(54) To find by what factor a number must be multiplied to become a perfect cube: resolve it into its elementary factors, and introduce factors which will ensure each elementary factor occurring 3, 6, or 9 times, &c.

(55) The *cube root* of a number is that number which multiplied twice by itself realises the given number.

(56) To find the cube root of a number: resolve it, when possible, into its elementary factors, and the cube root required will be the continued product of these factors repeated one-third as many times as they are in the cube number. (105)

(57) Other rules for extracting the cube root may be found in §§ 109, 111, 112.

(58) If we take one quantity, say one pound, as our unit, its sub-multiples, shillings, pence, and farthings, are called its *denominations*.

(59) If a quantity is expressed in one denomination it is called a *simple* quantity.

(60) If a quantity is expressed in several denominations it is called a *compound* quantity.

(61) Reduction is the process by which we express (i) a simple or a compound quantity in terms of its lower denomination, or (ii) a simple quantity in terms of its higher denomination: (i) is called *descending*, and (ii) *ascending* reduction.

(62) To express a number of a higher denomination in units of a lower denomination, multiply the number of the highest denomination in the proposed quantity by the number of units of the next lower denomination contained in one unit of the highest, and to the product add the number of that lower denomination, if there be any in the proposed quantity; repeat this process for each succeeding denomination till the required one is arrived at.

(63) To express a number of inferior denomination in units of a higher denomination:—Divide the given number by the number of units which connect that denomination with the next higher, and the remainder, if any, will be the number of surplus units of the lower denomination. Carry on this process till the denomination required is arrived at.

(64) Each of the above two rules is the converse of the other; the accuracy of any result obtained by either of them may be tested by working the result back again by the other rule.

(65) To reduce a fraction of one denomination to a lower denomination, and conversely :—Multiply or divide the numerical value of the fraction by the number of times the larger unit is contained in the smaller. (148)

(66) To reduce a fraction of one denomination to a compound quantity of lower denominations, and its converse :—Proceed by compound division ; or reduce both quantities to the lowest denomination, and place one over the other.

(67) To multiply a compound quantity by a proper fraction :—Multiply it by the numerator of the fraction, and then divide the product by the denominator.

(68) To multiply a quantity by a mixed number :—Multiply separately by the integer and by the fraction, and add the results together.

(69) To divide a quantity by a fraction :—Multiply by the denominator, and divide the product by the numerator.

(70) To divide a quantity by a mixed number :—Reduce the mixed number to a fraction, multiply by the denominator, and divide the product by the numerator.

(71) The question 'Reduce 13*s.* 4*d.* to the fraction of £1.,' may also appear in the form :—

(i.) If 13*s.* 4*d.* is the dividend, £1. the divisor, what is the quotient?
(ii) If £1. is the unit ; what is the measure of 13*s.* 4*d.* ?
(iii) What fraction of £1. is 13*s.* 4*d.* ?
(iv) Express 13*s.* 4*d.* as the fraction of £1.
(v) What part of £1. is 13*s.* 4*d.* ?
(vi) What is the ratio of 13*s.* 4*d.* to £1. ?

N.B.—(1) The unit, or the *quantity to be divided*, or that comes immediately after the words 'fraction of,' is the *denominator*, and the quantity *which you have* is the *numerator* in all these cases. (2) The concrete quantities may be reduced to their highest or lowest denominations.

(72) To express a decimal of a higher denomination in units of a lower denomination :—Multiply the decimal by the number of units of the next lower denomination contained in one unit of the highest ; similarly, reduce the decimal period of this product to the next lower denomination. Repeat the process till there is no decimal period, or the lowest denomination has been reached. (155)

(73) To reduce a compound quantity to a decimal of a higher denomination :—Take the number of units in the lowest denomination ; express this as a fraction, and then as a decimal of the next highest denomination ; to this decimal prefix the number of units of this denomination in the given quantity. Continue the process till the highest denomination required is arrived at. (156)

(74) To multiply a compound quantity by a decimal :—Reduce the compound quantity to its highest or lowest denominations, then multiply and, if necessary, express the product as a compound quantity. (156)

(75) To reduce one concrete quantity to the decimal of another concrete quantity :—Express both quantities in terms of the highest or lowest denomination mentioned, and proceed as in division of decimals, making the quantity that comes immediately after the words 'decimal of' the divisor. (157)

(76) To decimalise money :—The figure in the first decimal place will indicate the number of florins ; for an odd shilling, add 5 in the second place ; for 6*d.* over, add 25 in the second and third places ; and for 1*s.* 6*d.* over, 75 in the second and third places. Any number of farthings is the same number of thousandths of £1. + $\frac{1}{12}$ of $\frac{1}{2}$ of that number of thousandths.

(77) Reconversion into money :—(i) The first decimal place gives florins. (ii) A 5 (if any) in the second place gives 1*s.* (iii) The remaining figures in the second and third places give each $\frac{24}{25}$ of a farthing ; count them as farthings, rejecting 1 if they exceed 24. The remaining figures of the decimal yield less than a farthing, and may be disregarded.

(78) To find the cost of tons cwts. qrs. lbs. at per ton :—Call the cwts. shillings, every quarter 3*d.*, every 7 lbs. $\frac{3}{4}$*d.* ; decimalise and multiply. (163)

(79) To find the cost of cwts. qrs. lbs. at per cwt :—Call 1 cwt. £1., 1 qr. 5*s.*, 7 lbs. 1*s.* 3*d.* ; decimalise and multiply.

(80) To find the cost of lbs. oz. dwts. at per lb. :—Call 1 lb. £1., 1 oz. 1*s.* 8*d.*, 1 dwt. 1*d.* ; decimalise and multiply.

(81) To find the cost of ozs. dwts. grs. at per oz. :—Call 1 oz. £1., 1 dwt. 1*s.*, 1 qr. $\frac{1}{2}$*d.* ; decimalise and multiply.

(82) To find the cost of gals. qts. pts. and gills at per gallon :—Call each qt. 5*s.*, each pt. 2*s.* 6*d.*, and each gill 7$\frac{1}{2}$*d.*

(83) To find the cost of qrs. bus. pks. at per quarter :—Call each bus. 2*s.* 6*d.*, and each pk. 7$\frac{1}{2}$*d.*

(84) Practice is another way of multiplication, and is Simple or Compound, according as the quantity whose value is required is simple or compound.

(85) An *aliquot part* of any quantity is any measure of that quantity.

(86) *Ratio* is the relation which two quantities of the *same kind* bear to one another with respect to magnitude : of these two quantities (or terms) the first is called the *antecedent* and the second is called the *consequent.*

(87) We have the following sets of interchangeable terms :—

Numerator	Denominator	Fraction
Dividend	Divisor	Quotient
Antecedent	Consequent	Ratio

(88) A ratio of *greater inequality* is that in which the antecedent is greater than the consequent, and is therefore an improper fraction.

A ratio of *less inequality* is that in which the antecedent is less than the consequent, and is a proper fraction.

A ratio of *equality* is that in which the antecedent equals the consequent.

(89) Ratios are *compounded* by multiplying together the fractions by which they are represented.

(90) When two ratios are equal, the four terms are said to be in *Proportion*. The first and fourth are called *extremes*; the second and third are called the *means*.

(91) Three numbers are in *Continued Proportion* when the ratio of the first to the second equals the ratio of the second to the third.

The second number is called a *Mean Proportional* between the first and the third.

(92) All questions usually given under the head of Rule of Three or Double Rule of Three can be solved by the Rule of Cause and Effect, which is this :—first effect : first cause :: second effect : second cause.

The word 'effect' usually meaning work done, food or something consumed, money spent. (186)

(93) Care must be taken before the proportion is stated that all corresponding expressions are reduced to the same denomination.

(94) When money is lent in business, the borrower is charged a certain sum for the use of it; this is called *Interest*. The sum lent is called the *Principal*, and the sum returnable is called the *Amount*, which equals the Principal + the Interest.

(95) If P = principal, n = time, r = rate per cent., I = interest, and A = amount: (i) $P \times n \times r = 100 \times I$; (ii) $P(100 + nr) = 100 \times A$.

Three of these quantities are always given to find the fourth.

(96) The *Present Worth* of a sum of money due some time hence, is that Principal which, placed out at Interest for the given time at the given rate per cent., would amount to the given sum.

(97) The *Discount* on a sum of money is the difference between the sum and its Present Worth.

(98) If P = the sum of money, I = the interest on it, D = the discount on it,

$$\frac{D}{I} = \frac{100}{100+nr} : D = \frac{Pnr}{100+nr} : P(I-D) = I \cdot D : I - D = \frac{Pn^2r^2}{100(100+nr)}.$$

(99) I is called in business transactions 'Banker's discount,' and is always greater than D, the 'true discount;' hence the banker has an advantage: the distinction between them is, that the *Interest* is calculated on the Amount, while the *Discount* is calculated on the Present Worth of the Amount.

(100) A *bill of exchange* is a written instrument in which one person orders another to pay him a sum of money on a certain date. (202)

(101) A *promissory note* is a written instrument in which a person promises to pay another a sum of money at a specified time.

(102) A bill of exchange or promissory note always runs 3 days beyond the time specified, called *days of grace.*

(103) In *Compound Interest* the interest is not drawn, but is added from time to time, as it becomes due, to the principal, and the amount forms a new principal for the next period; and the difference between the final amount and the original principal is called the *Compound Interest.*

(104) If $P =$ the principal, $r =$ rate per cent., $n =$ time, $A =$ amount, then $P(1{\cdot}0r)^n = A$: any three of these being given the fourth can be obtained.

(105) To find the Compound Interest: find the amount of £1. for one year at the given rate per cent.; multiply this by itself as many times as there are years; then the product of the decimal part of this result and the principal will be the Interest required.

(106) To find the Compound Interest when the time is a mixed number of years: find the interest for the integral number of years; then for one year more, find the fractional value of the difference between the two results, and add it to the former one.

(107) Given the rate per cent. of a given quantity, to find the value of it: multiply the quantity by the rate per cent., and divide by 100. (211)

(108) To find how much per cent. one quantity is of another: multiply the part by 100, and divide by the whole quantity.

(109) Given the percentage on a quantity, to find the rate per cent.: multiply the percentage by 100, and divide by the quantity.

(110) The percentage and rate being given, to find the quantity: multiply the percentage by 100, and divide by the rate per cent.

(111) In profit and loss, the 1st buying price : 1st selling price :: 2nd buying price : 2nd selling price.

(112) The *Stocks* or Public Funds are the State Debts, arising from money borrowed from individuals, on the guarantee of a certain fixed rate of interest for every £100. This interest is called a *dividend.*

(113) The price of £100. stock varies from time to time; when it is exactly £100. it is said to be at *par*; if below £100., at a *discount*; and if above £100., at a *premium.*

(114) *Stock* money is the amount on which the interest is paid; *sterling* value is the market value per cent. If the principal is *in* the funds it is *stock* money; if it is *not in* the funds it is *sterling* money.

(115) The various kinds of questions that arise in stocks are :—

(i) Find the value of £P in the r per cents. at p.

$$Ans. = \frac{P \times p}{100}.$$

(ii) How much stock can be purchased by investing £P in the r per cent. at p?

$$Ans. = \frac{P \times 100}{p}.$$

(iii) How much is lost by buying £P stock at p_1 and selling at p_2?

$$Ans. = \frac{P(p_1 - p_2)}{100}.$$

(iv) Find the income obtained from investing £P in the r per cents. at p?

$$Ans. = \frac{P \times r}{p}.$$

(v) Find the income obtained from £P in the r per cents. at p

$$Ans. = \frac{P \times r}{100}.$$

(vi) Find the difference in income arising from the investment of £P in the r_1 per cents. at p_1, and the r_2 per cents. at p_2.

$$Ans. = \frac{P(r_1 p_2 \sim r_2 p_1)}{p_1 p_2}.$$

(vii) What rate of interest is obtained on money invested in the r per cents at p?

$$Ans. = \frac{100 \times r}{p}.$$

(viii) To find which is the better investment—in the r_1 per cents. at p_1, or in the r_2 per cents. p_2: compare the two fractions $\frac{r_1}{p_1}$, $\frac{r_2}{p_2}$.

(ix) I invest £P in the r_1 per cents. at p_1, and when they are at p_2 I sell out and invest the money in the r_2 per cents. at p_3; find the alteration in my income.

$$Ans. = \frac{P}{p_1 p_3}(r_1 p_3 \sim r_2 p_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad (219)$$

(116) Calculations on stock are applicable to *shares* by multiplying the price of the shares by some factor which will make one share equivalent to the £100. stock. (220)

(117) If a room is h feet high, l feet long, b feet broad, and w the width of the carpet in feet, and p^1 the price per yard, and P the whole cost in pounds,

(i) Area of floor $= l \times b$.

(ii) Quantity of carpet required for the floor $= \frac{b \times l}{w}$.

(iii) $P = \frac{b \times l \times p}{w \times 3 \times 12 \times 20}$.

(118) If w be the width of the paper required for the walls, p the price in pence per yard, and P the whole cost of paper in pounds,

(i) Area of the walls $= 2h(l+b)$.

(ii) Length of paper required $= \frac{2h(l+b)}{w}$.

(iii) $P = \frac{2h(l+b)p}{12 \times 20 \times 3w}$.

N.B.—Before applying these formulæ be sure that all the dimensions are expressed in feet.

(119) The volume of the above room is $V = h \times l \times b$.

(120) To find when the faster of two moving points will overtake the slower one: divide the distance they are apart by the difference of their velocities.

(121) To find when two points moving in opposite directions will meet: divide the distance between them by the sum of their velocities.

(122) Applying these remarks to the Clock Problem:

(i) Find the time between 3 and 4 when the hands of a watch are together.

The start is 15 minutes, the difference of their velocities is 11, $\therefore \frac{15}{11}$ or $1\frac{4}{11}$ is the distance past 3 the hour-hand will move. $\therefore$ the time is $16\frac{4}{11}$ minutes past 3. *Generally* to find the number of minutes past the hour when the hands are together, add the start to the quotient of the start by 11.

(ii) To find the number of minutes past the hour when the hands are at right angles, let N be the result (i) of adding to, (ii) of subtracting from, the start 15 minutes, then the time will be $N + \frac{N}{11}$ minutes past the hour. (Two answers.)

(iii) To find the number of minutes past the hour when the hands are in a straight line, let M be the result (i) of adding to, (ii) of subtracting from, the start 30 minutes; then the time will be $M + \frac{M}{11}$ minutes past the hour.

Or if x be the hour, the

$$\text{1st answer will be } x^\circ + \frac{60x'}{11}.$$

$$\text{2nd} \quad ,, \quad ,, \quad x^\circ + \frac{60(x \pm 3)'}{11}.$$

$$\text{3rd} \quad ,, \quad ,, \quad x^\circ + \frac{60(x \pm 6)'}{11}.$$

(123) *Exchange* is the reduction of a sum of money in the coinage of one country into its equivalent value in the coinage of another country at a given rate.

The standard rate of exchange, depending only on the intrinsic value of the coinage of two countries, is called the *par* of exchange.

The actual rate of exchange at any time, depending both on the intrinsic value of the coinage and on the state of trade between the two countries at that time, is called the *course* of exchange.

Arbitration of exchange is the determination of the rate of exchange between places by knowing their rates with intermediate countries. (221)

(124) The *Metric System* of weights and measures is now in use in many countries in Europe. The basis of all measurement is the *metre*, the ten-millionth part of the distance from the North Pole to the Equator, or equivalent to about 39·37 inches in English measure. In the formation of the multiples and submultiples the decimal system is followed; the Greek prefixes to any unit denoting the multiples, and the Latin prefixes the submultiples. Then the metre being the unit of length—

Its multiples are—

deca-metre,	denoting	10	metres.
hecto-metre	,,	100	,,
kilo-metre	,,	1,000	,,
myria-metre	,,	10,000	,,

Its submultiples are—

deci-metre,	denoting	$\frac{1}{10}$	metre.
centi-metre	,,	$\frac{1}{100}$	,,
milli-metre	,,	$\frac{1}{1000}$	,,

(125) The units of Metric Measure are—

(1) *Length* — the metre.
(2) *Surface* — the are = 100 square metres.

(3) *Solidity* — the stère = 1 cubic metre.
(4) *Capacity*— the litre = the cube of the tenth part of a metre.
(5) *Weight* — the gramme = the weight of distilled water which fills the cube of the hundredth part of a metre.
(6) *Money* — the franc—a coin weighing 5 grammes, 9 parts silver, 1 part copper.

(126) The following are rough approximates in English measures :—

The kilogramme is about 2¼ lb. avoirdupois.
The litre ,, 1¾ pints.
The kilometre ,, 5 furlongs.
The hectare ,, 2½ acres.
25 francs ,, 1*l.*

(127) The following table contains the weights and measures chiefly used, with their English equivalents :—

1 yard . . = ·91438347 metre.
1 foot . . = ·30479449 metre.
1 inch . . . = ·02539954 metre.
1 mile . . = 1·6093149072 kilometre.
1 grain troy . = ·06479895 gramme.
1 pound avoird. = ·4535926525 kilogramme.
1 ton . . . = 1·01604754l6 tonne.
1 acre . . = ·4046711 hectare.
1 gallon . . = 4·54102 litres.
1 metre . . = 1·09363307 yard.
1 metre . . = 3·2808992 feet.
1 metre . . = 39·3707904 inches.
1 kilometre . . = ·621383 mile.
1 gramme . = 15·43234874 grains.
1 kilogramme . = 2·20462125 lbs. avoird.
1 hectare . = 2·47114 acres.
1 litre . . . = ·220215 gallon.

				Multiply by,	or Divide by
(128) To turn	yards	into	metres .	·9143862	1·09363
,,	miles	,,	kilometres .	1·609315	·6213824
,,	sq. yds.	,,	centiares .	·83612	1·196
,,	acres	,,	hectares .	·404671	2·471143
,,	gallons	,,	litres . .	4·54345	·220097
,,	bushels	,,	hectolitres .	·363476	2·751211
,,	lbs. av.	,,	lbs. troy .	1·215277	·822857l4
,,	grains	,,	grammes .	·06479895	15·4323487
,,	lbs. av.	,,	kilogrammes	·45359265	2·2046212
,,	cwts.	,,	quintals .	·508023765	1·9684118
,,	tons	,,	tonnes .	1·01604753	·9842059

(129) For ordinary purposes francs are valued at 25 per £.

To find in one operation the price in English money :—

			Multiply by or	Divide by
Per yard,	given price per metre	in francs	·03625	27·5862
,, sq. yard	,, sq. metre	,,	·03315	30·1649
,, acre	,, hectare	,,	·016044	62·3275
,, gallon	,, litre	,,	·1765735	5·66335
,, bushel	,, hectolitre	,,	·0144113	69·3897
,, lb. avoird.	,, kilolitre	,,	·01798432	55·6038536
,, cwt.	,, quintal	,,	·020142488	49·646298
,, ton	,, tonne	,,	·040282976	24·823149

(130) The following notes on the reduction of a fraction to a decimal will be found useful.

Let $\frac{a}{b}$ be a fraction in its lowest terms, and let it be reduced to a decimal.

(1) The character of the decimal will depend solely on b.

(2) If $b = 2^n$ or 5^n or $2^n \times 5^m$, $\frac{a}{b}$ will terminate.

(3) If $b = 2^n$ or 5^n, there will be n decimal places.

(4) If $b = 2^n \times 5^m$, there will be n or m places according as $n > < m$.

(5) If b is prime to 10, $\frac{a}{b}$ will give a pure circulator with a number of figures in the period, which number is invariable, and depends solely on b.

(6) If b is a prime, this number is a measure of $b - 1$.

(7) If $b = 2^n \times c$, $5^n \times c$, or $2^n \times 5^m \times c$, c being prime to 10, the decimal will be a mixed circulator, having in the first and second cases n, and in the third n or m (whichever is greater) non-recurring figures, followed by the number of recurring figures proper to c.

(8) If b is prime to 10, and compounded of *different* primes, the number of recurring figures in the period is the L.C.M. of the number proper to these several primes.

(131) A rectangle is a four-sided figure whose opposite sides are parallel and equal, and its angles right angles.

(132) A triangle is a figure bounded by three straight lines.

(133) A trapezium is a four-sided figure whose opposite sides are neither parallel nor equal.

(134) A circle is a figure bounded by one line called the circumference, which is everywhere equally distant from a point within it called the centre.

(135) The diameter of a circle is a line passing through the centre and meeting the circumference both ways. The half of a diameter is a radius.

(136) A sector is a part of a circle bounded by an arc and two radii.

(137) A cylinder is a long circular body of uniform diameter, having equal parallel circular ends, and may be formed by the revolution of a rectangle on one of its sides which remains fixed, and is called the axis of the cylinder.

(138) A cone is a round pyramid with a circular base.

(139) A parallelopiped is a solid having six rectangular sides, every opposite pair of which are equal and parallel.

(140) A cube is a solid having six equal sides at right angles to each other.

(141) The circumference of a circle equals the diameter multiplied by 3·1416.

(142) The radius of a circle equals the circumference multiplied by ·159155.

(143) The radius of a circle equals the square root of the area multiplied by ·56419.

(144) The side of an inscribed equilateral triangle equals the diameter of the circle multiplied by ·86.

(145) The side of an inscribed square equals the diameter of the circle multiplied by ·7071.

(146) The side of an inscribed square equals the circumference of the circle multiplied by ·225.

(147) The circumference of a circle multiplied by ·282 equals one side of a square of the same area.

(148) The side of a square equals the diameter of a circle of the same area multiplied by ·8862.

(149) The solidity of a sphere equals its surface multiplied by one-sixth its diameter.

(150) The surface of a sphere equals the square of the diameter multiplied by 3·1416.

(151) The solidity of a sphere equals the cube of the diameter multiplied by ·5236.

(152) The diameter of a sphere equals the cube root of its solidity multiplied by 1·2407.

(153) The side of an inscribed cube equals the radius multiplied by 1·1547.

(154) The solidity of a cone or pyramid equals the area of its base multiplied by one-third of its altitude.

ARITHMETIC.

I. DEFINITIONS.

1. *Arithmetic* is the science of numbers.

2. *Quantity* is a term applied to anything which can be increased or diminished.

3. A *unit* is one, or an individual thing of the quantity considered: all other numbers are found from it by addition.

4. Numbers are either *abstract* or *concrete*, according as the unit involved is not or is applied to some particular object.

5. All numbers are expressed by the following ten characters, either singly or in combination, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9: read—nought, zero or cypher, one, two, three, four, five, six, seven, eight, nine. These characters are also called symbols, significant figures, digits.

6. *Notation* is the art of expressing any given number by these figures. *Numeration*, the art of reading them when so expressed.

7. The following signs are also made use of in Arithmetic:—

$+$ (plus) denotes the operation of addition.

$-$ (minus) denotes the operation of subtraction.

$\times$ (into) denotes the operation of multiplication.

$\div$ (by) shows that the number preceding it is to be divided by the one which follows it.

$=$ (equal) that the numbers between which it stands are equal to each other.

$\because$ = because. $\therefore$ = therefore.

II. NUMERATION.

8. The numbers expressed by 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, are written ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen. In 17, for instance, the 1 represents one bundle of 10 units; the 7 representing 7 units, and 17 represents the result of adding these units together, making seventeen units, and so for the rest.

Again, the number expressed by 26 represents the result of adding 2 bundles of ten units or ten, and six units or twenty-six units. In the same way any number between 9 and 100 is represented by two figures, the right-hand digit or figure in the unit's place, showing how many units there are; the figure in the second place showing how many bundles of ten units or tens are to be added on to make up the required number.

All numbers between ninety-nine and one thousand are expressed by three figures. The figure in the third place represents the number of bundles of one hundred units, or hundreds; the figure in the second place represents the number of tens; the figure in the first place the number of units, which, when added together, make up the given number. Thus the number expressed by 576 is written five hundred and seventy-six.

9. Any number expressed by more than three figures is best dealt with thus:—Draw a line separating the figures in the third and fourth places, then another separating those in the sixth and seventh places, then another between those in the ninth and tenth places, &c., thus the whole number is divided into periods, each period consisting of three figures. Over the 1st period write hundred, over the 2nd write thousand, over the 3rd write millions, over the 4th write thousand, over the 5th write billions, then proceed as in the following example:—

Write in such form the number expressed by 203056300072010.

Billions	Thousand	Millions	Thousand	Hundred.
203	056	300	072	010

The number expressed in words is,

Two hundred and three billion, fifty-six thousand three hundred million, seventy-two thousand and ten.

From what has been said it will be seen that a figure increases in value ten-fold as it moves from a lower to a higher place.

Exercise 1.

Write in words the numbers expressed by the following figures:—

(1) 6, 16, 60, 600, 606, 660, 7007, 17017, 18008, 1717.

(2) 40, 26, 34, 92, 68, 990, 704, 12000, 85040, 80078.

(3) 107, 913, 705, 6003, 4808, 2077, 4570, 840020, 461001.

(4) 1102070050, 90040, 304040004, 603050, 7003040.

(5) 8000001002, 500401, 30007040, 506000020, 4000505.

(6) 9000000600, 10020001, 400004050, 9050060004.

(7) 1659291, 2333331, 1648127, 2067687, 2230626.

(8) 2294129927, 20566726566, 173367270, 9101781675.

(9) 3417663240, 27416327796, 7313243616, 123456789.

(10) 1002001, 400004090, 31300070671, 10100100001.

III. NOTATION.

10. The method by which we represent a number, expressed in words, by figures is called Notation, and it is this :—

Arrange the divisions as before with their proper headings,

Billions	Thousand	Millions	Thousand	Hundred.

Place in each division the figures required, being careful to fill up any intervening space by zeros, thus :—

Express in figures, fifty-three billion fifty-three million fifty-three thousand and fifty-three.

Billions	Thousand	Millions	Thousand	Hundreds.
53	000	053	053	053

and the number expressed in figures is 53000053053053.

Exercise 2.

Express in figures the following numbers :—

(1) Twenty, thirty-five, forty, eighty-five, ninety-eight, sixty-three.

(2) Three hundred and twenty-seven, eight hundred and ninety-six, four hundred and thirty-five, eight hundred and twenty-seven, eight hundred and one.

(3) Three thousand three hundred and forty-three, four thousand nine hundred and eighty-one, seven thousand eight hundred and sixty, two thousand three hundred and forty, six thousand nine hundred and eighty, one thousand one hundred and eleven, one thousand three hundred and ten, two thousand one hundred and ninety-eight.

(4) Eighty thousand and eight ; seventy thousand and seventy ; fourteen thousand and fourteen ; ninety thousand and nineteen ; fifteen thousand and fifteen ; thirty thousand and three ; sixty thousand and sixty.

(5) Six hundred and seventy-eight thousand two hundred

and one; three hundred and ninety-five thousand and four; three hundred and ninety-five thousand and forty-one; two hundred thousand and one; eight hundred and seventy-six thousand five hundred and forty-two.

(6) Six hundred and three thousand and fifty; seven million three thousand and forty; eight thousand million one thousand and two; five hundred thousand four hundred and one; thirty million seven thousand and forty; five hundred and six million and twenty; four million five hundred and five; eight thousand million six hundred; ten million twenty thousand and one.

(7) Nine billion nine thousand and nine million and nine; eleven hundred and one billion, eleven hundred and one million, eleven hundred and one thousand eleven hundred and eleven; thirteen billion, thirteen hundred and thirteen million, thirteen hundred and thirteen thousand, thirteen hundred and thirteen.

For further examples, see Miscellaneous Examples A.

IV. ADDITION.

11. Addition is the collection of several numbers into one number called the *sum*: the several numbers are called the '*addenda*.'

Example.

The method to be adopted will be best explained by the following questions: find the sum of £3. 5*s*. 7*d*., £4. 0*s*. 2*d*., £5. 8*s*. 6*d*., £7. 5*s*. 4*d*., £1. 9*s*. 0*d*.; proceed thus, arrange the addenda under one another.

£	*s*.	*d*.
3	5	7
4	0	2
5	8	6
7	5	4
1	9	0

Add the pence column, the sum is 19 pence, or 1*s*. 7*d*.; write the 7 under the pence column, and carry the shilling to the shillings, which now amount to 28 shillings, or £1. 8*s*.; place the 8 shillings under the shillings column, and carry the pounds to the pounds column, which now yields £21. Therefore the sum required is £21. 8*s*. 7*d*. Special care must be taken that only quantities of like kind be added together. Applying this method to an example: Find the sum of 357, 402, 586, 754, 190. Arrange the addenda under one another in three columns, over which place U for units, T for tens, and H for hundreds, thus :—

H.	T.	U.
3	5	7
4	0	2
5	8	6
7	5	4
1	9	0
22	8	9

Add the units column, the sum is 19 units, or 1 ten and 9 units; place the 9 under the unit column and carry the one ten to the tens column, which now amounts to 28 tens, or 2 hundreds and 8 tens; place the 8 tens under the tens column, and carry the 2 hundred to the hundreds, which now amounts to 22 hundreds, or 2 thousand 2 hundred.

12. The accuracy of any addition sum can be tested by 'casting out nines':—

357	6
402	6
586	1
754	7
190	1
2289	3

Divide the sum of the digits in each of the addenda by 9, and write the remainder to the right of the vertical line as in example: add these remainders together, and the digits of the answer of the sum: divide these two results by 9, and if the remainders are different the sum is wrong.

If a class once understands how to decompose numbers, as shown above, no difficulty will be found in the hardest addition sum.

N.B.—The addition table must be learned thoroughly by heart like any other table.

Whenever practicable it will greatly benefit the class to dictate the sums, or to have the questions expressed as far as possible in words.

Exercise 3.

Find the sum of—

(1) 27186, 43512, 59138, 70269.

(2) 12758, 6254, 24382, 49603.

(3) 259132, 704271, 2138, 54142.

(4) 912843, 71286, 485012, 517400.

(5) 27561, 2487, 46935, 367, 2894.

(6) 341, 92768, 1004, 21238, 76, 7428.

(7) 739451, 43267, 849130, 693513, 79484, 107325.

(8) 64935039, 37290015, 95206813, 5984315, 19327117, 94863574.

(9) 74296583, 1027968, 70624, 4968735, 26587469, 42897.

(10) 6543, 807, 54091, 9999, 38.

(11) 9873, 361, 10458, 94, 7290.

(12) 4019, 50037, 88, 9900, 390. (13) 4907, 356, 4520, 38271.

(14) 8376214, 5976414, 635523, 46877, 39256479, 99783, 246804.

(15) 6142, 250, 2035, 367, 47498, 5809, 597, 196, 6071, 38276, 3148.

(16) 310531, 973572, 86127, 740537, 810453, 168179, 620438, 975162.

V. SUBTRACTION.

13. The process by which we take a number from a larger number is called subtraction, and will be explained best by considering the following examples :—

Case (1) From £7. 8*s*. 10*d*. take £6. 6*s*. 5*d*.

£7. 8*s*. 10*d*. is called the *minuend*; £6. 6*s*. 5*d*. the *subtrahend*, and the remainder is called the difference.

Writing them down as we did in addition,

£	*s*.	*d*.
7	8	10
6	6	5
1	2	5

First, take the 5*d*. from the 10*d*. in the pence column, and the remainder is 5*d*., which we place in the pence column ; next take the 6 shillings from the 8 shillings, and place the remainder, 2 shillings, in the shillings column ; next take the £6 from the £7, and the remaining £1 is placed in the pounds column : then the required difference is £1. 2*s*. 5*d*.

Case 2. Find the difference between £6. 8*s*. 11*d*. and £7. 6*s*. 5*d*.

Arrange as before.

£	*s*.	*d*.
7	6	5
6	8	11
	17	6

We cannot take 11*d.* from 5*d.*, therefore we must change one of the 6 shillings into 12 pence, which gives us now 17 pence; taking now the 11*d.* away we have 6*d.* left; next take the 8 shillings away from the 25 shillings, having previously changed one of the 7 pounds into shillings, and we have 17 shillings left; next take the 6 pounds from the 6 pounds, and we have nothing left: thus the required difference is 17 shillings and 6 pence.

Applying the same method to the question of finding the difference between 765 and 389, we arrange the numbers thus:

$$\begin{array}{ccc|c} \text{H.} & \text{T.} & \text{U.} & \\ 7 & 6 & 5 & 0 \\ 3 & 8 & 9 & 2 \\ \cline{1-3} 3 & 7 & 6 & 7 \end{array}$$

And say 9 from 15, 6, placing it in the units column; 8 from 15, 7, placing it in the tens column; 3 from 6, 3, place it in the hundreds column, and so on for any number of digits. The accuracy of the result is tested thus:—

The vertical column is obtained as in *addition*, but as 2 cannot be taken from 0, add 9 to 0, and the remainder is 7; if the two remainders are different the sum is wrong.

14. The various ways in which the sign − (minus) is used should be noticed; thus 9−7 may be read: (**1**) as subtract or take away 7 from 9; (**2**) by how much does 9 exceed 7; (**3**) what is the difference between 9 and 7? (**4**) what number must be added to 7 to make 9?

Let each sum in the following collection of examples be set in all these ways.

Exercise 4.

Find the value of the following:—

(**1**) 15−7. (**2**) 12−5. (**3**) 25−7. (**4**) 32−9. (**5**) 57−23. (**6**) 96−42. (**7**) 739−434. (**8**) 7372−3454. (**9**) 78069−16736. (**10**) 86004−19363. (**11**) 496736−279687. (**12**) 69086−47979. (**13**) 100100−3656. (**14**) 161−59. (**15**) 7648200069−868704368. (**16**) 6101507601061−3806790989. (**17**) 683216432−89532179. (**18**) 4723698007−494789127. (**19**) 8054010657811−76909748598. (**20**) 84331306 − 56421808. (**21**) 998765432 − 123456789. (**22**) 256714894−93553760. (**23**) 501505010678−794090589. (**24**) 97040341061−933701697.

VI. MULTIPLICATION.

15. Multiplication is a shortened form of addition, and is the process by which we obtain the sum of two or more numbers which are equal. If we multiply 4 by 3, we denote the operation by placing the sign × between the 4 and the 3, thus 4×3, which means that the 4 is to be repeated 3 times, and this yields the sum 12, $\therefore 4 \times 3 = 12$. Again, 3×4 means that 3 is to be repeated 4 times, which also yields the sum 12. ∴ we conclude that $4 \times 3 = 3 \times 4$, ∵ both of them = 12.

In the statement that $4 \times 3 = 12$, 4 is called the ***multiplicand***, 3 the ***multiplier***, and 12 the ***product***. Also 3 and 4 are each of them *factors* of 12.

16. (*a*) If we repeat 7 ten times the sum is 70, ∴ the product of 7 and 10 is found by adding zero to the 7; this is true of any number, ∴ to multiply any number by 10, add zero to the multiplicand: that is $57 \times 10 = 570$.

(*b*) If more than 2 factors are multiplied together, the result is called the continued product of those factors; thus 24 is the continued product of 2, 3, and 4, $\because 24 = 4 \times 6$ and $6 = 2 \times 3$; $\therefore 24 = 2 \times 3 \times 4$: whence also it appears that multiplication by any number is equivalent to a continued product by its factors.

(*c*) If a factor is repeated twice, three, or more times, the product obtained is called the 2nd, 3rd, 4th, &c. power of that factor; *e.g.* $\because 4 = 2 \times 2$, $8 = 2 \times 2 \times 2$, $16 = 2 \times 2 \times 2 \times 2$. 4, 8, 16 are called the 2nd, 3rd, and 4th powers of 2. Hence, since 100, 1000, 10000, &c. are the 2nd, 3rd, 4th powers of 10, to multiply any number by them add to the multiplicand as many zeros as there are powers of 10.

(*d*) Again, to multiply a number by another such as 9000 multiply first by the 9 and annex 3 zeros to the product.

(*e*) Since $12 \times 8 = 96$, and $96 = 72 + 24 = 12 \times 6 + 12 \times 2$. $\therefore 12 \times 8 = 12 \times 6 + 12 \times 2$, *i.e.* multiplying by any number is the same as multiplying by parts of that number and adding the results.

17. To multiply by any two figures, as 825×75.

75 consists of $70 + 5$, ∴ by **16** (*e*) we may multiply 1st by 70, then by 5, and add the results together. Thus:—

$$\begin{array}{r} 825 \\ \underline{5} \\ 4125 \end{array} \qquad\qquad \begin{array}{r} 825 \\ \underline{70} \\ 57750 \end{array}$$

$\therefore 4125 + 57750 = 61875$ is the product.

The usual form of the sum is—

$$\begin{array}{r@{\;}l}
825 & \\
\underline{75} & \\
4125 & = 825 \times 5 \\
\underline{57750} & = \underline{825 \times 70} \\
61875 & = 825 \times 75
\end{array}$$

18. To multiply by any number of more than two figures, *e.g.* 876 × 567.

$$\begin{array}{r} 876 \\ \underline{7} \\ 6132 \end{array} \qquad \begin{array}{r} 876 \\ \underline{60} \\ 52560 \end{array} \qquad \begin{array}{r} 876 \\ \underline{500} \\ 438000 \end{array}$$

$\therefore$ 876 × 567 = 438000 + 52560 + 6132 = 496692.

This will explain the usual form of the sum—

$$\begin{array}{r}
876 \\
\underline{567} \\
6132 \\
5256 \\
\underline{4380} \\
496692
\end{array}$$

To test the accuracy of the work—

'Cast out nines' from the multiplicand and multiplier; 'cast out nines' from the product of the remainders, then if this remainder is different from the remainder obtained by 'casting out nines' from the answer the result is wrong.

Exercise 5.

(1) Multiply 123456789 by 1, 2, 3, 4, 5, 6, 7, 8, 9.

(2) „ 13579 by 11, 13, 15, 17, 19.

(3) „ 24680 by 12, 14, 16, 18, 20.

(4) „ 125 by 105, 1500, 115, 150000.

(5) Values of 5184596 × 27, 8321074 × 87, 15901263 × 709.

(6) Product of 970052 × 40072, 9864302 × 300071, 32804070 × 409300.

(7) 469830 × 369, 3523725 × 2583, 463098 × 7380, 2778588 × 9867.

(8) Find the continued product of 15 × 16 × 17, 1100 × 101 × 1001, 2300 × 70010 × 2003.

(9) Find the squares or 2nd powers of 15, 17, 19, 114, 625, 897, 100, 543.

(10) Find the cubes or 3rd powers of 16, 18, 20, 25, 115, 907, 791, 987.

VII. DIVISION.

19. Division is the reverse of multiplication, being a shortened form of subtraction, and is the process by which we ascertain how many times one given quantity is contained in another given quantity of the same kind.

In the case $16 \div 2 = 8$, 16 is called the dividend, 2 the divisor, and 8 the quotient. The dividend = product of the divisor and quotient.

20. If we multiply 876 by 8 we saw that it might be worked thus—

$$800 \times 8 + 70 \times 8 + 6 \times 8 = 6400 + 560 + 48 = 7008.$$

∴ reversing the process $7008 \div 8$ may be written in the expanded form—

$$6400 \div 8 + 560 \div 8 + 48 \div 8 = 800 + 70 + 6 = 876.$$

or, as it is generally expressed—

$$\begin{array}{r} 8\underline{)7008} \\ 876 \end{array}$$

Again—

$$8006 \times 25 = 8000 \times 25 + 6 \times 25 = 200000 + 150 = 200150.$$

$$\therefore\ 200150 \div 25 = 200000 \div 25 + 150 \div 25 = 8000 + 6 = 8006.$$

Working this out at full length, the operation of division will be seen.

$$\begin{array}{r} 25)200150(8006 \\ \underline{200000} \\ 150 \\ 150 \end{array}$$

∴ Answer is 8006.

Also $825 \times 573 = 825 \times 500 + 825 \times 70 + 825 \times 3$
$= 412500 + 57750 + 2475 = 472725$

$\therefore\ 472725 \div 825 = 412500 \div 825 + 57750 \div 825 + 2475 \div 825$
$= 500 + 70 + 3 = 573.$

This may be exhibited thus, omitting unnecessary cyphers—

$$\begin{array}{r} 825)472725(573 \\ \underline{4125} \\ 6022 \\ \underline{5775} \\ 2475 \\ 2475 \end{array}$$

i.e. we first find how many times 825 is contained in 4727, viz. 5 which we place in the quotient, the product of 5 and 825 we subtract from 4727 and to the remainder 602 we annex the next figure of the dividend 2; then ∵ 825 is contained 7 times in 6022 we place 7 as the 2nd figure in the quotient and subtract the product of 7 and 825 from 6022, and to the remainder 247 add the next figure in the dividend 5; continue this operation till all the figures in the dividend have been brought down.

21. Since $9 \times 31 = 279$ and $9 \times 32 = 288$ all numbers between 288 and 279 if divided by 9 must leave a *remainder* which is less than the divisor; e.g. $286 \div 9 = 31$ and 7 over, or $286 = 31 \times 9 + 7$, i.e. the Dividend = Divisor × Quotient + the Remainder, or Dividend − Remainder = Divisor × Quotient This is universally true, and is a fact worthy of careful notice.

22. If we multiply 76 by 9000 and add 1678 to the product, then—

$$\begin{array}{r} 76 \\ \underline{9000} \\ 684000 \\ \underline{1678} \\ 685678 \end{array}$$

we obtain as result 685678; conversely if 685678 be divided by 9000, cut off as many of the figures at the right hand of the dividend as there are cyphers in the divisor, divide the remaining figures of the dividend by the figures preceding the final cyphers of the divisor, and for the total remainder affix to the remainder from the division the figures cut off from the dividend.

$$\begin{array}{r} 9000\overline{)685{,}678} \\ 76 - 1678 \end{array}$$

23. If we divide 192 by 3 the quotient is 64, divide this by 4 the quotient is 16, divide this by 2 the quotient is 8, which is the quotient we obtain by dividing 192 by 24 or $3 \times 4 \times 2$; hence we infer that if the divisor is composed of factors we can obtain the quotient by successive division by the factors. This method is called short division.

24. The process by which the remainder is obtained in short division by factors will be best illustrated by the following examples:—

Divide 70264 by 35.

$$\begin{array}{l} 5\underline{)70264} \\ 7\underline{)14052} \text{ and 4 units over} \\ \quad 2007 \text{ and 3 groups of fives or 15 units over.} \end{array}$$

∴ $70264 \div 35 = 2007$ and 19 units over.

Divide 5609 by 24, 24 = 2 × 3 × 4.

$$24\begin{cases} 6\begin{cases} 2\underline{)5609} \\ 3\underline{)2804} \quad \text{1 unit over} \end{cases} \\ 4\underline{)\ 934} \quad \text{and 2 groups of 2 or 4 units over} \\ \quad\ \ 233 \quad \text{and 2 groups of 6 or 12 units over} \end{cases}$$

∴ 5609 ÷ 24 = 233 and 17 units over.

Divide 7419 by 960 or by 4 × 12 × 20. If we call 7419 farthings, what precedes will be easily understood.

4)7419 farthings

12)1854 pence and 3 farthings over

20) 154 shillings and 6 pence, or 24 farthings over

7 pounds and 14 shillings, or 168 pence, or 672 farthings over.

∴ 7419 ÷ 960 = 7 and 699 over.

25. To test the accuracy of a sum in division if it is 'exact:' 'Cast out the nines' from the divisor and quotient, multiply the two remainders together, cast out the nines from the product, now cast out the nines from the dividend, and if the 2 last remainders are different, the sum is wrong. If there be a remainder, *i.e.* if the division be inexact, subtract the remainder from the dividend, and proceed as before.

Exercise 6.

(1) Divide 123456789 by 1, 2, 3, 4, 5, 6, 7, 8, 9.

(2) Divide 7654325 by 15, 36, 64, 84, 18, 45, 70, 88, 24, 60, 80, 96, by factors.

(3) Value of 3765897 ÷ 23, 34568135 ÷ 357, 76549139 ÷ 543.

(4) Value of 7563625 ÷ 5, by 50, 500, 5000, 50000.

(5) 95708654 ÷ 298 : 5946784379 ÷ 492 : 327840937575 ÷ 589 and by 716.

(6) 3278459367742 ÷ 833 : 294683125 ÷ 865 ; 8740434091488 ÷ 876.

(7) 6327980014 ÷ 144, 1728, 240, 480, 121, in each case by factors.

(8) Find the dividend in each of the following, given the

divisor, quotient, remainder respectively: 15, 276, 13; 16, 36724, 11; 18, 2378, 9; 21, 20174, 18.

(9) Find the divisor in each of the following, given the quotient, remainder, dividend respectively: 105531, 35, 724972: 844380, 85, 569024971; 849, 20, 2825780.

Miscellaneous Examples A on the Preceding Rules.

(1) Write down in figures six hundred million thirteen hundred and four thousand seventeen hundred and five; two million six thousand and seven.

(2) What does the figure 2 mean in the numbers, 12, 21, 2037, 2006007? What are its local and intrinsic values?

(3) Express in words and in figures how much greater the value of one 5 is than the other in the number 658457.

(4) Why is the addition of a cypher to the right of a whole number equivalent to the multiplication of that number by ten?

(5) Subtract nine hundred and thirty-one thousand two hundred and eighteen from one million three hundred and eight thousand four hundred and four.

(6) Explain the meaning of the terms *unit*, *number*, and distinguish between *abstract* and *concrete* numbers; give examples.

(7) Write down two million five hundred and two, and subtract from it nine hundred and nine thousand seven hundred and ten.

(8) Add together the following numbers: Three hundred and forty thousand and fifty; five million nine hundred and twenty-two thousand and nine; seven hundred and four thousand three hundred and four; twenty thousand and five; sixty-five thousand six hundred. Subtract from the sum three hundred and ninety-seven thousand eight hundred and ninety-seven, and write your answer in words.

(9) Add together ten thousand and ninety; one million one thousand and one; nine hundred thousand one hundred; and subtract twenty thousand two hundred and ten from their sum. Express your result in words.

(10) Subtract ten thousand and fifty from forty-one thousand and thirty-three; then to the remainder add sixteen thousand five hundred and seventy-two. Express the result in words.

(11) Add together fourteen million eight thousand and fifty-

one hundred and three thousand and nine; from the sum subtract two million twenty thousand and eighty-seven. Write the answer in words.

(12) Subtract two hundred and forty-three thousand six hundred and one from five million seven hundred thousand and fifty: multiply the result by twenty-three. Give your answer in words.

(13) Divide three million sixty two thousand and forty by one thousand and twenty, and divide the quotient by sixty-three by short divisions.

(14) Multiply seventy thousand and twenty by ten thousand and ten, and divide the product by three thousand five hundred, by short divisions.

(15) Add together sixteen million and sixty, forty million four hundred thousand and four, two hundred and nine thousand and two, and ninety-one thousand five hundred and one, and divide the result by ninety-nine.

(16) Find the difference of seventy-seven million and seven, and four hundred and ninety-nine thousand four hundred and forty, and divide the result by ninety-nine.

(17) Write down in words the difference between four hundred and thirty-five million nine thousand and sixty-six and fifty million ninety-six thousand seven hundred and eighty-four.

(18) Explain the process of 'borrowing one' in subtraction. Subtract seven million and eighty-six from seventy four million nine hundred and thirty-two.

(19) Assuming the value of the digits when they stand separately, what other assumption as to their value has to be made before all the numbers up to one hundred can be represented by them?

(20) Write down in words the difference between seven hundred and ninety-two million and eighty-six and three hundred thousand and two; and between five hundred and forty-five thousand three hundred and eighty-four, and three hundred million two hundred.

(21) Add together three million three thousand five hundred and seventy-one; thirty-nine thousand and eighteen; eight hundred thousand three hundred and ninety-one; five thousand and sixty; four hundred and ninety-five thousand and three. From the sum take away one million five hundred and eighty-five thousand four hundred and seventy; and divide the remainder by thirty-nine.

(**22**) Add together two million five hundred and seventy-one; ninety thousand and eighteen; five thousand and sixty; nine hundred and ninety-five thousand two hundred and fifty-seven; and divide the result by fifty-one.

(**23**) Define notation. Write in figures (*a*) seven hundred and fifty-six thousand and thirty; (*b*) six hundred and fifty-five thousand two hundred and twenty-six. Find the sum of these numbers, also their difference, and divide the sum by the difference.

(**24**) Define numeration. Express in figures (*a*) five hundred thousand six hundred and seventeen; (*b*) nine thousand and eight. Find the product and the difference of these numbers. Write the sum of these results in words.

(**25**) Describe the system of decimal notation. Multiply 67 by 92, and explain each step of the process.

(**26**) Divide four million seven hundred and twenty-two thousand three hundred and ninety-seven by one hundred and eight, employing the method of short division. Express the result in words, and explain the process by which the remainder is obtained.

(**27**) Find the sum of the 21 odd numbers which follow 15432.

(**28**) Divide the difference of the squares of 9409 and 10609 by the sum of the squares of 97 and 103.

(**29**) Find the sum of the 21 odd numbers which follow 14566.

(**30**) Divide the difference of the squares of 9604, and 10404, by the sum of the squares of 98 and 102.

Miscellaneous Examples B.

(**1**) What number divided by seven will give seven thousand and fifty-nine?

(**2**) In an examination there were four hundred and three candidates—two hundred and seventeen failed. How many passed?

(**3**) Of a population of six millions, two thousand and ninety-four are adults. How many children are there?

(**4**) If a man walk six miles an hour, how long will he take to walk eighteen thousand and thirty miles?

(**5**) What was the age of a man who was born in one thousand seven hundred and seventy-five, and died in one thousand eight hundred and sixty-three?

(6) A and B together have two thousand pounds; A has one hundred and ninety-three. How much has B?

(7) Two ships have one hundred and two hundred oxen on board respectively; during the voyage, thirty-six of the first and twenty-four of the second die. How many reach land alive in all?

(8) At an election the defeated candidate polls four hundred and seventy-three votes, one hundred and twenty-seven less than his rival. How many votes did the successful candidate poll?

(9) When A was 33, B was born. What is B's age when A is 79?

(10) Out of an army of eighty-eight thousand five hundred and thirty-five, every ninth man is killed. How many survived?

(11) If I have 713453 oranges and only 3900 boxes to pack them in, how many shall I have over?

(12) If a farmer buy five oxen at twelve pounds each, and six sheep at six pounds each, what will he have left out of one hundred pounds?

(13) If three hundred and seventy-nine thousand five hundred and forty-three pounds be raised from forty-two towns, what will each contribute supposing one town pay thirty-one pounds extra?

(14) If a horse and carriage cost one hundred and fifty pounds, and the horse cost twice as much as the carriage, what did the horse cost?

(15) In two hundred and fifty-two days how many weeks are there?

(16) In twenty-seven thousand and forty-eight half-crowns how many pounds?

(17) If I spend forty pounds a month for twenty months, what shall I have left out of two thousand pounds, and how long will it last me at the same rate of expenditure?

(18) How many sixpences in thirty thousand and forty half-crowns?

(19) A man died in one thousand eight hundred and eighty-two, aged eighty-three years. In what year was he born?

(20) How many threepenny-pieces in one sovereign?

Miscellaneous Examples C.

(1) What number multiplied by twenty-seven will give the same product as fifteen multiplied by sixty-three?

(2) How many years in one million one hundred days?

(3) What number multiplied by five and divided by three gives fifteen as result?

(4) What number must be added to 343418 to make 765439?

(5) How many men can I give £4153 to, if each man have £15? How much will the first man have if he take the remainder as well as his share?

(6) A book contains ten thousand seven hundred and sixty-four lines, and sixty-four lines in a page. How many pages are there?

(7) What money must I have to pay 8 men, 12 women, and 15 children, giving each man twice as much as a woman, and each woman three times as much as a child, a child's share being £3?

(8) Divide £360 among A, B, C, giving A £15 more than B and B £10 more than C.

(9) A man buys 30 articles at £13 each, and after losing 8 of them, sells the remainder so as to gain £50 on the whole. What does he sell them for each?

(10) A mixture is made of 25 gals. of spirit at 23*s.* a gal., 16 gals. at 43*s.*, 42 gals. at 25*s.*, and 21 gals. at 25*s.*, what is the price per gallon of the whole, if 16 gals. of water are mixed with it?

(11) A mile contains 5280 feet, how many times will a wheel, whose circumference is 11 feet, turn in going a mile?

(12) 1 ton contains 2240 lbs. What is the value of 2 tons at 2*s.* 6*d.* a lb.?

(13) If I save 6*d.* a week, what is that in a year?

(14) 1 mile = 1760 yards, 1 yard = 3 feet, 1 foot = 12 inches. How many inches in 1 mile?

(15) I bought 40 articles at 6*d.* each, 56 articles at 2*s.* 6*d.* each, and 80 other articles, and spent altogether £12, what was the price of the last articles?

(16) A farmer has £500: he buys 20 lambs at £4 each, 30 pigs at £5 each, 2 cows at £15 each, how many sheep at £6 each can he buy?

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(**17**) If in mixing 24 gallons at 9*s.* a gal. with 19 gals. at 8*s.*, one gallon is lost, what must I sell the remainder at per gal. to gain 10*s.*?

(**18**) I exchange 7 lambs at £8 each, and seven times as many sheep, which cost twice as much as a lamb, for the same number of cows, what did I give for a cow?

(**19**) If 12 articles cost £3, what is the price of a gross?

(**20**) If 9 men earn £12 in a given time, how much will 45 men earn?

(**21**) If 45 men dig 108 acres in 5 days, how many will 5 men dig in the same time?

(**22**) Prove that the product of the sum and difference of 25 and 27 is the same as the difference between their squares.

(**23**) After paying an income-tax of 2*s.* 6*d.* in the £, my net income is £1400, what is my gross income?

(**24**) If a man owes £1600 and has only £200, how much can he pay in the £?

(**25**) I have an equal number of coins which consist of half-sovereigns, crowns, half-crowns, florins, shillings, and sixpences, and the whole sum amounts to £21, how many of each have I?

(**26**) Divide £48. 15*s.* among 4 persons, so that one may have 15*s.* more than each of the rest.

(**27**) Divide £600 between A, B, C, so that A may have as much as C, and B three times as much as C.

(**28**) The velocity of sound is 1100 feet a second, how long would it take to travel 25 miles?

(**29**) If I can row 6 miles an hour in still water, and a stream run at a rate of 4 miles an hour, how long will it take me to row a distance of 10 miles up and down stream?

(**30**) The distance from A to B is 150 miles: a train whose rate is 30 miles an hour starts from A and arrives at B in 8 hours, stopping 6 minutes at each of the intervening stations, and being delayed 2 hours by an accident. How many stations did it stop at?

(**31**) A walks 6 miles an hour, B 4 miles an hour: if B start 20 miles in front of A, how long will A take to catch B, and what distance will each have walked?

(**32**) 2 trains moving with velocities of 40 and 30 miles per hour respectively are 140 miles apart. When and where will they meet?

(**33**) If the fore-wheel of a carriage be 8 feet in circumference, and the hind-wheel 11 feet, how many revolutions will the former make more than the other in 12 miles?

(**34**) A hare takes 6 leaps each of 3 feet in length in a second, a dog takes 4 leaps each of 5 feet in length in a second, how long will it take the dog to catch the hare, which is 200 yards in front of him, and how many leaps does each take before the hare is caught?

(**35**) If (in Problem 29) it take me 10 hours to row up the stream, how long will it take me to row back?

(**36**) Prove that the difference between the cubes of 2 consecutive numbers = sum of their squares + their product.

VIII. SOME PROPERTIES OF NUMBERS.

26. A number which is divisible by no factor but itself is called a *prime* number: as 5, 3, 7, 23.

A number which contains other factors besides itself is called a *composite* number: as 15, which is composed of the factors 5 and 3.

Let the following rules be learnt by heart:—

(**1**) A number is divisible by 2 if the last figure is even.

(**2**) A number is divisible by 3 if the sum of the digits will divide by 3.

(**3**) A number is divisible by 4 if the last 2 figures will divide by 4.

(**4**) A number is divisible by 5 if the last figure is 5 or 0.

(**5**) A number is divisible by 6 if it satisfy tests (**1**) and (**2**).

(**6**) A number is divisible by 8 if the last 3 figures will divide by 8.

(**7**) A number is divisible by 9 if the sum of the digits will divide by 9.

(**8**) A number is divisible by 10 if the last figure is 0.

(**9**) A number is divisible by 11 if the difference between the sums of its digits added up alternately, beginning at the units place, is divisible by 11.

The divisibility of a number by 7 is best found by trial.

[It is found convenient in the reduction of a number into its prime factors to indicate the fact that a factor is contained a certain number of times by placing a figure, called the index, which denotes the number of times the factor occurs in the power, over the number, and to the right: thus $8 = 2 \times 2 \times 2 = 2^3$, or $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$.]

27. To discover the factors of a composite number, or to resolve a number into its prime factors, we apply the tests given above, and divide the number by that factor as many times as possible, until the final quotient is a prime number: the prime factors required will be these several factors and the last quotient.

E.g. to find the factors of 5760:—

We see from test (1) it will divide by 2.

2)5760
2)2880
2)1440
2)720
2)360
2)180
2)90
3)45
3)15
5

$\therefore\ 5760 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^7 \times 3^2 \times 5$.

28. It is well to draw attention to the fact that after applying as far as possible the above tests, we may get a quotient which is not of necessity a prime number, as it may contain 13 17, 19, &c., as factors; these must be determined by trial. Remembering that $11^2 = 121$, $13^2 = 169$, $17^2 = 289$, $19^2 = 361$, we see that no number less than 400 can have a factor greater than 19; less than 300 greater than 17; less than 200 greater than 13; less than 100 greater than 10. In other words, in seeking prime factors of any number we need only try prime numbers until the quotient equals or is less than the divisor.

Exercise 7.

Resolve into prime factors:—

(1) 84, 48, 32, 45, 120, 141, 512, 1760, 5760, 212, 504, 112, 360.

(2) 18, 182, 456, 3996, 132, 238, 920575, 89012.

(3) 7840, 6840, 1909680, 345, 1620, 1820, 3000, 13734.

(4) 30086, 93940, 1546992, 7417784, 579826952.

i. Highest Common Factor.

29. A number is a factor or measure of another number when it is contained in that number an exact number of times; and a number which is a measure of each of two or more numbers is called their *common measure*, and the greatest of these is called their *greatest common measure*, or *highest common factor*, which are indicated by the letters G.C.M.: H.C.F.

30. To find the H.C.F. of 2 numbers.

Resolve both numbers into their prime factors, and the product of all the prime factors which they have in common is the H. C. F. required.

Exercise 8.

Find H.C.F. of:

(1) 8, 14; 12, 30; 32, 48; 48, 128; 16, 1024; 64, 256.

(2) 512, 240; 112, 28; 2100, 2240; 1760, 990; 77, 231; 210, 1008.

(3) 48, 88; 27, 63; 63, 120; 289, 340; 621, 805; 17104, 27794.

The same method applies to the following:—

(4) 78, 130, 195; 24, 60, 84, 128; 252, 315, 420, 504.

(5) 136, 204, 357, 459; 128, 192, 320, 368, 432; 3864, 3404, 3657.

(6) 3555, 4977, 6636; 15561, 11115, 13585; 833, 1785, 1309.

(7) 365, 511, 803; 232, 290, 493; 492, 1476, 1763; 148, 444, 592, 703.

31. Next consider the 2 numbers 39672, 45885

2)39672 5)45885
9)19836 7)9177
4)2204 3)1311
551 437

3 is a common factor and ∴ must be a factor of the result; the other factors 2, 5, 7, not being common, may be rejected, and

the problem is now reduced to finding the H.C.F. of 437, 551, which is thus obtained :—

```
437)551(1
    437
    ———
    114)437(3
        342
        ———
         95)114(1
             95
             ——
             19)95(5
                95
```

∴ H.C.F. is $3 \times 19 = 57$.

It will be observed that we have divided the greater of the 2 numbers by the less, and the divisor by the remainder, and repeated the process, until the remainder is zero: the last divisor giving the H.C.F. required.

By this rule the H.C.F. of 2 numbers can be obtained at once; but it will be found that the labour is considerably shortened by employing the 'tests.'

32. If the H.C.F. of more than two numbers is required, the shortest method to adopt will be to subtract the two largest numbers, the two smallest numbers, and find the H.C.F. of the two remainders. It may happen that one of the numbers is odd, and the H.C.F. thus obtained may be even. In such case divide it by 2, and the quotient will give the answer.

E.g., Find the H.C.F. of 57, 95, 133, 171.

The difference of 95 and 57 is 38, and of 171, 133 is 38 ∴ 19 is the H.C.F. required.

The following examples will furnish additional exercise in the above methods.

Exercise 9.

Find the H.C.F. of:

(1) 60, 144. (2) 96, 324. (3) 420, 5400. (4) 598, 1058.
(5) 1407, 2211. (6) 1792, 2048. (7) 6409, 7395. (8) 2433, 13787.
(9) 8398, 29393. (10) 19527, 23667. (11) 23760, 26136.
(12) 527751, 645029. (13) 100110, 31866.
(14) 1099901, 1067803. (15) 731, 2329, 3502.
(16) 3240, 1224, 2068. (17) 1125, 1575, 3150.
(18) 286720, 1310309. (19) 28431, 216. (20) 24720, 4155.

(**21**) 7056, 7392. (**22**) 6327, 23997. (**23**) 12321, 54345.
(**24**) 24720, 3755. (**25**) 352495, 51156. (**26**) 352495, 12673.
(**27**) 21879, 14872. (**28**) 15496, 12665. (**29**) 4781, 6147.
(**30**) 7535, 11645. (**31**) 33495, 106260. (**32**) 169037, 66429.
(**33**) 5568981, 506382. (**34**) 15463, 554470. (**35**) 23760, 26136.
(**36**) 6023, 15466. (**37**) 114135, 220661. (**38**) 1073, 1421.
(**39**) 629, 777, 1073. (**40**) 1495, 2145, 2795. (**41**) 357, 425.
(**42**) 391, 667, 713. (**43**) 333, 414, 711. (**44**) 2597, 3445.
(**45**) 6715, 8927. (**46**) 493, 1073, 1537. (**47**) 5989, 7571, 8927.
(**48**) 8128, 14816, 75288, 8472. (**49**) 1216, 832, 424.
(**50**) 35035, 41580, 24255.

ii. Lowest Common Multiple.

33. A number is said to be a *multiple* of another number when it contains that number an exact number of times; thus 12, 24, 48 are multiples of 3. A common multiple of two or more numbers is a number which contains each of them exactly; or a number of which each of them is a factor; thus 45 is a common multiple of 9 and 15.

The least common multiple, written L.C.M., of two or more numbers is the least number which contains each of them as factors. From this definition of L.C.M. we deduce the following rule to find it.

Resolve each number into its prime factors, selecting only the highest power of each factor which occurs in any one of the numbers, and find the continued product of the factors so selected.

Ex. 1. Thus to find the L.C.M. of 56, 64, 70, 84, 112—

$$\begin{aligned} 56 &= 7 \times 8 = 7 \times 2^3 \\ 64 &= 8 \times 8 = 2^6 \\ 70 &= 7 \times 10 = 7 \times 5 \times 2 \\ 84 &= 7 \times 4 \times 3 = 7 \times 2^2 \times 3 \\ 112 &= 7 \times 16 = 7 \times 2^4 \end{aligned}$$

The different factors are 2, 3, 5, 7, and the highest power of 2 is 6.

$$\begin{aligned} \therefore \text{L.C.M.} &= 2^6 \times 3 \times 5 \times 7 \\ &= 64 \times 3 \times 5 \times 7 = 6720. \end{aligned}$$

Ex. 2. Again, find L.C.M. of 112, 189, 175, 35, 54, 16—

$$\begin{aligned} 112 &= 7 \times 16 = 7 \times 2^4 \\ 189 &= 7 \times 27 = 7 \times 3^3 \\ 175 &= 7 \times 25 = 7 \times 5^2 \end{aligned}$$

$$\begin{aligned} 35 &= 7 \times 5 \\ 54 &= 9 \times 6 = 2 \times 3^3 \\ 16 &= 2^4 \end{aligned}$$

The different factors are 2, 3, 5, 7, and their highest powers are 4, 3, 2, 1 respectively. ∴ L.C.M. $= 2^4 \times 3^3 \times 5^2 \times 7 = 75600$.

34. In this operation we might have left out lines 4 and 6 without affecting the result; if, therefore, any one of the numbers is a factor of another we may omit it.

Sometimes it is convenient to arrange the work thus—

$$\begin{array}{r|rrrrrr} 2) & \cancel{16}, & \cancel{35}, & 54, & 112, & 175, & 189 \\ \hline 2) & & & \cancel{27}, & 56, & 175, & 189 \\ \hline 7) & & & & 28, & 175, & 189 \\ \hline & & & & 4, & 25, & 27 \end{array}$$

∴ L.C.M. $= 2 \times 2 \times 7 \times 4 \times 25 \times 27 = 2^4 \times 3^3 \times 5^2 \times 7 = 75600$.

35. Comparing these two methods will help to explain the reason for the following *rule*, which is the one usually given to find the L.C.M.:—

Set down the different numbers in order, omitting any that are factors of others; divide by the lowest factor common to two or more of them; placing the quotient or the numbers not divisible by the factor immediately underneath the original numbers; repeat the process with the numbers thus obtained until you get a line of numbers prime to one another; then the continued product of the divisors and the numbers in this line will be the L.C.M. required.

NOTE.—It is clear that the L.C.M. of two numbers prime to one another is their product, and that the L.C.M. of two numbers is their product divided by their H.C.F. Therefore, to find the L.C.M. of two large numbers it is generally well to find their H.C.F., to divide one of the numbers by it and to multiply the quotient by the other.

Exercise 10.

Find the L.C.M. of—

(1) 2, 3, 4, 5, 6. (2) 4, 5, 6, 8, 12. (3) 12, 8, 42, 56.
(4) 3, 7, 9, 63. (5) 52, 39, 78, 117. (6) 12, 15, 20, 27.
(7) $13 \times 17 \times 19$, $17 \times 19 \times 21$, $19 \times 21 \times 13$. (8) 105, 504, 3080.
(9) 504, 792, 3080. (10) 352495, 51156. (11) 352495, 12673.

(12) 4781, 6147. **(13)** 7535, 11645. **(14)** 8468, 6351, 19053, 14819. **(15)** 9217, 1418, 4963, 709. **(16)** 4004, 2548, 728, 4732. **(17)** 259, 222, 74, 185. **(18)** 209, 133, 95, 57. **(19)** 58, 81, 153, 42. **(20)** 5908, 5612, 3047.

IX. FRACTIONS.

36. Up to this point we have been dealing with whole numbers, taking units as our standard and considering it capable of *repetition*. We will now consider it as capable of *division*; and to make the meaning of what we shall say clearer, we will fix on some particular unit, say one pound, so that one pound will be represented by the symbol 1; and two pounds, three pounds, four pounds, &c. by the symbols 2, 3, 4, &c. But we know that we can divide one pound into two half-sovereigns, four crowns, eight half-crowns, ten florins, twenty shillings, &c.; and what we shall do is to discover a means by which we can represent each of these different coins or parts of a pound. Now, when mention is made of a half-sovereign, it is understood that the unit, the one pound, has been divided into 2 parts, and to signify this we put a 2 under the 1 and draw a line between them, thus $\frac{1}{2}$; and we say that a half-sovereign will be represented by $\frac{1}{2}$ if one pound is the unit, or generally we denote half of anything by the symbol $\frac{1}{2}$.

In the same way, when we speak of a crown it is understood that the pound has been divided into 4 parts, and therefore $\frac{1}{4}$ will represent one crown, or a quarter of a pound, or one-fourth of a pound; or generally $\frac{1}{4}$ means a quarter. Hence $\frac{1}{8}$ represents half-a-crown, $\frac{1}{10}$ represents a florin, $\frac{1}{20}$ represents one shilling, and the symbols $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{20}$ read one-eighth, one-tenth, one-twentieth, show by the figures under the line that the whole has been divided into 8, 10, 20 parts respectively.

Again, if we take one shilling as our unit and therefore represent it by 1, and divide the shilling into sixpences, fourpenny pieces, threepenny pieces, and pence, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$ will denote respectively one sixpence, one fourpence, one threepence, one penny; and 2, 3, 4, will denote two shillings, three shillings, four shillings.

From what has been said, it is clear that if we wish to show into how many parts a whole has been divided we place the figure representing the number under the line.

Next, how shall we represent 2 fourpenny pieces, 3 threepenny pieces, or 5 pence? 2 fourpenny pieces may be obtained either by the addition of one fourpenny piece to one fourpenny piece, or by the division of two shillings into three equal parts and taking one of them; in either case we represent it by $\frac{2}{3}$. $\frac{2}{3}$ therefore denotes either that the whole has been divided into 3 equal parts and 2 of them taken, or that 2 wholes have been divided into 3 equal parts and one of them taken. Similarly $\frac{3}{4}$ = three threepenny pieces, or generally $\frac{3}{4}$ denote that we have divided the whole into 4 parts and taken 3 of them; and $\frac{5}{12}$ = five pence; or generally $\frac{5}{12}$ denotes that we have divided the whole into 12 parts and taken 5 of them.

From these remarks it will be seen that the figure above the line tells us how many parts of the whole are taken, or how many wholes or units are taken and divided.

These symbols we have been investigating are called *fraction-symbols* or *fractions*; the number above the line is called *numerator*, the number below the line is called *denominator*.

Hence we get the following definitions: a fraction is a portion of a whole or unit and consists of 2 parts called *terms*, the *denominator* showing into how many portions the unit is divided, the *numerator* showing how many of those portions are to be taken. Thus $\frac{5}{6}$ is a fraction, 5 is the numerator, 6 the denominator.

Exercise II.

Considering £1 as unit, what is the value of:—

(1) $2\frac{1}{2}$, $3\frac{3}{4}$, $4\frac{7}{8}$, $5\frac{3}{10}$, $4\frac{5}{12}$, $\frac{7}{6}$, $5\frac{2}{3}$, $\frac{15}{20}$, $\frac{6}{8}$, $\frac{3}{4}$, $\frac{9}{12}$, $\frac{12}{16}$, and express symbolically:—

(2) One half-sovereign, two half-sovereigns, two crowns, three half-crowns, four pounds six shillings and eight pence, seven florins, twelve shillings, five crowns, eight half-crowns, ten shillings, twenty shillings, forty sixpences.

Considering one shilling as the unit, what is the value of:—

(1) $\frac{1}{12}$, $\frac{2}{12}$, $\frac{1}{6}$, $\frac{3}{12}$, $\frac{1}{4}$, $\frac{4}{12}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{6}{12}$, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{7}{12}$, $\frac{8}{12}$, $\frac{4}{6}$, $\frac{2}{3}$, $\frac{9}{12}$, $\frac{3}{4}$, $\frac{10}{12}$, $\frac{5}{6}$, and express symbolically:—

(2) Two shillings and two pence, three shillings and three pence, four shillings and four pence, five shillings and five pence, six shillings and six pence, seven shillings and seven pence, eight shillings and eight pence, nine shillings and nine pence, ten shillings and ten pence, eleven shillings and eleven pence, twelve shillings and twelve pence.

Considering one penny as the unit, what is the value of:—

(1) $5\frac{1}{2}$, $4\frac{3}{4}$, $2\frac{1}{4}$, $\frac{2}{4}$, $\frac{2}{2}$, $\frac{4}{4}$.

And how would you express:—

(2) Sixpence farthing, sevenpence halfpenny, ninepence threefarthings, one penny, four farthings, two halfpennies.

37. We know that one half-sovereign = two crowns = four half-crowns = five florins = ten shillings = twenty sixpences, and that if one pound be the unit, we could express these equivalents thus:—

$$\tfrac{1}{2}=\tfrac{2}{4}=\tfrac{4}{8}=\tfrac{5}{10}=\tfrac{10}{20}=\tfrac{20}{40}.$$

Noticing, then, that each of the last five fractions can be obtained from the first by multiplying its numerator and denominator by two, four, five, ten, and twenty respectively, and that if they be written in reverse order, the last five fractions could be obtained by dividing the numerator and denominator of the first fraction by the same five numbers respectively, we infer the truth of the following rules:—

(1) If the numerator and denominator of a fraction be multiplied by the same number, the value of the fraction is not altered.

(2) If the numerator and denominator of a fraction be divided by the same number, the value of the fraction is not altered.

38. A fraction is said to be in its lowest terms when its numerator and denominator are prime to each other.

Hence, given any fraction we reduce it to its lowest terms by removing the H.C.F. of its numerator and denominator. Of course, to determine the H.C.F. we apply the methods suggested in Article 30.

Example: Reduce to its lowest terms $\tfrac{42840}{47880}$.

$$\frac{42840}{47880}=\frac{17\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{5}}{19\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{5}}=\frac{17}{19}.$$

Exercise 12.

Reduce to their lowest terms the following fractions:—

(1) $\frac{60}{84}$. (2) $\frac{35}{63}$. (3) $\frac{105}{135}$. (4) $\frac{108}{144}$. (5) $\frac{112}{192}$. (6) $\frac{91}{117}$. (7) $\frac{135}{180}$.
(8) $\frac{192}{304}$. (9) $\frac{1296}{6561}$. (10) $\frac{1015}{1566}$. (11) $\frac{516}{2107}$. (12) $\frac{6409}{7395}$. (13) $\frac{9495}{15615}$.
(14) $\frac{272}{3600}$. (15) $\frac{9324}{14274}$. (16) $\frac{1004}{1056}$. (17) $\frac{10648}{12584}$. (18) $\frac{192}{5120}$. (19) $\frac{29061}{38289}$.
(20) $\frac{1792}{2048}$. (21) $\frac{2433}{13787}$. (22) $\frac{19527}{23667}$. (23) $\frac{527751}{645029}$. (24) $\frac{1099901}{1067803}$.
(25) $\frac{9193}{20677}$. (26) $\frac{4155}{24720}$. (27) $\frac{29393}{8398}$. (28) $\frac{9019}{96721}$. (29) $\frac{11385}{16335}$.
(30) $\frac{3300}{4235}$. (31) $\frac{1003}{1829}$. (32) $\frac{1197}{2471}$. (33) $\frac{10307}{94637}$. (34) $\frac{370125}{212346}$.
(35) $\frac{729}{1917}$. (36) $\frac{1739}{2726}$. (37) $\frac{3817}{5205}$. (38) $\frac{10894}{19693}$. (39) $\frac{1645}{2209}$. (40) $\frac{1166}{2067}$.
(41) $\frac{2014}{3551}$. (42) $\frac{8251}{14718}$. (43) $\frac{3575}{4719}$. (44) $\frac{1261}{44232}$. (45) $\frac{10759}{20405}$.
(46) $\frac{29810}{99999}$. (47) $\frac{6216}{999999}$. (48) $\frac{8925345}{10881585}$. (49) $\frac{5544}{8008}$. (50) $\frac{48016}{108036}$.

(51) $\frac{25025}{35035}$. (52) $\frac{13068}{15444}$. (53) $\frac{2919}{4865}$. (54) $\frac{5568}{12992}$. (55) $\frac{2229}{9659}$.
(56) $\frac{4015}{6935}$. (57) $\frac{7821}{8217}$. (58) $\frac{7040}{7372}$. (59) $\frac{5184}{1008}$. (60) $\frac{64961}{68419}$. (61) $\frac{23489}{26071}$.

39. If we wish to compare a half-sovereign, a crown, a half-crown, a florin, and a shilling, we can do so by reducing them all to the same kind of coin which they each contain an exact number of times, such as sixpences, threepences, or pence: the simplest way of doing this is to change them all into sixpences, when they would be represented by 20, 10, 5, 4, 2 sixpences. Or, taking one pound as our unit, we see that in order to compare the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{20}$, we must reduce all the fractions to a common denominator 40—in this case the least common denominator—and they would be represented by $\frac{20}{40}$, $\frac{10}{40}$, $\frac{5}{40}$, $\frac{4}{40}$, $\frac{2}{40}$, or with the usual notation $\frac{20,\ 10,\ 5,\ 4,\ 2}{40}$

40. This will explain the following rule for comparing fractions:—

Reduce all the fractions to a common denominator, *i.e.* find the L.C.M. of all the denominators.

Divide this L.C.M. by each of the denominators in turn and multiply the quotients severally by the corresponding numerators.

Arrange these products in ascending or descending order of magnitude over a horizontal line under which is written the L.C.M.

41. A whole number may be represented as a fraction by placing it over a denominator 1. Thus 2, 7, 9 may be written $\frac{2}{1}$, $\frac{7}{1}$, $\frac{9}{1}$:

Ex. 1. Represent 5 with denominator 7.

Here $5=\frac{5}{1}$, and to make the denominator 7 we must multiply the denominator 1 by 7, but if we wish the fraction to retain its value, we must also multiply the numerator by 7 **37.** (1); thus:

$$5=\frac{5\times 7}{1\times 7}=\frac{35}{7}$$

Ex. 2. Represent $\frac{5}{7}$ with denominator 56:—

$$56=7\times 8\ \therefore\ \frac{5}{7}=\frac{5\times 8}{7\times 8}=\frac{40}{56},$$

or divide the 56 by 7 and multiply by 5, and place the product over 56.

Ex. 3. Represent $\frac{15}{17}$, $\frac{65}{102}$ as equivalent fractions with their lowest common denominator.

The L.C.M. of 17, 102 is 102.

$$\begin{array}{r} 17\underline{)102} \\ 6 \\ \underline{15} \\ 90 \end{array} \qquad \begin{array}{r} 102\underline{)102} \\ 1 \\ \underline{65} \\ 65 \end{array}$$

∴ the above fractions may be written $\frac{90,\ 65}{102}$.

Ex. 4. Compare the values of $\frac{3}{20}$, $\frac{7}{40}$, $\frac{11}{70}$.

The L.C.M. of 20, 40, 70 is 280.

$$\begin{array}{r} 20\underline{)280} \\ 14 \\ \underline{3} \\ 42 \end{array} \qquad \begin{array}{r} 40\underline{)280} \\ 7 \\ \underline{7} \\ 49 \end{array} \qquad \begin{array}{r} 70\underline{)280} \\ 4 \\ \underline{11} \\ 44 \end{array}$$

∴ arranging them in descending order of magnitude, they are written:

$$\frac{49,\ 44,\ 42}{280},$$

or in their original form $\frac{7}{40}$, $\frac{11}{70}$, $\frac{3}{20}$.

42. It is clear that the greater number of parts we divide a whole into, the smaller those parts must be, consequently we infer that of two fractions having the same numerator, that one is the greater which has the smaller denominator. Another way then of working Ex. 4 would be to

Reduce the fraction to a common *numerator* and then giving that fraction the greatest value which has the least denominator.

Thus, the above fractions may be written $\frac{231}{1540}$, $\frac{231}{1320}$, $\frac{231}{1470}$, which arranged in descending order of magnitude would be $\frac{231}{1320}$, $\frac{231}{1470}$, $\frac{231}{1540}$, or written in their original form, we have, as before $\frac{7}{40}$, $\frac{11}{70}$, $\frac{3}{20}$.

Exercise 13.

Express with denominators, 10, 63, 144, 1728 the following:—

(1) 7, 9, 15, 24, 144, 36, 78, 72, 90, 5760, 480, 2240.

Express in their lowest common denominator the fractions:—

(1) $\frac{5}{6}$, $\frac{7}{8}$, $\frac{7}{12}$, $\frac{9}{10}$, $\frac{11}{16}$, $\frac{13}{30}$. (2) $\frac{3}{16}$, $\frac{5}{28}$, $\frac{59}{112}$, $\frac{113}{2240}$, $\frac{17}{20}$. (3) $\frac{5}{7}$, $\frac{13}{24}$, $\frac{17}{60}$. (4) $\frac{11}{12}$, $\frac{2}{3}$, $\frac{5}{11}$, $\frac{39}{40}$, $\frac{7}{8}$. (5) $\frac{8}{9}$, $\frac{3}{4}$, $\frac{4}{5}$. (6) $\frac{13}{144}$, $\frac{8}{9}$, $\frac{5}{121}$, $\frac{39}{40}$. (7) $\frac{133}{1728}$, $\frac{26}{27}$. (8) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$. (9) $\frac{23}{24}$, $\frac{19}{20}$, $\frac{11}{12}$. (10) $\frac{15}{16}$, $\frac{13}{14}$, $\frac{27}{28}$, $\frac{3}{4}$, $\frac{19}{20}$, and arrange the

above fractions (1) by reducing them to a common denominator; (2) by reducing them to a common numerator.

43. When we speak of 3 half-crowns, one pound being the unit, we say that it is properly called a fraction of one pound, ∵ it can be subtracted from one pound: whereas, 11 half-crowns cannot be subtracted from one pound, and ∴ is improperly called a fraction of a pound; in other words $\frac{3}{8}$ is a proper fraction—$\frac{11}{8}$ is an improper fraction: or more generally a proper fraction is one in which the numerator is less than the denominator, and an improper fraction is one in which the numerator is greater than the denominator.

For instance, 3 half-sovereigns, one pound being the unit, would be represented by $\frac{3}{2}$. But 3 half-sovereigns = £1. 10*s.* = $1\frac{1}{2}$ symbolically ∴ $\frac{3}{2} = 1\frac{1}{2}$. Again, 7 crowns = £1. 15*s.* = $1\frac{3}{4}$: and 7 crowns = $\frac{7}{4}$, ∴ $\frac{7}{4} = 1\frac{3}{4}$. In both these cases $\frac{3}{2}$, $\frac{7}{4}$ are improper fractions, and are simplified by making the quotient the whole number, the remainder the numerator, and the divisor the denominator of the fraction. The result is called a ***mixed number.*** Hence the reason for the following rules:—

44. (1) To reduce a mixed number to an improper fraction, multiply the integer, or whole number, by the denominator and add in the numerator, for the numerator of the fraction, retaining for the denominator the original denominator.

(1) To reduce an improper fraction to a mixed number, divide the numerator by the denominator, setting down the quotient as the whole number, the remainder as the numerator of the fractional part of the mixed number, the denominator being the denominator of the original fraction.

Exercise 14.

Reduce the following mixed numbers to improper fractions:—

(1) $5\frac{3}{7}$. (2) $25\frac{13}{48}$. (3) $11\frac{116}{117}$. (4) $17\frac{144}{1728}$. (5) $37\frac{3333}{4240}$, and the following improper fractions to mixed numbers:—

(6) $\frac{27}{4}$. (7) $\frac{261}{8}$. (8) $\frac{4364}{220}$. (9) $\frac{1798}{1232}$. (10) $\frac{1671}{1423}$

i. Addition.

45. We now proceed to apply these elementary principles to the first four rules.

Ex. 1. One half-sovereign + one crown + one half crown + one florin + one shilling = 20 sixpences + 10 sixpences + 5 sixpences + 4 sixpences + 2 sixpences = 41 sixpences = £1. 0*s.* 6*d.*, if one pound be the unit $= \frac{41}{40} = 1\frac{1}{40}$.

Or symbolically—$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{10} + \frac{1}{20}$

$$= \frac{20 + 10 + 5 + 4 + 2}{40} = \frac{41}{40} = 1\frac{1}{40}$$

Comparing these 2 results the truth is established.

Ex. 2. If we add together 3 crowns, 7 half-crowns, 16 shillings, and 9 florins, we get £3. 6*s.* 6*d.*, which we express symbolically as $3\frac{13}{40}$. Hence to add $\frac{3}{4} + \frac{7}{8} + \frac{4}{5} + \frac{9}{10}$, express each fraction with the denominator 40, the L.C.M. of the denominators 4, 5, 8, 10.

$$\begin{array}{r} 4\overline{)40} \\ \hline 10 \\ 3 \\ \hline 30 \end{array} \qquad \begin{array}{r} 8\overline{)40} \\ \hline 5 \\ 7 \\ \hline 35 \end{array} \qquad \begin{array}{r} 5\overline{)40} \\ \hline 8 \\ 4 \\ \hline 32 \end{array} \qquad \begin{array}{r} 10\overline{)40} \\ \hline 4 \\ 9 \\ \hline 36 \end{array}$$

and we have—

$$\frac{30 + 35 + 32 + 36}{40} = \frac{133}{40} = 3\frac{13}{40},$$

which is the result we obtained above. In the following exercise, first write down the value of each fraction, add the sum together, and represent the result symbolically; next express each fraction as equivalent fractions, with their lowest common denominator, add the new numerators together, place this sum over the common denominator, and reduce if necessary to a mixed number. The answers should be the same.

Exercise 15.

(1) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$. (2) $\frac{1}{6} + \frac{3}{8} + \frac{7}{10} + \frac{9}{20}$. (3) $\frac{2}{3} + \frac{4}{5} + \frac{7}{8} + \frac{19}{20}$.
(4) $\frac{5}{8} + \frac{9}{10} + \frac{3}{5} + \frac{5}{6}$. (5) $\frac{7}{10} + \frac{9}{40} + \frac{11}{20} + \frac{1}{3}$. (6) $\frac{1}{3} + \frac{2}{5} + \frac{5}{8} + \frac{30}{40}$.

ii. Subtraction.

46. To subtract a fraction from a greater fraction, the rule is: Express each fraction as equivalent fractions with their lowest common denominator, subtract the new numerators, and place the difference over the common denominator. Reduce the result to lowest terms if necessary

Thus, subtract $\frac{9}{40}$ from $\frac{7}{8}$:

(1) $\frac{7}{8} = 17$*s.* 6*d.*; $\frac{9}{40} = 9$ sixpences = 4*s.* 6*d.* The difference between 4*s.* 6*d.* and 17*s.* 6*d.* is 13 shillings $= \frac{13}{20}$.

(2) $\frac{7}{8}-\frac{9}{40}=\frac{35-9}{40}=\frac{26}{40}=\frac{13}{20}$

In this way let the following exercise be worked.

Exercise 16.

Find the difference between :

(1) $\frac{1}{2}$ and $\frac{1}{3}$. (2) $\frac{1}{3}$ and $\frac{1}{4}$. (3) $\frac{1}{4}$ and $\frac{1}{5}$. (4) $\frac{1}{5}$ and $\frac{1}{6}$. (5) $\frac{1}{6}$, $\frac{1}{8}$. (6) $\frac{1}{8}$, $\frac{1}{10}$. (7) $\frac{1}{10}$, $\frac{1}{20}$. (8) $\frac{1}{20}$, $\frac{1}{40}$. (9) $\frac{1}{8}$, $\frac{1}{20}$. (10) $\frac{5}{8}$ and $\frac{5}{6}$.

iii. Multiplication.

47. $\frac{1}{8}$ is our symbol for half-a-crown, and we know that $\frac{1}{5}$ of half-a-crown is *6d.*, or in other words $\frac{1}{5}$ of $\frac{1}{8}=\frac{1}{40}$; also $\frac{1}{5}$ is our symbol for 4 shillings, and $\frac{1}{8}$ of 4 shillings = *6d.* ∴ $\frac{1}{8}$ of $\frac{1}{5}=\frac{1}{40}$. Again, $\frac{1}{5}$ of 7 half-crowns = $\frac{1}{5}$ of 17*s.* *6d.* = 3*s.* *6d.* = $\frac{7}{40}$, or $\frac{1}{5}$ of $\frac{7}{8}=\frac{7}{40}$. And since $\frac{1}{5}$ of 7 half crowns = 3*s.* *6d.*, $\frac{3}{5}$ of 7 half-crowns = 3 × 3*s.* *6d.* = 10*s.* *6d.* = $\frac{21}{40}$, ∴ $\frac{3}{5}$ of $\frac{7}{8}=\frac{21}{40}$. Hence this rule for multiplication of fractions.

The numerator and denominator of the resulting fraction is the product of the several numerators, and of the several denominators respectively. Reduce if necessary to a mixed number.

In order that the resulting fraction may be in its lowest terms, common factors should be removed from the numerator and denominator before effecting the multiplication, and they will be more easily discovered if the work is thus arranged :

Find value of $\frac{1}{8}$ of $2\frac{2}{3}$ of $15\frac{3}{5}=\frac{1}{\cancel{8}}\times\frac{\cancel{8}}{\cancel{3}}\times\frac{\overset{26}{\cancel{78}}}{5}=\frac{26}{5}=5\frac{1}{5}$ *Ans.*

It will be observed that mixed numbers must be reduced to improper fractions before this rule can be applied.

Fractions, as above, connected by the sign 'of' are called *compound fractions.*

Exercise 17.

Reduce to simple fractions :—

(1) $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{10}{18}$. (2) $\frac{3}{4}$ of $5\frac{1}{7}\times\frac{14}{21}$ of 15. (3) $5\frac{1}{6}$ of $\frac{13}{62}$ of $\frac{18}{52}$ of 28. (4) $\frac{4}{5}$ of $\frac{3}{7}$ of $10\frac{1}{2}$. (5) $2\frac{4}{5}$ of $4\frac{1}{8}$ of $1\frac{3}{8}$. (6) $4\frac{2}{3}$ of $\frac{2}{15}$ of $2\frac{1}{8}$ of $1\frac{2}{7}$. (7) $6\frac{1}{9}$ of $\frac{9}{11}$ of $8\frac{1}{2}$ of $1\frac{2}{5}$ of $5\frac{5}{11}$ of $2\frac{1}{3}$ of $\frac{8}{17}$. (8) $5\frac{3}{5}$ of $\frac{15}{28}$ of $\frac{7}{8}$ of $2\frac{1}{3}$ of 9.

iv. Division.

48. We can take away 5 shillings twice from half-a-sovereign; in other words, $\frac{1}{2} \div \frac{1}{4} = 2$; and 12 fourpenny-pieces are contained twice in 8 shillings, which written in symbols would be $\frac{8}{20} \div \frac{12}{60} = 2$; but $\frac{1}{2} \times \frac{4}{1} = 2$ $\therefore$ $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1}$; and $\frac{8}{20} \times \frac{60}{12} = 2$ $\therefore$ $\frac{8}{20} \div \frac{12}{60} = \frac{8}{20} \times \frac{60}{12}$.

This perhaps will appear more clear from the observation in Art. 19.

To prove that $\frac{8}{9} \div \frac{7}{16} = \frac{8}{9} \times \frac{16}{7}$:

$\frac{8}{9}$ is the dividend, $\frac{7}{16}$ is the divisor, and $\frac{8}{9} \times \frac{16}{7}$ is the quotient $\therefore$ by Art. 19 $\frac{8}{9} = \frac{8}{9} \times \frac{16}{7} \times \frac{7}{16} = \frac{8}{9}$, by rules of multiplication;

or generally
$$\frac{a}{b} = \frac{a}{b} \times \frac{d}{c} \times \frac{c}{d}$$
$$\therefore \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

49. Hence rule for division of fractions:

Invert the divisor and proceed as in multiplication.

Exercise 18.

Value of—

(1) $\frac{3}{7} \div \frac{9}{5}$. (2) $\frac{3}{4} \div \frac{16}{12}$. (3) $\frac{15}{28} \div \frac{11}{9}$. (4) $\frac{42}{65} \div \frac{56}{39}$. (5) $\frac{339}{978} \div \frac{2513}{6846}$. (6) $23\frac{1}{3} \div 11\frac{2}{5}$. (7) $\frac{3}{5} \div 1\frac{4}{5}$. (8) $\frac{4}{7}$ of $\frac{5}{9}$ of $18 \div \frac{1}{3}$ of $\frac{10}{11}$ of $2\frac{2}{7}$. (9) $\frac{3}{5}$ of $\frac{8}{9} + \frac{6}{7}$ of $\frac{3}{4}$. (10) $\frac{1}{7}$ of $2\frac{1}{3}$ of $6\frac{7}{8} \div 4\frac{2}{3}$. (11) $\frac{2}{11}$ of $27\frac{1}{2} \div \frac{3}{10}$ of $21\frac{1}{4}$.

v. Complex Fractions.

50. We have now investigated the ordinary rules of fractions: all we have left is to consider some special cases, and the methods of dealing with them.

Ex. 1. Add together $2\frac{1}{2} + 3\frac{3}{4} + 4\frac{7}{8} + 5\frac{3}{10} + 4\frac{5}{12}$.

If we make one pound the unit, these fractions represent £2. 10*s*., £3. 15*s*., £4. 17*s*. 6*d*., £5. 6*s*. 0*d*., £4. 8*s*. 4*d*., and their sum is required.

£	*s*.	*d*.
2	10	0
3	15	0
4	17	6
5	6	0
4	8	4
£20	16	10

= £20 and 202 pence, or $20\frac{101}{120}$.

51. This suggests the method of adding mixed numbers.

Find the sum of the whole numbers: in this case 18; add all the fractions together. Now the L.C.M. is 120.

$$\begin{array}{ccccc} 2\underline{)120} & 4\underline{)120} & 8\underline{)120} & 10\underline{)120} & 12\underline{)120} \\ 60 & 30 & 15 & 12 & 10 \\ & \underline{3} & \underline{7} & \underline{3} & \underline{5} \\ & 90 & 105 & 36 & 50 \end{array}$$

and $60 + 90 + 105 + 36 + 50 = 341$ ∴ sum of fractions $= \frac{341}{120} = 2\frac{101}{120}$. Hence the whole sum $= 18 + 2\frac{101}{120} = 20\frac{101}{120}$, which agrees with result above.

Ex. 2. Find the difference between $8\frac{4}{5}$ and $12\frac{5}{8}$:

$12\frac{5}{8} =$ £12. 12*s.* 6*d*, $8\frac{4}{5} =$ £8. 16*s.*, if one pound is the unit, and the difference between £12. 12*s.* 6*d.*, and £8. 16*s.* is £3. 16*s.* 6*d.*, or $3\frac{33}{40}$.

52. This suggests the method of subtraction of 2 mixed numbers.

Express both the fractional parts with a common denominator, subtract the whole numbers, and if the fractional part of the minuend be less than that of the subtrahend, reduce the whole numbers by unity, which must be added to the minuend.

Thus:

$$12\frac{5}{8} - 8\frac{4}{5} = 4 + \frac{25}{40} - \frac{32}{40} = 3 + \frac{65 - 32}{40} = 3\frac{33}{40}, \text{ as before.}$$

53. A *complex* fraction is one of which the numerator or denominator is a fraction or a mixed number; as $\frac{5\frac{1}{2}}{7\frac{3}{4}}$, $\frac{\frac{5}{9}}{\frac{12}{16}}$, $\frac{\frac{3}{4}}{7}$, and are simplified by division. These may be written:—

$$5\tfrac{1}{2} \div 7\tfrac{3}{4} = \tfrac{11}{2} \times \tfrac{4}{31} = \tfrac{22}{31}$$
$$\tfrac{5}{9} \div \tfrac{12}{16} = \tfrac{5}{9} \times \tfrac{16}{12} = \tfrac{80}{108} = \tfrac{20}{27}$$
$$\tfrac{3}{4} \div 7 = \tfrac{3}{4} \times \tfrac{1}{7} = \tfrac{3}{28}$$

Exercise 19.

Express in one line and simplify—

(1) $\frac{\frac{1}{2}}{3 \times \frac{3}{4}}$. (2) $\frac{9\frac{7}{9}}{4\frac{4}{5}}$. (3) $2\frac{1}{11} \times 2\frac{12}{23}$. (4) $8\frac{3}{4} \div 1\frac{5}{9}$. (5) $\frac{15\frac{3}{5}}{23\frac{2}{5}}$.

(6) $\frac{3\frac{3}{7} \text{ of } 2\frac{11}{12} \times 11}{\frac{2}{33} \text{ of } 8\frac{9}{14}}$. (7) $\frac{4\frac{1}{2} \text{ of } 48 \text{ of } \frac{5}{27}}{5 \text{ of } 5\frac{3}{5} \text{ of } 1\frac{3}{7}}$. (8) $\frac{99}{1016} \div \frac{\frac{11}{12}}{7\frac{1}{16}}$.

(9) $\frac{12}{4\frac{1}{5}} \times \frac{\frac{4}{5}}{\frac{2}{3}} \times \frac{7}{24}$. (10) $\frac{3\frac{1}{3} \text{ of } \frac{4}{7} \text{ of } 2\frac{2}{5}}{1\frac{5}{7} \text{ of } 1\frac{1}{2} \text{ of } 1\frac{7}{9}}$. (11) $\frac{3\frac{1}{5} \text{ of } 2 \text{ of } 6\frac{1}{4}}{16 \text{ of } 3 \div 1\frac{1}{5}}$.

(12) $\frac{5}{4}$ of $2\frac{13}{25} \div \frac{1}{1\frac{1}{5}} \times \frac{5}{9} \div 1\frac{1}{3}$.

(13) $\frac{\frac{2}{3} \text{ of } \frac{1}{3} \div \frac{5}{7} \text{ of } \frac{3}{5}}{\frac{2}{7} \div \frac{7}{8}} \times \frac{5}{14}$ of $5\frac{1}{4}$ of $\frac{185}{216} \div 1\frac{10}{27}$.

(14) $\frac{\frac{5}{8} \text{ of } \frac{3}{7} \text{ of } \frac{1}{2} \text{ of } \frac{14}{15}}{1\frac{1}{8} \text{ of } 1\frac{2}{3} \text{ of } \frac{3}{4} \text{ of } \frac{5}{9} \text{ of } 12}$.

54. A fraction like $\frac{1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}}{1+\frac{1}{2}-\frac{1}{3}-\frac{1}{4}}$ is most easily simplified by finding the L.C.M. of all the denominators involved in the expression, and multiplying each term of the numerator and denominator by it; thus L.C.M. of 2, 3, 4 is 12; multiplying each term, then, by 12, it may be written—

$$\frac{12-6+4-3}{12+6-4-3}=\frac{7}{11}.$$

Exercise 20.

Simplify, as above, the following:—

(1) $\frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}-2\frac{1}{7}}$. (2) $\frac{\frac{2}{3}+\frac{5}{7}}{\frac{5}{6}+\frac{3}{7}}$. (3) $\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}{\frac{1}{2}-\frac{4}{9}+\frac{5}{12}-\frac{7}{16}}$. (4) $\frac{2\frac{1}{4}-1\frac{3}{5}}{8\frac{1}{2}+1\frac{9}{10}}$.

(5) $\frac{4}{5+\frac{6}{7}}$. (6) $\frac{1}{2+\frac{1}{2}}$. (7) $\frac{5}{6+\frac{7}{8}}$. (8) $\frac{2\frac{1}{2}+\frac{1}{6}}{2\frac{2}{3}-\frac{1}{8}}$. (9) $\frac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{4}+\frac{5}{6}}$.

55. A fraction written in the form—

$$\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8+\frac{9}{10}}}}}$$

is called a continued fraction, and the method of simplifying such a fraction is this: begin from the bottom—

$$\frac{7}{8+\frac{9}{10}}=\frac{70}{80+9}=\frac{70}{89};$$

we may now write it—

$$\cfrac{1}{2+\cfrac{3}{4+\cfrac{5}{6+\frac{70}{89}}}}$$

repeat the process, and it becomes—

$$\cfrac{1}{2+\cfrac{3}{4+\frac{445}{604}}}=\frac{1}{2+\frac{1812}{2861}}=\frac{2861}{7534}.$$

Exercise 21.

(1) $\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{4}}}}$ (2) $\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{5+\frac{5}{6}}}}$ (3) $\cfrac{3}{4+\cfrac{5}{6+\cfrac{7}{8+\frac{9}{10}}}}$

(4) $\cfrac{4}{5+\cfrac{6}{7+\cfrac{8}{9+\frac{10}{11}}}}$ (5) $\cfrac{2}{2+\cfrac{3}{6+\cfrac{4}{12+\frac{5}{13}}}}$ (6) $\cfrac{6}{9+\cfrac{9}{12+\cfrac{12}{15+\frac{15}{18}}}}$

(7) $\cfrac{12}{16+\cfrac{20}{24+\cfrac{28}{32+\frac{36}{40}}}}$ (8) $\cfrac{8}{10+\cfrac{12}{14+\cfrac{16}{18+\frac{20}{22}}}}$

56. In some questions brackets are introduced; the solution of such questions must be begun by finding the value of any expression within the brackets, beginning at the innermost one first, and substituting this value for the expression.

Ex. 1.—Thus $\{(7+5)\div(4-2)\}\div16$ may be simplified in this way:—

$$(7+5)=12,\ (4-2)=2$$
$$\therefore\ (7+5)\div(4-2)=12\div2=6$$
$$\therefore\ \{(7+5)\div(4-2)\}\div16=6\div16=\tfrac{6}{16}=\tfrac{3}{8}$$

Ex. 2.—Again, $(2\tfrac{1}{2}-\tfrac{1}{3})$ of $(3\tfrac{1}{2}-\tfrac{1}{4})$.

$$2\tfrac{1}{2}-\tfrac{1}{3}=2\tfrac{1}{6}=\tfrac{13}{6}.\quad 3\tfrac{1}{2}-\tfrac{1}{4}=3\tfrac{1}{4}=\tfrac{13}{4}.$$
$$\therefore\ (2\tfrac{1}{2}-\tfrac{1}{3})\text{ of }(3\tfrac{1}{2}-\tfrac{1}{4})=\tfrac{13}{6}\text{ of }\tfrac{13}{4}=\tfrac{169}{24}=7\tfrac{1}{24}$$

Ex. 3.—$1\div\{2+3\div[4+5\div(6+\tfrac{7}{8})]\}$—another way of stating Ex. 21:—

$$6+\tfrac{7}{8}=\tfrac{55}{8}\quad\therefore\ 5\div(6+\tfrac{7}{8})=5\times\tfrac{8}{55}=\tfrac{8}{11}$$
$$\therefore\ [4+5\div(6+\tfrac{7}{8})]=4+\tfrac{8}{11}=\tfrac{52}{11}$$
$$\therefore\ 2+3\div[4+5\div(6+\tfrac{7}{8})]=2+3\div\tfrac{52}{11}=2+\tfrac{33}{52}=2\tfrac{33}{52}=\tfrac{137}{52}$$
$$\therefore\ 1\div\{2+3\div[4+5\div(6+\tfrac{7}{8})]\}=1\div\tfrac{137}{52}=\tfrac{52}{137}$$

57. The beginner will be saved much confusion, and the most complicated fractional exercise be solved without difficulty, by closely attending to these directions :—The different fractions are separated by signs +, or −; and there must be always one more fraction than the number of these signs; of course a fraction which does not contain them is simplified in one line (Ex. 19). Write down, one under the other, in separate lines, the different fractions; reduce each to its simplest form; then write these results as they occur in the expression separated by the necessary signs.

Ex. 1.—Simplify $1\frac{28}{36} + (\frac{1}{6} \times \frac{7}{12}) - \frac{1}{2}$ of $(\frac{3}{4} \div \frac{2}{3})$.

Here there are 2 signs +, −; ∴ there are 3 fractions which, written in separate lines, are—

$$1\frac{28}{36} = 1\frac{28}{36}$$
$$\frac{1}{6} \times \frac{7}{12} = \frac{7}{72}$$
$$\frac{1}{2} \text{ of } (\frac{3}{4} \div \frac{2}{3}) = \frac{1}{2} \times \frac{3}{4} \times \frac{3}{2} = \frac{9}{16}$$

The expression may now be written—

$$1\frac{28}{36} + \frac{7}{72} - \frac{9}{16} \qquad \text{L.C.M. } 144$$
$$= +\frac{112 + 14 - 81}{144} = 1\frac{45}{144} = 1\frac{5}{16}.$$

Ex. 2.—Simplify :—

$$\frac{3}{14} \times \frac{5\frac{5}{6} - 1\frac{5}{18}}{\frac{1}{9} \text{ of } \frac{111}{2}} \div \frac{\frac{2}{7}(\frac{58}{3} + 21\frac{2}{3})}{\frac{1}{1 + \frac{3}{8}} + \frac{12}{13} \text{ of } \frac{91}{14}}$$

$$5\frac{5}{6} - 1\frac{5}{18} = 4\frac{15 - 5}{18} = 4\frac{5}{9} = \frac{41}{9}$$

$$\frac{58}{3} + 21\frac{2}{3} = \frac{58 + 65}{3} = \frac{123}{3}$$

$$\frac{1}{1 + \frac{3}{8}} = \frac{8}{11}\,;\ \frac{\overset{6}{\cancel{12}}}{\cancel{13}} \text{ of } \frac{\overset{7}{\cancel{91}}}{\underset{7}{\cancel{14}}} = 6$$

$$\therefore \frac{1}{1 + \frac{3}{8}} + \frac{12}{13} \text{ of } \frac{91}{14} = 6\frac{8}{11} = \frac{74}{11}$$

∴ Original expression may be written—

$$\frac{3}{14} \times \frac{\frac{41}{9}}{\frac{1}{9} \text{ of } \frac{111}{2}} \div \frac{\frac{2}{7} \times \frac{123}{3}}{\frac{74}{11}},$$

which, as there is no sign + or −, can be solved in one line, thus :—

$$\frac{\cancel{3}}{\cancel{14}} \times \frac{\cancel{41}}{\cancel{9}} \times \frac{\cancel{9}}{1} \times \frac{\cancel{2}}{\underset{3}{\cancel{111}}} \times \frac{\overset{2}{\cancel{74}}}{11} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{\cancel{3}}{\underset{3}{\cancel{123}}} = \frac{1}{11}.$$

Miscellaneous Examples D.

Simplify (1) $14\left(\frac{1}{3}-\frac{4}{21}+\frac{1}{2}-\frac{7}{11}\right)$. (2) $\frac{1}{2}$ of $\dfrac{2+\frac{1}{2}-\frac{1}{3}}{2\frac{1}{3}-1\frac{1}{4}}$.

(3) $\frac{1}{12}\times\frac{38}{39}\times 2\frac{1}{7}\times\dfrac{2\frac{1}{4}}{4\frac{3}{4}}\div\frac{5}{14}$. (4) Add $\frac{2}{5}$, $6\frac{5}{7}$, $\frac{13}{9}$ and $\frac{8}{21}$.

(5) Subtract $3\frac{3}{4}$ from $4\frac{3}{7}$. (6) $\frac{2}{5}$ of $7\frac{1}{2}\times\frac{1}{8}\times 11\frac{1}{5}\div 4\frac{4}{5}$.

(7) $11\frac{3}{5}\times\dfrac{\frac{5}{14}}{4\frac{1}{7}}$. (8) $5\frac{2}{3}$ of $\frac{1}{15}\div\left(6\frac{1}{5}-\frac{2}{25}\right)\times\frac{81}{10}$. (9) $\dfrac{5\frac{1}{3}+\frac{15}{7}+\frac{5}{6}}{11\frac{8}{21}-3\frac{1}{14}}$.

(10) $\dfrac{8\frac{4}{5}\times 2}{3\frac{54}{55}+4\frac{9}{11}}$. (11) $\left(\frac{1}{9}\text{ of }1\frac{1}{4}\text{ of }4\frac{1}{2}\right)\div\left(\frac{5}{56}\text{ of }1\frac{1}{3}\text{ of }3\frac{1}{2}\right)-\left(3\frac{1}{4}+4\frac{1}{3}\right)$
$\div\left(6\frac{1}{2}+1\frac{1}{12}\right)$. (12) $\frac{11}{5}$ of $\left(2\frac{1}{2}+3\frac{1}{3}-4\frac{1}{4}\right)\div\left(5\frac{1}{5}+7\frac{1}{12}\right)$.

(13) $\dfrac{4\frac{2}{9}\times 6\frac{3}{7}}{38}\div\dfrac{2\frac{1}{2}}{7}$. (14) $\frac{1}{3}+\frac{1}{5}-\frac{6}{7}+\frac{2}{35}$.

(15) $\left\{\left(4\frac{1}{7}-2\frac{1}{4}\right)\div\left(6\frac{1}{2}+2\frac{1}{7}\right)\right\}\div\left\{\left(\frac{3}{7}+\frac{1}{2}\right)\div\left(\frac{5}{7}-\frac{1}{2}\right)\right\}$.

(16) $\dfrac{2\frac{5}{7}}{5}-10\frac{5}{6}\div 1\frac{12}{13}+\frac{3}{7}$ of $3\frac{3}{4}$ of $3\frac{4}{5}$. (17) $7\frac{3}{4}+\frac{5}{7}$ of $4\frac{2}{3}\div 2\frac{1}{2}-13\times\frac{7}{65}$.

(18) $\frac{18}{17}\left(1-\frac{64}{81}\right)+\frac{8}{11}\times\frac{1}{6}\times\left(\frac{1}{2}+\frac{5}{12}\right)+\left(1\frac{1}{2}\text{ of }9\frac{1}{3}\right)\div\left(2\frac{3}{4}\text{ of }7\frac{7}{11}\right)$.

(19) $\frac{7}{12}\times 4\frac{1}{3}-\dfrac{4\frac{1}{4}}{3}+\left(1\frac{3}{5}-\frac{4}{15}\right)\div\left(\frac{2}{3}-\frac{2}{15}\right)\times\dfrac{5\frac{3}{5}}{9\frac{1}{3}}$.

(20) $\dfrac{3\frac{1}{2}-1}{15\frac{1}{3}+5\frac{1}{2}-3\frac{1}{3}}\div\dfrac{1-\frac{3}{4}}{1+\frac{3}{4}}$. (21) $\dfrac{1\frac{4}{17}\times 6\frac{4}{5}}{3\frac{3}{5}-1\frac{2}{25}}+\frac{2}{11}$ of $\left(2\frac{2}{5}-\frac{3}{4}\right)-\dfrac{1\frac{3}{5}}{12}$.

(22) $3\frac{1}{3}\div\left\{\dfrac{1-\frac{15}{16}}{\frac{1}{5}-\frac{1}{8}}\right\}$. (23) $\dfrac{3\frac{1}{4}-2\frac{1}{3}}{3\frac{1}{6}+1\frac{1}{2}}\div 1\frac{4}{7}$.

(24) $\frac{2}{7}\times\dfrac{1-\frac{2}{7}}{2}+\frac{4}{5}\times\frac{1}{10}+\frac{3}{5}\left(\frac{1}{2}+\frac{11}{14}\right)+\frac{3}{7}\left(\frac{2}{7}+\frac{4}{5}\right)$.

(25) $1\div\left(1+\dfrac{7}{6\frac{2}{3}}\right)+7\div 13\frac{2}{3}$. (26) $\dfrac{3}{8-\dfrac{7}{2-\frac{3}{4}}}+\dfrac{5}{6-\dfrac{5}{2-\frac{5}{6}}}$

(27) $\dfrac{1\frac{3}{5}}{3+\dfrac{1}{3\frac{1}{3}}}+\dfrac{1\frac{2}{3}\text{ of }4\frac{2}{7}}{1\frac{2}{9}\text{ of }3\frac{6}{7}}$.

(28) $\dfrac{5\frac{2}{5}\text{ of }7\frac{2}{9}}{8\frac{7}{24}-3\frac{5}{12}}$. (29) $\dfrac{\frac{2}{9}\text{ of }1\frac{2}{3}\text{ of }2\frac{2}{5}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$.

(30) $\dfrac{1\frac{2}{3}+\frac{3}{5}}{\frac{2}{3}\text{ of }2\frac{1}{2}}\times\dfrac{10-7\frac{1}{2}}{3\frac{1}{3}-1\frac{1}{5}}\times 1\frac{1}{15}$. (31) $1+\dfrac{3}{\dfrac{1+3}{1+\frac{1}{2}}}$

(32) $1\frac{1}{6} \text{ of } 5\frac{1}{4} + \dfrac{4\frac{1}{2} - \frac{3}{10}}{1\frac{1}{15}} - \dfrac{6\frac{3}{4}}{\frac{1}{2} \text{ of } 2\frac{2}{3}}$.

(33) $(3\frac{1}{2} \text{ of } 5\frac{1}{6}) + \frac{2}{3} \text{ of } (6\frac{1}{4} - 1\frac{1}{2}) - 2\frac{1}{6} \div \frac{2}{3}$

(34) $\dfrac{4\frac{1}{6}}{\frac{7}{16} \text{ of } 3\frac{1}{5}} - \left(\frac{2}{7} \text{ of } \dfrac{2\frac{2}{3} + 1\frac{1}{6}}{2 - \frac{4}{25}}\right)$. (35) $\frac{2}{3} \text{ of } \dfrac{3 - 1\frac{1}{4}}{2\frac{2}{5}} + \dfrac{5\frac{1}{8}}{16\frac{2}{5}} \times \frac{1}{3}$.

(36) $\frac{1}{4} \text{ of } \{(4\frac{7}{9} \text{ of } 6\frac{2}{3}) + 7\frac{26}{27}\} \{(3\frac{2}{5} - 3\frac{1}{6}) \div (3\frac{3}{5} + 2\frac{1}{6})\}$.

(37) $2\,(\frac{29}{20} - 1\frac{19}{45}) \div \left(131 - \dfrac{943}{7\frac{1}{5}}\right) - 1\frac{1}{207} \text{ of } 5\frac{11}{13} \text{ of } 10\frac{7}{20} \text{ of } \frac{5}{19} \text{ of } \frac{1}{16}$.

(38) $\frac{6}{7} + \dfrac{7\frac{3}{4} - 6\frac{1}{2}}{2\frac{1}{3} \times 9}$. (39) $\dfrac{1\frac{2}{3} \text{ of } 1\frac{1}{4}}{3\frac{2}{5} + \dfrac{5}{1\frac{19}{41}}} + \dfrac{4}{5 + \dfrac{6}{7 + \frac{8}{9}}}$.

(40) $(4\frac{1}{5} \text{ of } 1\frac{1}{14} \text{ of } \frac{2}{11}) \times \dfrac{1\frac{5}{8} - \frac{1}{4}}{3 \times \frac{3}{8}}$. (41) $\dfrac{\frac{1}{2} + \frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4} - \frac{1}{5}} + 1\frac{13}{23}$.

(42) $\left\{\dfrac{1}{2 - \dfrac{3}{4 - \frac{5}{6}}} - \dfrac{1}{2 + \dfrac{3}{4 + \frac{5}{6}}}\right\} \times 10\frac{5}{9}$.

(43) $1\frac{1}{11} - \dfrac{1 - \frac{7}{22}}{2 - \frac{1}{3}} + \dfrac{1\frac{2}{5}}{3\frac{1}{2}} - \dfrac{5\frac{5}{8}}{6\frac{1}{4}} \text{ of } \left(\frac{1}{5} - \dfrac{\frac{1}{2} - \frac{1}{3}}{4\frac{3}{4} - 3\frac{2}{9}}\right)$.

(44) $\dfrac{1\frac{1}{2}}{4\frac{1}{3}} + \dfrac{3\frac{1}{3} - 2\frac{5}{9}}{4\frac{1}{2} + 5\frac{9}{10}} + \frac{3}{26} \text{ of } 5\frac{1}{4}$.

(45) $\frac{3}{8} \div 6 + \frac{63}{128} \text{ of } \frac{1}{5} \text{ of } 2\frac{2}{3} + \dfrac{1}{6\frac{6}{7}} \div \frac{7}{8} + \frac{27}{81}$. (46) $\frac{1}{4} \text{ of } \frac{7}{8} \text{ of } \left(\dfrac{2\frac{1}{3}}{3\frac{1}{2}} + \dfrac{4\frac{1}{2}}{5\frac{1}{4}}\right)$.

(47) $(\frac{8}{15} + \frac{7}{18} - \frac{17}{105} + \frac{1}{3}) \div (\frac{11}{30} - \frac{8}{35} + \frac{11}{15})$. (48) $\dfrac{\frac{1}{7} - \frac{1}{3}\,(\frac{1}{2} + \frac{1}{7})}{1 + 1 \div (2 - \frac{1}{4})}$.

(49) $\dfrac{7\frac{1}{7} - 3\frac{1}{8}}{6 + 4\frac{3}{4}} \div \dfrac{5\frac{1}{3} + 1\frac{1}{7}}{6\frac{1}{7} - 2\frac{1}{3}}$. (50) $\frac{1}{2} \text{ of } \frac{1}{3} \times \frac{3}{4} \text{ of } \frac{4}{3} \div (\frac{7}{6} + \frac{1}{3} \text{ of } 20)$.

(51) $\dfrac{3\frac{1}{5} - 2\frac{1}{3}}{5\frac{2}{7} + 1\frac{1}{9}}$. (52) $\dfrac{1\frac{1}{8}}{1\frac{3}{14}} + \{(\frac{3}{17} \times 1\frac{1}{2}) - 18\frac{8}{9}\}$.

(53) $(\frac{1}{3} + \frac{2}{11} + \frac{7}{24} - \frac{1}{3} \text{ of } \frac{2}{11} \text{ of } \frac{7}{24}) \div (1 - \frac{1}{3} \text{ of } \frac{2}{11} - \frac{2}{11} \text{ of } \frac{7}{24} - \frac{7}{24} \text{ of } \frac{1}{3})$.

(54) $\dfrac{1\frac{1}{2}}{8\frac{1}{2}} \text{ of } \dfrac{1\frac{1}{2} + 1\frac{1}{3}}{2\frac{3}{8} - 1\frac{1}{6}} + \dfrac{2\frac{1}{5} + 4\frac{1}{3}}{1\frac{1}{5} - \frac{1}{9}} \text{ of } \frac{1}{9}$. (55) $\dfrac{2\frac{2}{3} - 1\frac{1}{2}}{2\frac{2}{3} + 1\frac{1}{2}} + \dfrac{2\frac{3}{5} \text{ of } 9}{2\frac{4}{11} \text{ of } 2\frac{3}{4} \text{ of } 5}$.

(56) $\dfrac{3\frac{1}{3} - 1\frac{1}{2} \text{ of } 1\frac{1}{5} - 1\frac{1}{7}}{(3\frac{1}{3} - 1\frac{1}{2}) \text{ of } (1\frac{1}{5} - 1\frac{1}{7})} \times \dfrac{3\frac{1}{3} - 1\frac{1}{2} \text{ of } (1\frac{1}{5} - 1\frac{1}{7})}{(3\frac{1}{3} - 1\frac{1}{2}) \text{ of } 1\frac{1}{5} - 1\frac{1}{7}}$.

(57) $(3\frac{1}{4} \text{ of } 4\frac{1}{3}) \div \{(2\frac{1}{2} - \frac{1}{3}) \text{ of } (3\frac{1}{2} - \frac{1}{4})\}$.

(58) $\dfrac{4 - \frac{5}{9} \text{ of } 4\frac{2}{13} + 6 \div 4\frac{2}{3}}{18\frac{20}{91} \div 8 - \frac{2}{3} \text{ of } 1\frac{6}{7}}$. (59) $\dfrac{2\frac{69}{88} - \frac{5}{13} \text{ of } 4\frac{8}{11} + 3\frac{11}{12} \div 7\frac{8}{9}}{3\frac{3}{11} \text{ of } 10\frac{1}{12} - 3\frac{7}{21} \div 4\frac{4}{9}}$.

(60) $\dfrac{\frac{2}{3} \text{ of } 5\frac{4}{7} - 3\frac{2}{5} \div 1\frac{2}{15} + 10\frac{5}{7}}{3\frac{3}{7} - \frac{2}{9} \text{ of } 6\frac{3}{7} - \frac{18}{19}}$. (61) $\dfrac{5\frac{3}{4} - \frac{3}{7} \text{ of } 15\frac{3}{4} + 2\frac{2}{35} \div 1\frac{11}{25}}{\frac{3}{4} \text{ of } 7\frac{3}{7} - 5\frac{3}{5} \div 3\frac{4}{15}}$.

X. DECIMAL FRACTIONS.

58. A fraction of which the denominator is some power of 10 is called a decimal fraction, or simply a decimal. Thus $\frac{7}{10}$, $\frac{89}{100}$, $\frac{17}{1000}$, are decimal fractions.

59. A shorter and simpler method of representing such fractions will appear from the following considerations. In the number 7777 we know that each digit 7 diminishes in value one-tenth as it approaches the units place, the 1st 7 denoting 7000, the 2nd 7 denoting 700, the 3rd 7 denoting 70, and the last 7, 7 units, or in other words, when we remove a digit from left to right we divide it by 10. If now we put a ·, called a decimal point, after the units place, we may continue to write other digits as 7777·7777, to which we must consistently attach the values $\frac{7}{10}$, $\frac{7}{100}$, $\frac{7}{1000}$, $\frac{7}{10000}$, respectively, read seven-tenths, seven hundredths, seven thousandths, seven tens of thousandths. Conversely the above fractions represented singly in this notation would appear as ·7, ·07, ·007, ·0007. In the same way 1234 represents 1000 + 200 + 30 + 4, and ·1234 represents $\frac{1}{10} + \frac{2}{100} + \frac{3}{1000} + \frac{4}{10000} = \frac{1234}{10000}$.

60. The digits to the left of the point are integers, the digits to the right are fractions, and as zeros *prefixed* to an integer do not alter its value, so zeros *annexed* to a decimal do not alter its value. This last statement is easily proved :

$$\cdot 7 = \tfrac{7}{10}\,;\ \cdot 70 = \tfrac{70}{100} = \tfrac{7}{10}\,;\ \cdot 700 = \tfrac{700}{1000} = \tfrac{7}{10}\ \therefore\ \cdot 7 = \cdot 70 = \cdot 700.$$

61. A mixed number, partly integral, partly fractional, can be represented as an improper fraction thus: $7{\cdot}8 = 7\frac{8}{10} = \frac{78}{10}$; $314{\cdot}07 = 314\frac{7}{100} = \frac{31407}{100}$, the numerator of which consists of the number made up by writing all the digits in their proper order, the denominator being the product of as many factors of ten as there are digits after the point.

62. The *order* of a decimal is determined by the number of digits after the point, and from the above remarks it is clear it may be increased by annexing zeros to its digits. Thus ·007, a decimal of the 3rd order, appears in the 5th order, written ·00700, without changing its value.

Exercise 22.

Write as vulgar fractions in their lowest terms, stating the *order* of each :—

(1) ·1, ·01, 1·01, 10·1, 101·01. (2) ·2, ·02, 2·02, 20·2, 202·02.
(3) ·3, ·03, ·0003. (4) ·5, ·05. 5·5, 5·05, 50·5, 500·05.
(5) ·6, ·06, 6·06, ·0006, 600·060. (6) ·8, ·08, ·0008.

And as decimals :

(7) $\frac{1}{10}$, $\frac{12}{100}$, $\frac{123}{1000}$. (8) $15\frac{7}{10^3}$, $\frac{157}{10^4}$, $\frac{1570}{10^6}$. (9) $\frac{123456789}{10^5}$.

i. Addition.

63. If U. represents units, T. tens, H. hundreds, TH. thousands, XTH. tens of thousands, CTH. hundreds of thousands, etc. ; and t = tenths, h = hundreds, th = thousands, etc., we may, as in Article 2, proceed at once to obtain the result.

XTH.	TH.	H.	T.	U.	t.	h.	th.	xth.
	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	9
			1	2	3	4	5	
		1	2	3	4	5	6	
1	3	7	1	6	0	4	7	7

Ex. Find value of 1234·5678 + 12345·6789 + 12·345 + 123·456.

Place the digits of each number in its proper place in the above columns, add them up as in Art. 11, and the sum is 13716·0477.

After a time this tabular form can be dispensed with ; it will help, however, to extend the idea of numeration, and will clear up any misleading notion about the 'position of the point,' which must be kept in a vertical line.

Placing zeros in the above blank spaces, as we already know from Art. 60, will not affect the result.

We might have obtained the result likewise by first expressing each term of the addenda as vulgar fractions, summing them, and writing the answer as a decimal. The following examples should be worked in *both* these ways :—

Exercise 23.

Add together :—

(1) 21·3706, 1·89426, 57, ·0012.
(2) 15·243, 13·8465, 6·8, 5·8281 12·4764, 83·072.

(3) 1·8954, 671·89, 1200, 5, 35·14732
(4) 2·368, 41·289, ·268, 4·23, 71·651.
(5) ·0012, 2·26823, ·001, 5·6125. (6) ·15625, 2·465, ·00192, 52·76.
(7) 5·68912, 20·756, 19·52, ·847, 159·2.
(8) 39·43, 27·62815, ·8469, ·00176.

ii. Subtraction.

64. To find the difference between two decimals, arrange them in a tabular form as in Art. 63; then proceed as in simple subtraction.

Ex. 1. Thus find the difference between 34·2869 and 8·4389:

T.	U.	t.	h.	th.	xth.
3	4	2	8	6	9
	8	4	3	8	9
2	5	8	4	8	0

Ans. = 25·848.

Ex. 2. Find the difference between 2 and 1·9876.

T.	U.	t.	h.	th.	xth.
	2	0	0	0	0
	1	9	8	7	6
	0	0	1	2	4

Ans. = ·0124.

Both these examples might have been done by writing the fractions as vulgar fractions, then subtracting, and expressing the remainders as decimals.

Exercise 24.

Find the value of the following (employing both methods):—

(1) 12·3706 − 9·243. (2) 15·243 − 1·8954.
(3) 1·8954 − ·026891. (4) 29·74 − 5·328. (5) 13·8465 − 1·89426.
(6) 671·89 − 5·68912. (7) 20·756 − 19·52. (8) 57 − ·0057.
(9) 1200 − 159·2. (10) 10 − ·0012.

The next examples are a combination of these two rules:

Exercise 25.

Simplify by summing the + quantities, then the − quantities, and subtracting the results:

(1) 24·271 − 3·6485 + 15·271 − 13·256 − 14·125 + ·4875 − 9.

(**2**) 52 + ·52 – 17·8946 – 30·254 – ·5 + 21·12 + ·0086 – 24·9999.

(**3**) 2·465 + 1·21 + (3·2 – 2·89) + 2·465 + 1·21 – (3·2 – 2·89).

(**4**) 709·63 – ·8514 + 1234·057 – 18·5 + 72·065 – 19·7234 + 81·963 – 1·7.

(**5**) 2107·5462 – 17·19382 + 18 – ·18 + 150·7 – 1·507 + 216·9 – ·83472.

(**6**) 1201·6 – 43·598 + 47·106 – 8·271 + 1 – ·001 + 827·43 – 97·6387.

(**7**) 2123·5 – ·0078 + 62·97 – ·63845 + 2172·81 – 31·629 + 41·78 – ·29643.

(**8**) 56·429 + ·799 – 5·218 – ·8 + 9·005 – 7·462 + 6·047 – 5·9863.

(**9**) 850·007 – ·000985 + ·00537 – 270·8796 + ·09999 – ·101 + 10 – ·0002.

(**10**) Add together ·130055, 900, 57·1, 13·34, ·0000397, and take the result from 1314·9.

iii. Multiplication.

65. If we multiply 678 by 3, we can obtain the result by the addition of 678, 3 times, the sum being 2034.

TH.	H.	T.	U.
	6	7	8
			3
2	0	3	4

so 6·78 × 3 = 6·78 + 6·78 + 6·78 = 20·34; or represented as a sum in multiplication:

H.	T.	U.	t.	h.	
		6	7	8	
		3			
	2	0	3	4	*Ans.* = 20·34.

Again—678 × 30 = 20340.

XTH.	TH.	H.	T.	U.
		6	7	8
			3	0
2	0	3	4	0

and 6·78 × 30 = 203·4.

H.	T.	U.	t.	h.	
		6	7	8	
	3	0			
2	0	3	4		*Ans.* = 203·4.

From these two last examples it is clear that when we multiply by a digit in the 'tens place,' we multiply as if the digit were in the units place, and shift all the digits of the product one place to the left. So the product of multiplicand and a digit in the H. column would be shifted two places to the left, and so on. Whence, when we set down the product of the multiplicand by a digit in the *t.* column, we must shift it one place to the *right*, if by a digit in the *h.* column, two places to the *right*, and ∴ 6·78 × ·3 = 2·034 would be represented thus :

U.	t.	h.	th.
6	7	8	
	3		
2	0	3	4

Ans. = 2·034.

From these remarks the arrangement of the figures in the sum below will be understood.

Find value of 7·64 × 8·9 :

T.	U.	t.	h.	th.
	7	6	4	
	8	9		
	6	8	7	6
6	1	1	2	
6	7	9	9	6

Ans. = 67·996.

It would have been more 'consistent,' perhaps, to have multiplied first, as we have been in the habit of doing by the unit figure in the multiplier. Thus :

T.	U.	t.	h.	th.
	7	6	4	
	8	9		
6	1	1	2	
	6	8	7	6
6	7	9	9	6

We will work one more example : ·476 × ·0137.

Place zeros where necessary :

U.	t.	h.	th.	xth.	cth.	m.	km.
	4	7	6				
	0	1	3	7			
	0	0	4	7	6		
			1	4	2	8	
				3	3	3	2
	0	0	6	5	2	1	2

Ans. = ·0065212.

From these examples, the reason for this rule of multiplication will be evident.

Rule :—

Multiply the factors together, disregarding the points, and make the number of places in the product equivalent to the sum of those in the factors.

The truth of this rule might also have been deduced by changing the decimals into vulgar fractions, and then multiplying, expressing the product as a decimal. Thus: ·3 × ·07 × ·005 =

$$\frac{3}{10} \times \frac{7}{10^2} \times \frac{5}{10^3} = \frac{105}{10^6} = \cdot 000105.$$

The former method seemed preferable, as extending the idea of numeration. The accuracy of the work may be tested by 'casting out nines.'

Exercise 26.

Find the value of :

(1) 6·14 × 4 ; 4·28 × 8 ; 8·742 × 6 ; 66·45 × 9 ; 41·633 × 15 (factors) ; 6·28 × 84.

(2) 48·21 × 33 ; 94·538 × 42 ; 7·812 × 36 ; 51·7358 × 64 ; 6·24 × 86 ; 5·0829 × 35.

(3) 6·28 × 53 ; 14·54 × 87 ; 53·21 × 65 ; 52·762 × 19 ; 4·0684 × 23 ; 15·36 × 41.

(4) 4·756 × 6·24 ; 9·2 × 94·385 ; 5·5 × 6·832 ; 12·5 × 3·42 ; 567·25 × 1·28 ; 96·42 × 11·5.

(5) ·023 × 32091 ; 9·6 × ·001875 ; 25 × ·000125 ; ·0112 × ·03965 ; ·0032 × ·0276 ; ·0025 × 813·4.

(6) ·0036 × 22000 ; 14 × 34·3168 ; ·125 × 30 ; ·6849 × 2·4 ; 290 × 284·7432.

(7) 3·653 × 12·85 ; 23115 × ·007456 ; 380500 × ·0023 ; ·51 × 72·5 ; ·006 × 30.

(8) 245·7 × 6·3 ; ·18349 × 62·5 ; ·013 × 220 ; 79·25 × 26153·4 ; 1·3 × 11·7.

(9) 6·5 × 267 ; ·5625 × 51472 ; ·0368 × 21·4 ; ·0028 × 2040000 ; ·023 × 32091.

(10) ·003125 × 34·3168; 647 × ·0104; 19·88 × ·0072; ·001405 × 2 400·07 × ·57.

(11) Find the continued product of 3·06, 23·4, ·5605, and divide it by 2·21 × 32·4 × ·1121.

(12) 9·876 × ·432 × 8000 divided by the continued product of ·823, 86·4.

(13) Prove that 4·2 raised to the 5th power $= (\cdot 7)^3 \times (6)^2 \times (7)^5 \times (\cdot 6)^3 \times 10$.

(14) Prove that $(3\cdot 5)^3 + (1\cdot 4)^3 - (4\cdot 9)^3 + 3 \times 4\cdot 9 \times 1\cdot 4 \times 3\cdot 5$
$= (1\cdot 5)^3 + (1\cdot 2)^3 - (2\cdot 7)^3 + 3 \times 1\cdot 5 \times 1\cdot 2 \times 2\cdot 7$.

(15) Prove that $(\cdot 2)^2 - (\cdot 3)^2 = (\cdot 2 + \cdot 3)\ (\cdot 2 - \cdot 3)$;
$(\cdot 3)^3 - (\cdot 2)^3 = (\cdot 3 - \cdot 2)\{(\cdot 3)^2 + \cdot 3 \times \cdot 2 + (\cdot 2)^2\}$;
$(\cdot 4)^3 + (\cdot 3)^3 = (\cdot 4 + \cdot 3)\{(\cdot 4)^2 - \cdot 4 \times \cdot 3 + (\cdot 3)^2\}$;
$(\cdot 5)^5 + (\cdot 6)^5 = (\cdot 5 + \cdot 6)\ \{(\cdot 5)^4 + (\cdot 5)^2 \times (\cdot 6)^2 - (\cdot 5)^3 \times (\cdot 6) - (\cdot 6)^3 \times (\cdot 5) + (\cdot 6)^4\}$.

iv. Division.

66. If we have to divide 244·608 by 7, 56, 168 we could proceed thus :—

		H.	T.	U.	t.	h.	th.	
	7)	2	4	4	6	0	8	
Divide this by 8	8)		3	4	9	4	4	∴ 244·608 ÷ 7 = 34·944
" " 3	3)			4	3	6	8	∴ 244·608 ÷ 56 = 4·368
" " 7	7)			1	4	5	6	∴ 244·608 ÷ 168 = 1·456
	8)				2	0	8	∴ 244·608 ÷ 1176 = ·208
					0	2	6	∴ 244·608 ÷ 9408 = ·026

From these equations we infer that if the divisor is an integer the quotient is of the same order as the dividend.

Exercise 27.

Value of :

(1) 173·056 ÷ 416 ; ·855 ÷ 57 ; 575·19 ÷ 693 ; 506·016 ÷ 753 ; 7704·256 ÷ 928 ; 174·096 ÷ 372.

(2) 53·720 ÷ 85 ; 2845·32 ÷ 786 ; 25248·8 ÷ 6824 ; 49·088 ÷ 59 ; 56·7009 ÷ 753 ; 5·780 + 85.

(3) 7·225 ÷ 85 ; 1·54596 ÷ 5946 ; 2918·992 ÷ 482 ; 1029·0321 ÷ 423 ; 11·28 ÷ 48 ; 1893·26 ÷ 523.

(4) 4789·875 ÷ 75 ; 7354·744 ÷ 5276 ; 18·2226 ÷ 502 ; 1030·005 ÷ 423 ; 24146·387 ÷ 337.

(5) 10·04 ÷ 251 ; 9·106 ÷ 58 ; 4310·4 ÷ 768 ; 6832·5 ÷ 625 ; 7·32 ÷ 16.

(6) 23·5÷1024; 50·7702÷283; 1000÷128; 332·52÷102; 663÷325.

N.B.—If necessary, borrow zeros from the dividend to complete the operation.

67. Again, we may write the division of 244608 by 9408 thus:—

	CTH.	XTH.	TH.	H.	T.	U.		T.	U.
9408)	2	4	4	6	0	8	(	2	6
	1	8	8	1	6				
		5	6	4	4	8			
		5	6	4	4	8			

And we notice that before we can begin the work, we have to employ the digits as far as the 'T. column,' ∴ the first digit in the quotient must be in the T. column too, ∴ the quotient must be 26.

68. This fact suggests the conclusion that

The first digit in the quotient must be placed in the same column as that in which stands the last digit of the dividend employed to commence the operation, and so for the others.

Bearing this in mind let us now proceed to divide 244·608 by 9408.

	H.	T.	U.	t.	h.	th.		h.	th.
9408)	2	4	4	6	0	8	(	2	6
	1	8	8	1	6				
		5	6	4	4	8			
		5	6	4	4	8			

The quotient is 2h. | 6th., or, written according to our notation, ·026.

69. We have already remarked that a digit increases or diminishes in value tenfold as it is shifted to the left or to the right; therefore when we move the 'point' one, two, three places left or right we practically divide the number, or multiply it by 10, 10^2, 10^3, etc. But we may multiply or divide the dividend and divisor by the same factor without affecting the quotient, ∴ we may move the 'point' either way in dividend and divisor without affecting the quotient. Hence the rule for division if the divisor be decimal:—

Move the point in the divisor to the end of its last digit, thus making it a whole number, then move the point in the dividend the same number of places. Next arrange the dividend in tabular form as in Art. 67, and proceed as there directed.

Exercise 28.

Value of:

(1) 36÷·012; 3·6÷12; 3·6÷1·2; ·4206969÷·0123.

(2) 84·375÷·00375; ·0000128÷·0016; ·251÷20·08.

(3) 21·97÷1·69, and by ·169, and by 1690 respectively.

(4) ·000080223÷·02057; ·144÷1200; ·0020304÷·00432.

(5) 4.8÷·0016; ·0169÷013; 49·7÷·0025; 00003÷·001.

(6) 106·5918÷3·714; 68·8085÷1·5; ·060248÷27·2.

(7) 17·25÷·0023; ·01653377÷·00029; 30·5118÷50·6.

(8) ·0040804÷2·02; 1215013·8÷2·023; ·000072072÷·00012.

(9) 3·12÷325; 312÷·0325; 64÷·08; 6·4÷80; ·064÷·008.

(10) ·024÷60; 24÷·006; 2·4÷·06.

(11) 1·69÷·013; 16·9÷13; 169÷1·3.

(12) 1·44÷1·2; 144÷·6; ·144÷·016.

(13) ·045÷·0015; 4·5÷150; ·45÷·15.

(14) 5·7÷2·5; 5·7÷19; and 2·375 by 250, by 2·25, and by ·00005.

(15) 3·52 by 2·2, by ·22, and by 22; 35·2 by ·22 and by ·022.

(16) 48·6 by 1·8 and by 18; 4·86 by ·18, by 1·8, and by ·0018.

70. The above rules may be established also by changing the decimals into vulgar fractions, simplifying and expressing the result as a decimal. The method will be seen from the following examples :—

(1) $3·75 \div ·25 = \frac{375}{100} \div \frac{25}{100} = \frac{375}{100} \times \frac{100}{25} = \frac{375}{25} = 15.$

(2) $·375 \div ·25 = \frac{375}{1000} \div \frac{25}{100} = \frac{375}{1000} \times \frac{100}{25} = \frac{15}{10} = 1·5.$

(3) $315·9 \div 29·25 = \frac{3159}{10} \div 29\frac{25}{100} = \frac{\overset{\overset{27}{\cancel{351}}}{\cancel{3159}}}{10} \times \frac{\overset{4}{\cancel{100}}}{\underset{\underset{\cancel{25}}{\cancel{325}}}{\cancel{2925}}} = \frac{108}{10} = 10·8$

Example 3 justifies the addition of cyphers to the dividend to complete the working.

(4) $·0025 \div ·05 = \frac{25}{10000} \div \frac{5}{100} = \frac{25}{10000} \times \frac{100}{5} = \frac{5}{100} = ·05.$

It would be a useful exercise to prove the results obtained in Ex. 28 by this method.

The accuracy of the result may also be tested by 'casting out nines.

71. To reduce a vulgar fraction to a decimal, we proceed exactly as in Ex. 27, the number of decimal places in the result being the same as that of cyphers employed to complete the operation.

Exercise 29.

Express as decimals :—

(1) $\frac{1}{2}$, $\frac{1}{5}$, $\frac{7}{8}$, $\frac{4}{5}$, $\frac{19}{25}$. (2) $\frac{7}{16}$, $3\frac{11}{25}$, $4\frac{13}{32}$, $5\frac{47}{64}$, $6\frac{97}{625}$.

Sometimes the result is obtained more quickly

By multiplying the fraction by a factor which will change the last digit of the denominator into zero, and continuing the process until it consists of powers of 10 only.

v. Repeating Decimals.

72. The following examples should be carefully noted: Express as decimals $\frac{17}{74}$, $\frac{13}{19}$, $\frac{12}{31}$.

Ex. 1.

```
74)17 (·2297
   148
   ---
    220
    148
    ---
     720
     666
     ---
      540
      518
      ---
       22
```

Which is the same as the first remainder, consequently the figures 297 in the quotient will be repeated. We denote this fact by placing a dot · over the first and last figure of the series. ∴ result = $\cdot 2\dot{2}9\dot{7}$.

Ex. 2.

```
19)13 (·68
   114
   ---
    160
    152
    ---
      8
```

Which is half of the first remainder, ∴ the next figure of the quotient will be half of the last one, the next half of this and so on ; the quotient therefore is ·684210526315789473684, &c., or $\cdot\dot{6}8421052631578947\dot{3}$.

Ex. 3

$$
\begin{array}{l}
31\cdot)12\ (\cdot387 \\
\underline{\ 93\ } \\
\ \ 270 \\
\ \ \underline{248} \\
\ \ \ \ 220 \\
\ \ \ \ \underline{217} \\
\ \ \ \ \ \ \ 3
\end{array}
$$

Which is a quarter of the dividend, ∴ as in the above example dividing the quotient by 4, we obtain as result $\cdot\dot{3}8709677419354\dot{8}$.

73. The above are examples of *recurring*, or *circulating*, or *repeating* decimals, which are defined

as decimals in which the digits are repeated. A *pure* circulator is one in which there are no figures except the recurring period, as Ex. 2 and 3; a *mixed* circulator in which there are other figures besides the recurring period, as in Ex. 1.

Exercise 30.

Express as decimals:

(1) $8\frac{5}{11}$, $4\frac{5}{12}$, $2\frac{17}{48}$, $\frac{1}{370}$, $7\frac{25}{26}$, $9\frac{5}{108}$. (2) $\frac{8}{29}$, $1\frac{18}{23}$, $2\frac{2}{43}$.

74. Since a terminating decimal must have as denominator some power of 10, it follows that no vulgar fraction which has a denominator composed of other factors than 2 or 5 can be expressed as a terminating decimal, and must therefore be a circulating decimal. In other words,

A vulgar fraction reduced to its lowest terms will produce a circulating decimal if its denominator contains any prime factor other than 2 or 5.

And as the remainder, in any division sum, must be less than the divisor, it follows that the number of circulating figures must be less than the number expressed by the denominator. Thus $\frac{5}{7}$, $\frac{16}{19}$, $\frac{15}{31}$ cannot have more than 6, 18, 30 different figures respectively in the quotient, or the several periods cannot consist of more than 6, 18, 30 different figures.

Exercise 31.

Express as decimals:

(1) $\frac{2}{3}$, $\frac{3}{7}$, $\frac{5}{11}$, $\frac{7}{13}$, $\frac{9}{17}$, $\frac{11}{19}$, $\frac{13}{23}$, $\frac{15}{29}$, $\frac{17}{31}$, $\frac{19}{37}$, $\frac{21}{41}$, $\frac{23}{43}$, $\frac{25}{47}$, $\frac{27}{53}$.

(2) $\frac{29}{59}$, $\frac{31}{61}$, $\frac{33}{67}$, $\frac{35}{71}$, $\frac{37}{73}$, $\frac{39}{79}$, $\frac{41}{83}$, $\frac{43}{89}$, $\frac{45}{97}$, $\frac{47}{101}$, $\frac{49}{103}$.

75. To reduce a recurring decimal to a vulgar fraction :—

Let $C = \cdot\dot{0}\dot{7}$.

$\therefore\ C = \cdot07070707$, &c.

$\therefore\ 100C = 7\cdot070707$, &c.

$\therefore$ subtracting $99C = 7$

$\therefore\ C = \frac{7}{99}$.

Again, let $C = \cdot\dot{8} = \cdot888$, &c.

$\therefore\ 10C = 8\cdot888$, &c.

$\therefore\ 9C = 8$ by subtraction.

$\therefore\ C = \frac{8}{9}$.

And let $C = \cdot\dot{7}4\dot{5} = \cdot745745$, &c.,

$\therefore\ 1000C = 745\cdot745$

$\therefore\ 999C = 745$

$\therefore\ C = \frac{745}{999}$.

76. To reduce a *pure* circulator to a vulgar fraction, place all the digits of the period in the numerator, and as many nines as there are recurring figures in the denominator.

Exercise 32.

Express as vulgar fractions in their lowest terms :—

(1) $\cdot\dot{5}$; $\cdot\dot{4}2857\dot{1}$; $\cdot\dot{2}8571\dot{4}$; $1\cdot\dot{4}$; $\cdot\dot{3}$; $\cdot\dot{7}2\dot{9}$; $\cdot\dot{1}\dot{8}$; $\cdot\dot{1}5\dot{6}$; $\cdot\dot{0}08\dot{9}$ $\cdot0322580645161 2\dot{9}$.

(2) $\cdot\dot{0}\dot{5}$; $\cdot\dot{0}2\dot{7}$; $\cdot\dot{6}$; $\cdot\dot{0}7692\dot{3}$; $\cdot\dot{1}9753086\dot{4}$; $\dot{7}6190\dot{4}$; $\cdot\dot{5}\dot{4}$; $\dot{2}4\dot{7}$; $\cdot\dot{0}42\dot{3}$; $\cdot\dot{4}20\dot{3}$.

77. Again, let $C = \cdot6\dot{7} = \cdot6777$, &c.

$\therefore\ 100C = 67\cdot777$, &c.

$\therefore\ 10C = 6\cdot777$, &c.

$\therefore$ subtracting these 2 equations, $90C = 67 - 6$

$$\therefore\ C = \frac{67-6}{90} = \frac{61}{90}$$

And if $C = \cdot03\dot{0}\dot{2} = \cdot03020202$, &c.

$10000C = 302\cdot020202$, &c.

$100C = 3\cdot020202$, &c.

$\therefore\ 9900C = 302 - 3$

$$\therefore\ C = \frac{302-3}{9900} = \frac{299}{9900}.$$

Hence to reduce a *mixed* circulator to a vulgar fraction, place in the numerator the difference between the number formed by the figures up to the end of the first period and the number formed by those that precede the first period; and in the denominator place 9 as many times as there are figures in the period, affixing as many cyphers as there are figures between the decimal point and the first period. Of course the whole numbers remain unaffected.

Exercise 33.

Express as vulgar fractions in their lowest terms :—

(1) $\cdot 5\dot{6}$; $\cdot 57\dot{2}$; $\cdot 3\dot{4}\dot{5}$; $\cdot 917897\dot{7}\dot{2}$; $\cdot 012\dot{3}\dot{6}$; $\cdot 9\dot{2}8571\dot{4}$; $\cdot 044\dot{3}$; $\cdot 0\dot{5}03\dot{1}$.

(2) $\cdot 0\dot{5}$; $\cdot 5\dot{4}\dot{5}$; $\cdot 208\dot{3}\dot{7}$; $\cdot 254\dot{6}2\dot{9}$; $\cdot 0\dot{3}013\dot{5}$; $\cdot 857423\dot{6}$; $\cdot 78\dot{9}30\dot{6}$.

(3) $4\cdot 25\dot{3}$; $53\cdot 002\dot{4}\dot{3}$, $7\cdot 2\dot{1}1\dot{1}$; $2\cdot 53\dot{0}\dot{6}$; $5\cdot 12\dot{2}9\dot{7}$; $3\cdot \dot{2}8571\dot{4}$.

(4) $6\cdot 34\dot{2}5\dot{9}$; $1\cdot 1\dot{9}2307\dot{6}$; $10\cdot 3\dot{0}9523\dot{8}$; $4\cdot 10\dot{0}4273\dot{5}$.

(5) $8\cdot \dot{4}598540\dot{1}$; $7\cdot 6\dot{5}3\dot{1}$; $9\cdot 093\dot{1}\dot{8}$; $2\cdot 0\dot{9}0\dot{9}$; $5\cdot 193\dot{1}\dot{8}$; $11\cdot 2\dot{8}\dot{7}$.

78. Let us now ascertain the meaning of $\cdot\dot{9}$, $\cdot 7\dot{9}$, $\cdot 65\dot{9}$, &c.

In the 1st case let $C = \cdot\dot{9} = \cdot 99$, &c.

$\therefore\ 10C = 9\cdot 9$, &c.

$\therefore\ 9C = 9$

$\therefore\ C = 1 \qquad \therefore\ \cdot\dot{9} = 1.$

In the 2nd case, let $C = \cdot 7\dot{9} = \cdot 7999$, &c.

$\therefore\ 100C = 79\cdot 99$, &c.

$\therefore\ 10C = 7\cdot 99$, &c.

$\therefore\ 90C = 72$

$\therefore\ C = \frac{72}{90} = \cdot 8.$

In the next case let $C = \cdot 65\dot{9}$

$\therefore\ 1000C = 659\cdot 999$, &c.

$\therefore\ 100C = 65\cdot 999$, &c.

$\therefore\ 900C = 594$

$\therefore\ C = \frac{594}{900} = \frac{66}{100} = \cdot 66$

Hence it will be seen that, as in case 1,—

A series of recurring nines immediately after the decimal point is equivalent to 1; and if such a series follow some non-recurring figures, as in cases 2 and 3, the last of these may be increased by 1, and the nines omitted.

79. The addition of circulating decimals is thus effected.

Find the L.C.M of the numbers of recurring figures in each of the addenda, and after making the decimals of the same order, carry on all the decimals further as many places as the number represented by the L.C.M. Thus, add together—

$$3{\cdot}57\dot{9},\ 8{\cdot}4\dot{7}\dot{6},\ 9{\cdot}3\dot{2}1\dot{5},\ 12{\cdot}46\dot{7}89\dot{4}.$$

Here the periods consist of 1, 2, 3, 4 figures respectively, and the L.C.M. of 1, 2, 3, 4 is 12, and the greatest order being 6 we shall have in all 18 places of decimals.

$$\begin{array}{r}
3{\cdot}579999999999999999\\
8{\cdot}476767676767676767\\
9{\cdot}321521521521521521\\
12{\cdot}467894789478947894\\
\hline
33{\cdot}846183987768146181
\end{array}$$

And the answer is $33{\cdot}84\dot{6}1839877681\dot{4}$, *i.e.* we have as many figures recurring in the sum as the number represented by the L.C.M.

80. The same method should be adopted in subtraction.

Ex. 1.—Find the value of $15{\cdot}0\dot{2}\dot{5} - 13{\cdot}\dot{2}4\dot{7}$.

Here the periods consist of 2 and 3 figures respectively, and the L.C.M. of 2 and 3 is 6. And the greater order being 3 we shall have nine places of decimals.

$$\begin{array}{r}
15{\cdot}025252525252\\
13{\cdot}247247247\\
\hline
1{\cdot}778005278
\end{array}$$

and the answer is $1{\cdot}7\dot{7}8005\dot{2}$, *i.e.* we have as many figures recurring in the remainder as the number represented by the L.C.M.

81. In multiplication and division of recurring decimals,

The recurring decimals should be converted into vulgar fractions, and when the product or quotient of these fractions has been found, it should be expressed as a decimal.

Ex. 1.—Find the value of $3{\cdot}\dot{7} \times 5{\cdot}\dot{4}\dot{9}$.

$$3{\cdot}\dot{7} = 3\tfrac{7}{9},\ 5{\cdot}\dot{4}\dot{9} = 5\tfrac{49}{99}. \quad \therefore\ 3{\cdot}\dot{7} \times 5{\cdot}\dot{4}\dot{9} = 3\tfrac{7}{9} \times 5\tfrac{49}{99} = \tfrac{34}{9} \times \tfrac{544}{99} = \frac{18496}{9 \times 9 \times 11}$$

$$\begin{array}{l}
9)\underline{18496}\\
9)\underline{2055{\cdot}1}\\
11)\underline{228{\cdot}345679012345679012|34}\\
\quad 20{\cdot}758698092031425364|75
\end{array}$$

$\therefore$ **Answer** $= 20{\cdot}\dot{7}5869809203142536\dot{4}$.

Ex. 2.—$1{\cdot}08\dot{3} \div 1{\cdot}3\dot{8}$.

$$1{\cdot}08\dot{3} = 1\tfrac{75}{900},\ 1{\cdot}3\dot{8} = 1\tfrac{35}{90}.$$

$$\therefore\ 1{\cdot}08\dot{3} \div 1{\cdot}3\dot{8} = \tfrac{975}{900} \div \tfrac{125}{90} = \tfrac{975}{1250} = \tfrac{7800}{10000} = {\cdot}78.$$

$\therefore$ Answer = ·78.

vi. Contracted Operations in Decimals.

82. In some cases of multiplication and division of decimals the answer is required to contain a certain number of decimal places, when the following contracted operation may be employed. Thus, required the product of 414·793, 23·7286 exact to the 3rd place:

TH.	H.	T.	U.	t.	h.	th.	xth.
	4	1	4	7	9	3	
		2	3	7	2	8	6
1	2	4	4	3	7	9	
8	2	9	5	8	6		
	2	9	0	3	5	5	1
			8	2	9	5	8
			3	3	1	7	6
				2	4	8	4
9	8	4	2	4	5	6	

Ans. 9842·457.

Ex. 2. Multiply 8·6534 by 9·3215 so as to have 4 places in the product.

	8·	6	5	3	4	
	9·	3	2	1	5	
7	7·	8	8	0	6	
	2·	5	9	6	0	2
		1	7	3	0	6
				8	6	5
				4	3	0
8	0·	6	6	2	6	

Ans. 80·6626.

In these examples it will be noticed we arrange the numbers under one another, units under units, &c., then multiply, 1st, by the units digit in the multiplier, placing the product under the last digit of the multiplicand, then, as we proceed with the digits of the multiplier, we omit in turn the digits in the multiplicand ∵ we obtain a product of greater order than is required. It would be well in all these operations to carry on the work one decimal place more than is required.

83. When the quotient of a division is required only to be true to a certain decimal place, a similar method may be adopted, thus:

Ex. 3 7·9362÷2·7451 to four places of decimals.

```
27451)79362(28910
      54902
      -----
 2745)24460
      21961
      -----
  274) 2499
       2470
       ----
    27)  28
         27
         --
     2)   1
```

Ans. 2·8910.

Here we retain the number of places in the dividend and obtain the different divisors by omitting in turn the last digit.

84. If in performing the operation of multiplication we reject a figure larger than 5, it is generally well to add unity to the next figure retained, and in division cut off figures from the divisor, multiplying each time the figure just cut off by the new figure of the quotient to ascertain the carriage, which is to be always the nearest ten.

85. In order to obtain greater accuracy in finding the continued product of several decimals, proceed thus :—

Arrange the factors in descending order of magnitude, move the points so that all the factors except the first are proper fractions, and retain in each product one more place than the assigned number.

86. In all cases where vulgar and decimal fractions are involved reduce the vulgar to decimal fractions before proceeding with the operation.

Thus, $7\frac{3}{4} + 5\cdot45 = 7\cdot75 + 5\cdot45 = 13\cdot2$.

Miscellaneous Examples E.

(1) Reduce to a fraction in its lowest terms—

$\cdot\dot{0}5263157894736842\dot{1}$.

(2) Find value of $\dfrac{\frac{5}{64} + \cdot078125}{\frac{3}{14} + \cdot2\dot{1}4285\dot{7}} : \dfrac{\frac{3}{32} + \cdot09375}{\frac{40}{33} + 1\cdot\dot{2}\dot{1}}$.

(3) Prove that $\dfrac{\cdot3\dot{4}6153\dot{8}}{2\cdot\dot{9}1\dot{8}} = \dfrac{9}{26} \div \dfrac{108}{37}$.

(4) Simplify $(\frac{1}{25}-\cdot\dot{0}3\dot{7})+(\frac{1}{4}-\cdot 04)+\frac{1}{27}$.

(5) Prove that $\frac{275\times\cdot 13\dot{0}\dot{9}}{12}+\frac{3}{\cdot 1875}=19$.

(6) Simplify $\frac{2}{3}+\frac{5}{8}+\frac{1}{16}-(\cdot 0625\times 16-1\frac{1}{6})$.

(7) Value of $(\frac{3}{4}+\frac{7}{8}+\frac{11}{16})\div 2\cdot 3125$.

(8) Prove that $\left(\frac{1\frac{3}{4}-\frac{7}{8}\text{ of }\frac{18}{28}}{\frac{5}{6}\text{ of }\frac{12}{26}+5\frac{1}{2}}\right)^2=\frac{247}{1224}\times\cdot 20\dot{1}797385620915032\dot{6}$.

(9) Value of $\frac{\cdot\dot{5}98\dot{4}\times 909\times 1121}{\cdot 672\dot{6}\times 544\times 111}$.

(10) $\frac{\cdot 1\dot{3}0\dot{9}\times 1665}{436}+\frac{\cdot 13\dot{0}\dot{9}\times 275}{72}-\cdot\dot{9}$.

(11) Express as a decimal $\frac{14\times 48+9\times 4+3}{21\times 12\times 4}$.

(12) „ „ $\frac{5\times 5\frac{1}{2}\times 3+4\times 3+2\frac{1}{2}}{220\times 3}$.

(13) „ „ $\frac{20\times 12+7\frac{1}{2}}{1760\times 3\times 12}$.

(14) „ „ $(\cdot 3211\times 21+\frac{3}{8}\text{ of }5+\cdot 6925)\times\cdot 0\dot{5}$.

(15) Find value of $\frac{\cdot 812\dot{1}\dot{2}\times\cdot\dot{0}0673\dot{4}\times 165}{33\times 134\times\cdot 02\dot{0}\dot{2}}$.

(16) Express as a decimal—

$$(4\times 20\times 12\times 4+17\times 12\times 4+9\times 4+3)\div 960,$$

and prove by expressing each fraction as a decimal and adding the results that it is equivalent to $4+\frac{17}{20}+\frac{9}{240}+\frac{3}{960}$.

(17) Express as decimals—

$$5,\ \frac{17}{20},\ \frac{3}{4\times 20},\ \frac{16}{28\times 4\times 20},\ \frac{13}{16\times 28\times 4\times 20},\ \frac{12}{16\times 16\times 28\times 4\times 20},$$

and add them together. Do the same with—

$$2,\ \frac{2}{20},\ \frac{11}{28\times 4\times 20},\ \frac{2}{16\times 28\times 4\times 20},\ \frac{4}{16\times 16\times 28\times 4\times 20},$$

and prove that the sum of the results is 8.

(18) Find value of $\frac{\frac{\cdot 5+\cdot 7}{\cdot 5-\cdot 7}-\frac{\cdot 7}{\cdot 5}}{1+\frac{\cdot 5+\cdot 7}{\cdot 5-\cdot 7}\times\frac{\cdot 7}{\cdot 5}}$.

(19) Find value of $(130 + \frac{18}{20} + \frac{6}{240}) \times {\cdot}06 \times (4 + \frac{9}{20} + \frac{1}{120})$ as a decimal; take 35 from it and express the answer as a fraction

(20) Simplify $(74 + \frac{19}{20} + \frac{105}{2400}) \times 3 \div (44 + \frac{19}{20} + \frac{111}{2400})$.

(21) Find value of $256{\cdot}5 \times (1{\cdot}05)^4 - \{55 + \frac{1}{4} + \frac{656475}{24000000}\}$.

(22) Find value of $\{2800 \times (1{\cdot}045)^3 - 3195\frac{1}{4}\}240$ as a decimal.

(23) $9091{\cdot}8074253125 \div (1{\cdot}0325)^4$.

(24) 200÷25·52568125 to 5 places. **(25)** 563·75 × 2·5÷102·5.

(26) 4858·14÷4·75 to 5 places. **(27)** 3897·75÷4·8.

Exercise 34.

(1) 27·14986 × 92·41035 to 4 places.

(2) 480·14936 × 2·72416 to 4 places.

(3) 2490·3048 × ·573286 to 5 places.

(4) 325·701428 × ·7218393 to 3 places.

(5) 2508·92806÷92·41035 to 4 places.

(6) 4109·2351÷230·409 to 4 places.

(7) 37·10438÷5713·96 to 5 places.

(8) 913·08÷2137·2 to 3 places.

(9) 2227·1132÷104263·7 to 7 places.

(10) ·13412 × 3·12 to 3 places.

(11) 24·356782 × 245·326 to 4 places.

(12) 39·947025÷11·2132 to 2 places.

(13) 34·952÷53·07 to 4 places. **(14)** 3·785÷21·24 to 4 places.

(15) 307·8÷23·8 to 3 places. **(16)** 8·747÷413·4 to 3 places.

(17) 91·4÷9020·4 to 4 places. **(18)** 4·37÷·0104 to 3 places.

(19) 91·6÷8931·61 to 4 places. **(20)** 43·2÷·0351 to 3 places.

(21) 84·5÷3936·2 to 4 places. **(22)** 56·64÷·0107 to 3 places.

(23) 37·52÷2871·3 to 6 places. **(24)** 507·97÷·0023 to 4 places.

(25) 39·49÷13·476 to 4 places. **(26)** 721·42÷21·9 to 4 places.

(27) ·034÷2·14 to 5 places. **(28)** 1÷9403 to 7 places.

vii. The Measures and Multiples of Fractions and Decimals.

87. If we remember our definitions of measure and multiple already given, viz. that if A is a measure of B, A must be contained an exact number of times in B, and then B is said to be a multiple of A, the reason of the following methods adopted will be apparent.

Ex. 1. Find the G.C.M. and L.C.M. of $\frac{9}{35}$, $\frac{12}{25}$, $\frac{27}{50}$.
Reduce them to a common denominator, 350, the fractions now appear as $\frac{90}{350}$, $\frac{168}{350}$, $\frac{189}{350}$; the G.C.M. of 90, 168, 189 is 3.
the L.C.M. „ „ „ 7560.
∴ G.C.M. of the fractions is $\frac{3}{350}$, the L.C.M. is $\frac{7560}{350}=\frac{108}{5}$.

Hence the rule to find the G.C.M. of vulgar fractions: Express them as equivalent fractions with a common denominator, then the G.C.M. required will be the fraction with the G.C.M. of the new numerator as its numerator, and its denominator the L.C.M. of the denominators.

88. Again, to find the G.C.M. and L.C.M. of decimals, the rule is

To make the order of the decimals the same; then find the G.C.M. or L.C.M. of these numbers, disregarding the decimal points; and mark off from the results as many places as this order consists of.

Ex. Find the G.C.M. and L.C.M. of 2·88, 3·6
2·88, 3·6 expressed in same order are 2·88, 3·60.
The G.C.M. of 288, 360 is 72.
The L.C.M. of 288, 360 is 1440.
∴ G.C.M. required is ·72, the L.C.M. is 14·4.

XI. POWERS AND ROOTS.

i. Squares and Square Roots.

89. When a number is multiplied by itself the product is called the square or the second power of that number, thus, $25=5\times 5=5^2$ is the square of 5. When this number is a whole number the product is a perfect square.

90. If we find the continued product of $2\times 2\times 3\times 3\times 7$, the result is 252; this is not a square number, ∵ it contains only one factor, 7, ∴ to make it a square number we must multiply it by another factor, 7. We can now deduce a method for solving this question: What is the least number by which 252 must be multiplied to become a square number?

Resolve the number into its elementary factors, and introduce factors which will ensure each elementary factor being repeated an even number of times

Exercise 35.

What factors will make the following numbers perfect squares?—

(**1**) 18. (**2**) 24. (**3**) 27. (**4**) 32. (**5**) 54. (**6**) 72. (**7**) 99. (**8**) 108. (**9**) 112. (**10**) 176. (**11**) 288. (**12**) 432. (**13**) 999. (**14**) 5760.

91. The square root of a number, symbolised by $\sqrt{}$, is that number which, multiplied by itself, produces the given number, thus, 5 is the square root of 25.

In order to find the square root of a given number, the most obvious method would be at present to resolve the number into its elementary factors, employing the 'tests of divisibility,' knowing that if the number is a perfect square each factor involved must be involved at least twice, and therefore the square root required will be the continued product of these factors repeated half as many times as they are in the square number.

Thus, to find the square root of 144 :

$$\begin{array}{rl} 2\underline{)144} & \\ 2\underline{)72} & \\ 2\underline{)36} & \\ 2\underline{)18} & \\ 3\underline{)\ 9} & \therefore \quad 144 = 2^4 \times 3^2. \\ 3 & \therefore \sqrt{144} = 2^2 \times 3 = 12. \end{array}$$

92. Many examples of course occur where this method is inapplicable ; but in many cases it is useful, and often the labour of finding the roots by the ordinary method is considerably shortened. Thus, find $\sqrt{54756}$. Here, applying tests we see it will divide by 2 and by 9, $\therefore$ we can divide it by two factors of 2 and 9.

$$\begin{array}{r} 4\underline{)54756} \\ 9\underline{)13689} \\ 9\underline{)1521} \\ 169 \end{array}$$

$$\therefore\ 54756 = 2^2 \times 9^2 \times 169 \quad \therefore\ \sqrt{54756} = 2 \times 9 \times \sqrt{169}$$

and square root of 169 we know, by a method to be explained $= 13$; $\therefore\ \sqrt{54756} = 2 \times 9 \times 13 = 234$.

Exercise 36.

Find by this method the square root of

(1) 36. (2) 144. (3) 81. (4) 64. (5) 324. (6) 1089. (7) 100. (8) 784. (9) 4356. (10) 11025. (11) 625. (12) 729. (13) 1296. (14) 193600. (15) 57600. (16) 32400. (17) 592900. (18) 4096. (19) 50625. (20) 7056. (21) 613089 given that 29 is a factor. (22) 141376. (23) 1960000. (24) 102400. (25) 562500. (26) 88209. (27) 19360000. (28) 22071204 (29 is a factor). (29) 6350400. (30) 238517136.

93. The squares of the first twenty numbers should be learnt by heart. We now proceed to the explanation of the general rule of extracting the square root of any number.

[1] To do this we shall adopt a new notation, which will easily explain itself, thus, 7 will be represented by 7, 70 by 7_1 or 7×10, 700 by 7_2 or 7×10^2.

The figures written below, or the subscript letters, denoting the number of factors of 10 taken, thus $7_5 = 700000$, $7_3 = 7000$, $7_2 = 700$, $7_1 = 70$; thus 3457 may be written $3_3 + 4_2 + 5_1 + 7$, and if the square of 3457 is required, the result might be represented thus:

$$\begin{array}{r}
3457 \\
\underline{3457} \\
24199 \\
17285 \\
13828 \\
\underline{10371} \\
11950849
\end{array}$$

Or, adopting this new notation, the operation would appear as below:

$$\begin{array}{l}
3_3 + 4_2 + 5_1 + 7 \\
\underline{3_3 + 4_2 + 5_1 + 7} \\
9_6 + 12_5 + 15_4 + 21_3 \\
\quad + 12_5 + 16_4 + 20_3 + 28_2 \\
\qquad + 15_4 + 20_3 + 25_2 + 35_1 \\
\underline{\qquad\quad + 21_3 + 28_2 + 35_1 + 49} \\
9_6 + 24_5 + 46_4 + 82_3 + 81_2 + 70_1 + 49
\end{array}$$

---

[1] From here to Art. 94 may be omitted by a beginner.

$\therefore$ the product $= 9000000 + 2400000 + 460000 + 82000 + 8100 + 700 + 49 = 11950849$.

Conversely, to find the square root of 11950849, we may proceed thus:

$$\text{The number} = 9_6 + 24_5 + 46_4 + 82_3 + 81_2 + 70_1 + 49.$$

$\sqrt{9_6} = 3_3$. Put this in the quotient and bring down the next two terms; twice the quotient is contained 4 times in the first term of the result, put the 4 in the quotient and also in the divisor, multiply them together, and to the difference between this product and the last portion of the dividend annex the 2 next terms, double the quotient again, and make the product the new divisor, and continue the process till there is no remainder.

$$\begin{array}{rl}
 & 9_6 + 24_5 + 46_4 + 82_3 + 81_2 + 70_1 + 49(3_3 + 4_2 + 5_1 + 7 \\
 & \underline{9_6} \\
6_3 + 4_2) & 24_5 + 46_4 \\
 & \underline{24_5 + 16_4} \\
6_3 + 8_2 + 5_1) & 30_4 + 82_3 + 81_2 \\
 & \underline{30_4 + 40_3 + 25_2} \\
6_3 + 8_2 + 10_1 + 7) & 42_3 + 56_2 + 70_1 + 49 \\
 & 42_3 + 56_2 + 70_1 + 49
\end{array}$$

The square root equals $= 3_3 4_2 5_1 7$, or 3457.

94. The above is generally worked thus:

Mark off the number into periods by putting a dot over the units, hundreds, tens of thousands, millions, &c., *i.e.* on every alternate digit beginning at the units place. Find the highest square number in the first period on the left: the corresponding root will be the first figure of the answer. Subtract the square number from the first period, and annex to the remainder the two figures of the next period. Double the root already found, and prefix it as a divisor to the number last mentioned. Place the quotient both as the second figure of the root, and also as an additional figure in the divisor. Multiply the divisor so completed by this last figure, subtract, and to the remainder annex the two figures of the next period, and proceed as before.

```
     11950849(3457
      9
    64)295
      256
   685)3908
       3425
  6907)48349
       48349
```

Exercise 37.

Extract the square root of—

(1) 17698849. **(2)** 6084. **(3)** 4096. **(4)** 841. **(5)** 56821444.
(6) 714025. **(7)** 9585216. **(8)** 45369. **(9)** 10816. **(10)** 74529.
(11) 33709636. **(12)** 5764801. **(13)** 10099684. **(14)** 522729.
(15) 7890481. **(16)** 8832784. **(17)** 197262025.
(18) 4745757682576. **(19)** 10246401. **(20)** 81830116.
(21) 197739844. **(22)** 186624. **(23)** 77841. **(24)** 9659664.
(25) 37491129. **(26)** 16949689. **(27)** 3534400. **(28)** 65561409.
(29) 99960004. **(30)** 24088464. **(31)** 119550669121.
(32) 368451428004. **(33)** 177241. **(34)** 4334724.
(35) 14356521. **(36)** 17338896. **(37)** 16353936.
(38) 3915380329. **(39)** 120409. **(40)** 4376464. **(41)** 57426084.
(42) 29506624. **(43)** 69355584. **(44)** 65415744. **(45)** 481636.
(46) 1022121. **(47)** 229704336. **(48)** 659565124.
(49) 25553025. **(50)** 282475249.

95. To find the square root of a decimal which is a terminating decimal, we may proceed as before, marking off in the root half the number of places as in the square.

Thus to find the square root of 68·492176—

```
       68·492176(8276
       64
   162)449
       324
  1647)12521
       11529
 16546)99276
       99276
```

Here, the square being of the *sixth* order, the root must be of the third. ∴ root = 8·276.

Exercise 38.

Find the square roots of—

(1) 1624·09. (2) 187·4161. (3) 40·1956. (4) 94·2841.
(5) 482·6809. (6) 66·585600. (7) ·25. (8) ·16. (9) ·0016.
(10) ·0729. (11) 1524·9025. (12) 48·8601. (13) 84·8241.
(14) 449·44. (15) ·0001. (16) 1·7161. (17) 22·1841. (18) 4·1209.
(19) 6577·21. (20) ·106929.

96. In extracting the root of a non-terminating decimal, unless otherwise directed, make the order of the square 10, the root then will be of the fifth order.

Thus find square root of ·7 to 5 places:

```
              ·70̇00̇00̇00̇00̇(8366600265
               64
             163)600
                 489
           1666)11100
                 9996
          16726)110400
                 100356
         167326)1004400
                 1003956
      167332002)      444000000
```

Ans. to 5 places = ·83666.

Continue the process.

```
                      334664004
     1673320046)10933599600
                 10039920276
    16733200525)  89367932400
```

thus $\sqrt{\cdot 7}$ to 10 places = ·8366600265.

97. Suppose now when we had obtained the first 6 in the root, we had, instead of adding 2 figures to the remainder 1104, and one to the divisor, cut off one figure at a time from the divisor, as in Art. 83, and proceeded by simple division, we should get 2 more places in the quotient, thus:

$1\cancel{6}\cancel{7}\cancel{2})1104(66$, carrying 1 from the rejected 2;

$$\begin{array}{r} 1003 \\ \hline 101 \\ 100 \\ \hline 1 \end{array}$$

so when we have obtained 5 places, 5 more can be obtained by division, thus:

$1\cancel{6}\cancel{7}\cancel{3}\cancel{3}\cancel{2})444(00265$

$$\begin{array}{rl} 334 & \\ \hline 109 & \quad \text{carrying 4 from rejected } 7 \times 6 = 42 \\ 100 & \\ \hline 9 & \\ 8 & \quad \text{carrying 3 from rejected 6.} \\ \hline 1 & \end{array}$$

Accordingly we may begin to cut off figures from the divisor as soon as more than half the number of significant figures required have been obtained.

Apply this method to find to nine places the square roots of—

Exercise 39.

(1) 2. (2) 3. (3) 5. (4) 6. (5) 8. (6) 10. (7) 11. (8) 12. (9) 13. (10) 14. (11) 15. (12) 17. (13) 18. (14) 19. (15). 20.

98. To find the square root of a fraction, reduce if necessary to an improper fraction, find the root of the numerator and root of the denominator: or express vulgar fraction as decimal, and proceed as in last 2 articles.

Thus $\sqrt{1\frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{\sqrt{49}}{\sqrt{25}} = \frac{7}{5} = 1\frac{2}{5}$;

or $\sqrt{1\frac{24}{25}} = \sqrt{1 \cdot 96} = 1 \cdot 4$.

99. $5 = \sqrt{25}$, $\therefore 5 \times 5 = \sqrt{25} \times \sqrt{25}$; but $25 = 5 \times 5$ $\therefore 25 = \sqrt{25} \times \sqrt{25}$; in other words squaring a square root is effected by removing the symbol. We might also show that

$$25 = \sqrt{625} = \sqrt{25 \times 25}.$$

$$\therefore \sqrt{25} \times \sqrt{25} = \sqrt{25 \times 25}.$$

i.e. the product of 2 or more factors {as $\sqrt{2} \times \sqrt{3} \times \sqrt{5} = \sqrt{30}$} is the root of their product.

100. To find the value of—

$$\sqrt{\tfrac{3}{7}} = \frac{\sqrt{3}}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{\sqrt{49}} \text{ by Art. 99} = \frac{\sqrt{21}}{7}.$$

Hence, to simplify such fractions,

Multiply numerator and denominator by the denominator, or some factor which will make it a square, then extract the root of the numerator and divide it by the denominator last obtained.

101. To find the square root of circulating decimals, either proceed as in Art. 97 or reduce to a vulgar fraction as in Art. 98.

Exercise 40.

In the following examples all decimals should be carried to 5 places.

Find value of—

(1) $\sqrt{2 + \sqrt{2}}$. (2) $\sqrt{(3 - \sqrt{3})}$. (3) $\sqrt{5 - \sqrt{5}}$. (4) $\sqrt{7 + \sqrt{7}}$.

What is your error in trying to prove the following 6 identities—

(5) $(8 + \sqrt{8})(8 - \sqrt{8}) = 56$. (6) $(5 + \sqrt{5})(5 - \sqrt{5}) = 20$.

(7) $(2 + \sqrt{3})(2 - \sqrt{3}) = 1$. (8) $\sqrt{\dot{.9}} = 1$.

(9) $(\sqrt{11} + \sqrt{10})(\sqrt{11} - \sqrt{10}) = 1$.

(10) $(\sqrt{15} + \sqrt{14})(\sqrt{15} - \sqrt{14}) = 1$. (11) $\sqrt{\tfrac{3}{5}}$. (12) $\sqrt{\tfrac{3}{2}}$.

(13) $\sqrt{40\tfrac{1}{36}}$. (14) $\sqrt{2\tfrac{1}{6}}$. (15) $\sqrt{\cdot 0000729}$. (16) $\sqrt{11\tfrac{37}{49}}$.

(17) $\sqrt{12\tfrac{4}{5}}$. (18) $\sqrt{28\tfrac{1}{3}}$. (19) $\sqrt{76\tfrac{14}{17}}$. (20) $\sqrt{\dfrac{1\cdot 28}{12\cdot 5}}$.

ii. Cubes and Cube Roots.

102. When a number is multiplied by itself twice, the continued product is called the cube of the number.

Thus, ∵ $5 \times 5 \times 5 = 125$, $7 \times 7 \times 7 = 343$; 125, 343 are called the cubes of 5, 7 respectively, or $125 = 5^3$, $343 = 7^3$.

103. If we find the continued product of $2 \times 2 \times 2 \times 3 \times 3 \times 7$, the result is 504, this is not a perfect cube, ∵ it contains only 2 factors of 3 and 1 of 7; ∴ to make it a perfect cube we must multiply by $2 \times 7 \times 7$.

Hence, to find by what factor a number must be multiplied to become a perfect cube, we resolve it into its elementary factors, and introduce factors which will ensure each elementary factor occurring 3, 6, or 9 times, &c.

Exercise 41.

What is the least factor which will make the following perfect cubes?—

(1) 6561. (2) 4096. (3) 16807. (4) 774144. (5) 1002375. (6) 36864. (7) 491520. (8) 814968. (9) 671088640. (10) 7283146752.

104. The cube root of a number, symbolised by $\sqrt[3]{\quad}$ or $(\quad)^{\frac{1}{3}}$ is that number which multiplied by itself twice realises the given number.

Thus ∵ $343 = 7^3$ ∴ 7 is the cube root of 343 written $\sqrt[3]{343}$ or $(343)^{\frac{1}{3}}$.

105. In order to find the cube root of a perfect cube, *i.e.* of a number whose cube root is an integer, our method at present would be to resolve the number into its elementary factors, employing the 'tests of divisibility,' knowing that if the number be a perfect cube each factor involved must be involved at least 3 times; and therefore the cube root required will be the continued product of these factors repeated one-third as many times as they are in the cube number.

Thus $125 = 5 \times 5 \times 5 = 5^3$ ∴ its cube root is 5.

$$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6.$$

∴ its cube root $= 3^2 = 9$.

106. Many cases occur where this method is inapplicable, but it is frequently useful, and often the labour of extracting the cube root is in this way shortened. Thus to find the cube root of 1601613, we see from the tests that 9 is a factor.

$$\begin{array}{r} 9\underline{)1601613} \\ 9\underline{)177957} \\ 9\underline{)19773} \\ 2197 \end{array}$$

$$\therefore\ 1601613 = 9^3 \times 2197 \quad \therefore\ \sqrt[3]{1601613} = 9 \times \sqrt[3]{2197}$$

and $\sqrt[3]{2197}$ we shall find by a method to be explained $= 13$.

$$\therefore\ \sqrt[3]{1601613} = 9 \times 13 = 117.$$

Exercise 42.

Find by this method the cube roots of—

(1) 4096. **(2)** 32768. **(3)** 262144. **(4)** 970299. **(5)** 250047. **(6)** 14832537993 of which 13 is a factor. **(7)** 14706125. **(8)** 2097152. **(9)** 16777216. **(10)** 130323843. **(11)** 322828856. **(12)** 134217728. **(13)** 122615327232 of which 23 is a factor.

107. The cubes of the first nine numbers are :—

1, 8, 27, 64, 125, 216, 343, 512, 729

∴ these numbers should be learnt by heart.

108*. We now proceed to explain the method of finding the cube root of any number which is a perfect cube. Adopting the notation of Art. 93—

$$\begin{array}{r} 2456 \\ \underline{2456} \\ 14736 \\ 12280 \\ 9824 \\ \underline{4912} \\ 6031936 \\ \underline{2456} \\ 36191616 \\ 30159680 \\ 24127744 \\ \underline{12063872} \\ 14814434816 \end{array}$$

---

* May be omitted by a beginner.

$$\begin{array}{l}
2_3 + 4_2 + 5_1 + 6 \\
2_3 + 4_2 + 5_1 + 6 \\
\hline
4_6 + 8_5 + 10_4 + 12_3 \\
\quad + 8_5 + 16_4 + 20_3 + 24_2 \\
\qquad\quad + 10_4 + 20_3 + 25_2 + 30_1 \\
\qquad\qquad\quad + 12_3 + 24_2 + 30_1 + 36 \\
\hline
4_6 + 16_5 + 36_4 + 64_3 + 73_2 + 60_1 + 36
\end{array}$$

$$\begin{array}{l}
4_6 + 16_5 + 36_4 + 64_3 + 73_2 + 60_1 + 36 \\
2_3 + 4_2 + 5_1 + 6 \\
\hline
8_9 + 32_8 + 72_7 + 128_6 + 146_5 + 120_4 + 72_3 \\
\quad + 16_8 + 64_7 + 144_6 + 256_5 + 292_4 + 240_3 + 144_2 \\
\qquad\quad + 20_7 + 80_6 + 180_5 + 320_4 + 365_3 + 300_2 + 180_1 \\
\qquad\qquad\quad + 24_6 + 96_5 + 216_4 + 384_3 + 438_2 + 360_1 + 216 \\
\hline
8_9 + 48_8 + 156_7 + 376_6 + 678_5 + 948_4 + 1061_3 + 882_2 + 540_1 + 216
\end{array}$$

$$(1)\quad \begin{array}{l|l}
3(2_3)^2 = & 12_6 \\
(3 \times 2_3 + 4_2)4_2 = & \quad + 24_5 + 16_4 \\
\cline{2-2}
 & 12_6 + 24_5 + 16_4 \\
3(2_3 + 4_2)^2 = & 12_6 + 48_5 + 48_4 \\
\{3(2_3 + 4_2) + 5_1\}5_1 = & \qquad\quad + 30_4 + 60_3 + 25_2 \\
\cline{2-2}
 & 12_6 + 48_5 + 78_4 + 60_3 + 25_2 \\
3(2_3 + 4_2 + 5_1)^2 & 12_6 + 48_5 + 108_4 + 120_3 + 75_2 \\
\{3(2_3 + 4_2 + 5_1) + 6\}6 = & \qquad\qquad\qquad 36_3 + 72_2 + 90_1 + 36 \\
\cline{2-2}
 & 12_6 + 48_5 + 108_4 + 159_3 + 147_2 + 90_1 + 36
\end{array}$$

$$(2)\quad \begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad [2_3 + 4_2 + 5_1 + 6 \\
8_9 + 48_8 + 156_7 + 376_6 + 678_5 + 948_4 + 1061_3 + 882_2 + 540_1 + 216 \\
8_9 \\
\hline
\quad 48_8 + 156_7 + 376_6 \\
\quad 48_8 + 96_7 + 64_6 \\
\hline
\qquad\quad 60_7 + 312_6 + 678_5 + 948_4 + 1061_3 \\
\qquad\quad 60_7 + 240_6 + 390_5 + 300_4 + 125_3 \\
\hline
\qquad\qquad\quad 72_6 + 288_5 + 648_4 + 936_3 + 882_2 + 540_1 + 216 \\
\qquad\qquad\quad 72_6 + 288_5 + 648_4 + 936_3 + 882_2 + 540_1 + 216
\end{array}$$

$\therefore$ cube root required is 2456.

109. This perhaps will explain the reason for the following contracted method. Mark off the number into periods of three figures, placing a dot on the unit figure in each period. The number of dots will be the number of figures in the root.

N.B.—(1) and (2) should be placed side by side with the horizontal lines of (2) in the same straight line with those of (1).

		$1\dot{4}\,|\,81\dot{4}\,|\,43\dot{4}\,|\,81\dot{6}\,|\,2456$
$3\times(20)^2=3\times10^2\times2^2=$	1200 . . (*a*)	8 . . . . (*m*)
		6814 . . . . (*r*)
$\{3\times20+4\}4=64\times4=$	256 . . (*d*)	
	1456 . . (*h*)	5824 . . . (*n*)
		990434 . . (*s*)
$3\times(240)^2=3\times10^2\times24^2=$	172800 . . (*b*)	
$\{3\times240+5\}5=725\times5=$	3625 . . (*e*)	
	176425 . . (*k*)	882125 . (*p*)
		108309816 . (*t*)
$3\times(2450)^2=3\times10^2+245^2=$	18007500 . (*c*)	
$\{3\times2450+6\}6=7356\times6=$	44136 . (*f*)	
	18051636 . (*l*)	108309816 (*q*)

(*a*), (*b*), (*c*), &c. are called respectively the 1st, 2nd, and 3rd trial divisors, and are the continued product of 3, 10^2, and the square of the number already in the root.

(*d*), (*e*), (*f*) are obtained by adding the new figure in the root to thirty times the number already in the root, and multiplying the sum by the new figure in the root.

(*h*), (*k*), (*l*) are the sums of *a*, *d*; *b*, *e*; *c*, *f* respectively.

(*m*) is the cube of the 1st figure in the root.

(*n*), (*p*), (*q*) are the products of (*h*), (*k*), (*l*) by the next figures in the root taken in order.

(*r*), (*s*), (*t*) are called the 1st, 2nd, 3rd remainders with the next figures of the cube number brought down.

The figures in the root are the quotient of *r* by *a*; *s* by *b*; *t* by *c*, &c. Care must be taken not to make them too large; *e.g.* if in $r\div a$ we had used 5, *d* would have become $65\times5=325$, and *h* would be 1525 and *r* 7625, a number obviously too large.

We may now proceed to examples, and the last set should be worked again this way.

Exercise 43.

Find the cube roots of—

(1) 97336. **(2)** 405224. **(3)** 941192. **(4)** 2985984.
(5) 134217728. **(6)** 273359449. **(7)** 167284151. **(8)** 731189187729.
(9) 10970645048. **(10)** 1371742108367626890260631.
(11) 2803221. **(12)** 8365427. **(13)** 806954491. **(14)** 7077888.
(15) 251239591. **(16)** 991026973. **(17)** 12812904. **(18)** 28372625.
(19) 20123648. **(20)** 8741816. **(21)** 48228544. **(22)** 34012224.
(23) 56623104. **(24)** 17173512. **(25)** 39651821.
(26) 122615327232. **(27)** 46656. **(28)** 15069223.
(29) 64481201. **(30)** 28991029248. **(31)** 12821119155125.

110. To find the cube root of a decimal fraction which terminates proceed as above, disregarding the point, and mark off from the root one-third as many places as are contained in the cube number. Thus $\sqrt[3]{1\cdot321}=1\cdot1$.

To find the cube root of a vulgar fraction, place the cube root of its numerator over the cube root of its denominator, or reduce to a decimal and employ rule above: or Art. 100.

111. The result in Art. 109 may also be practically obtained in this way—

(i.)	(ii.)	(iii.)
2		
2	4	14|814|434|816(2456
4	8	8
2	1200 . . . . . . (*a*)	6814
64 . . .	256 . . . . . . (*d*)	
4	1456 . . . . . (*h*)	5824
68 . . .	272 . . . . . (*A*)	990434
4	172800 . . . . (*b*)	
725 . . .	3625 . . . . (*e*)	
5	176425 . . . . (*k*)	882125
730 . . .	3650 . . . . (*B*)	108309816
5	18007500 . . . . (*c*)	
7356 . .	44136 . . . (*f*)	
	18051636 . . . (*l*)	108309816

Comparing these figures with Art. 109, those under (iii.) are the same, (ii.) are the same, with the exception of figures before line '*a*', between '*h*' and '*b*,' and '*k*,' '*e*.'

Hence this practical rule for cube root: Arrange the paper in 3 columns as above. Find the nearest cube root of first period. Place it in the quotient and also in (i.), multiply it by itself, and put the product under (ii.), to this add twice the square of the first figure of the root, and annex 2 cyphers—line '*a*.' Now add the first figure of root to its double, annex the second figure of the quotient, this gives '*d*' in (i.), multiply this again by the second figure of quotient, and we get '*d*' of (ii.), '*h*' = '*a*' + '*d*.' Line '*A*' in (i.) is obtained by adding the second figure in root to '*d*,' multiplying this sum by it, gives '*A*' in (ii.); '*b*' in (ii) = '*h*' + '*A*' with 2 cyphers annexed; '*e*' in (i.) is got by adding the last figure in root to '*A*' and annexing the new figure; this result multiplied by the new figure gives '*e*' in (ii.). *k* in (ii.) = '*b*' + '*e*.' Add the new figure to '*e*' of (i.), and multiply the result by it, and we get line '*B*.' '*C*' = '*k*' + '*B*' with two cyphers annexed. Line '*f*' in (i.) is got by adding to line '*B*' the last figure in quotient and annexing the new figure of root, and the product of this by the new figure gives line '*f*' in (ii.), '*l*' = '*c*' + '*f*' under (ii.). Column (iii.) is obtained as already shown in Art. 109.

112. After obtaining a few figures in the decimal part of the root, bring down no more zeros in remainder of (iii.), cut off *one* figure from the right of (ii.) and 2 figures from 1, and then work as in contracted multiplication and division of decimals.

Thus find cube root of 21035·8 to ten places of decimals.

(i.)	(ii.)	(iii.)
2		
2	4	21035·8 (27·60491055944
4	8	8
2	1200	13035
67	469	
7	1669	11683
74	518	1352800
7	218700	
816	4896	
6	223596	1341576
822	4932	11224 . . . . . .
6	2285280000	
82804	331216	
4	2285611216	9142444864
82808	331232	2081555136
4	2285942448	
|8|28|1$\cancel{2}$	74531	
	228601697$\cancel{2}$	2057415281
	7453$\cancel{1}$	24139855
	22860915$\cancel{1}$	
	8$\cancel{3}$	
	228609234$\cancel{4}$	22860923
	8$\cancel{3}$	1278932
	2286093$\cancel{2}$	1143046
	22861	135886
	2286	114305
	228	21581
	23	20575
		1006
		914
		92
		91
		1

Exercise 44.

Find the cube roots of—

(1) 64481·201. (2) 28991·029248. (3) ·000076765625.
(4) $\frac{13824}{15625}$. (5) $7\frac{6}{7}$ to 8 places. (6) 16 to 6 places. (7) $91\frac{1}{8}$.

Find to six places of decimals—

(8) Cube root of 2. (9) The fourth root of 2. (10) The fourth root of 97·41. (11) The sixth root of 21035·8. (12) The sixth root of 2. (13) The eighth root of 21035·8. (14) The eighth root of 2. (15) The ninth root of 21035·8. (16) The ninth root of 2. (17) Express (4) as a decimal, and find the cube root of the result.

XII. CONCRETE NUMBERS.

I. MEASURES OF MONEY.

113. 2 farthings . . . are 1 halfpenny.
4 farthings or 2 halfpence are 1 penny.
12 pence . . . . are 1 shilling.
20 shillings . . . are 1 pound, or sovereign.

The symbol £ placed before or over a number denotes pounds.
" *s.* " after " " shillings.
" *d.* " after " " pence.

Since 1 farthing is $\frac{1}{4}$ of a penny, 2 farthings or 1 halfpenny is $\frac{1}{2}$ of a penny, 3 farthings is $\frac{3}{4}$ of a penny; the symbols $\frac{1}{4}$*d.*, $\frac{1}{2}$*d.*, $\frac{3}{4}$*d.*, represent 1 farthing, 2 farthings, 3 farthings.

Thus 5 pounds 12 shillings and 7 pence 3 farthings is written £5. 12*s.* $7\frac{3}{4}$*d.* Sometimes farthings is denoted by q, thus 3 farthings is written 3q.

114. If now we take one pound as our unit, its submultiples, shillings, pence, and farthings are called its denominations.

If a quantity is expressed in one denomination only it is called a *simple* quantity—as £7.

If a quantity is expressed in several denominations it is called a *compound* quantity—as £12. 13*s.* $11\frac{3}{4}$*d.*

i. Reduction.

115. REDUCTION is the process by which we express (1) a simple or a compound quantity in terms of its lower denominations, or (2) a simple quantity in terms of its higher denominations.

(1) Is sometimes called descending, and (2) ascending reduction.

116. As an example of (1), to reduce pounds to shillings, to find how many shillings there are in £7, and in £9. 15*s.*, the process is thus arranged:

$$\begin{array}{r} \text{£} \\ 7 \\ 20 \\ \hline 140 \end{array} \qquad\qquad \begin{array}{rr} \text{£} & s. \\ 9 & 15 \\ 20 & \\ \hline 195 & \end{array}$$

Since £1 contains 20 shillings, £7 will contain 7 × 20, or 140 shillings; and £9. 15*s.* contains 9 × 20 + 15, or 180 + 15, or 195 shillings; *i.e.* to change pounds into shillings, we multiply the number of pounds by 20 and add to the product the number of shillings.

Exercise 45.

Reduce to shillings:

(1) £10. (2) £14. (3) £16. (4) £34. (5) £38. (6) £30.
(7) £490. (8) £514. (9) £498. (10) £16. 5*s.* (11) £18. 7*s.*
(12) £17. 9*s.* (13) £8. 19*s.* (14) £15. 16*s.* (15) £17. 17*s.*
(16) £64. 15*s.* (17) £69. 19*s.* (18) £79. 17*s.*

117. Next, to reduce a sum of money to pence, we reduce as above the pounds to shillings, then multiply the number of shillings thus obtained by 12 and add to the product the number of pence. Thus to reduce 6*s.* 7*d.*; £5. 13*s.* 4*d.* to pence, we proceed in this way:

$$\begin{array}{rr} s. & d. \\ 6 & 7 \\ 12 & \\ \hline 79 & \end{array} \qquad\qquad \begin{array}{rrr} \text{£} & s. & d. \\ 5 & 13 & 4 \\ 20 & & \\ \hline 113 & & \\ 12 & & \\ \hline 1360 & & \end{array}$$

£5 contains 5 × 20, or 100 shillings ∴ £5. 13*s.* contains 113 shillings, and 113 shillings contains 113 × 12 or 1356 pence, ∴ £5. 13*s.* 4*d.* contains 1360 pence.

Exercise 46.

Reduce to pence:

(1) 6*s.*, 7*s.*, 8*s.*, 9*s.*, 10*s.*, 11*s.*, 12*s.* (2) 6*s.* 6*d.*, 7*s.* 7*d.*, 8*s.* 8*d.*, 9*s.* 9*d.* (3) £2. 3*s.* 4*d.*, £3. 4*s.* 5*d.*, £4. 5*s.* 6*d.*, £5. 6*s.* 7*d.*, £6. 7*s.* 8*d.*, £7. 8*s.* 9*d.*, £8. 9*s.* 10*d.* (4) £12. 11*s.* 1*d.*, £13. 12*s.* 2*d.*, £14. 13*s.* 3*d.*, £15. 14*s.* 4*d.*, £16. 15*s.* 5*d.*, £17. 16*s.* 6*d.* (5) £23. 0*s.* 6*d.*, £35. 10*s.* 11*d.*, £46. 15*s.* 9*d.*, £57. 0*s.* 3*d.*, £68. 14*s.* 5*d.*, £79. 11*s.* 2*d.*

(6) Change to pounds 123456, 789543, 567843, 157678 guineas.

(7) Change to guineas 399854, 568943, 179055, 33347 pounds.

(8) Change to pounds 157643, 678954, 176653, 35789 guineas.

118. Next, to reduce a sum of money to halfpence, we reduce as above the sum to pence, multiply this number by two, and to the product add the number of halfpence; thus to reduce £5. 6*s.* 7½*d.* to halfpence, arrange the work in this way:

$$\begin{array}{rcc}
\pounds & s. & d. \\
5 & 6 & 7\frac{1}{2} \\
20 & & \\
\hline
106 & & \\
12 & & \\
\hline
1279 & & \\
2 & & \\
\hline
2559 & &
\end{array}$$

∴ £5. 6*s.* 7½*d.* contains 2559 halfpence.

Exercise 47.

Reduce to halfpence:

(1) £3. 16*s.* 8*d.*, £4. 15*s.* 5*d.*, £5. 16*s.* 6*d.*, £6. 17*s.* 7*d.*, £7. 18*s.* 8*d.*, £8. 19*s.* 9*d.*

(2) £13. 16*s.* 9½*d.*, £14. 17*s.* 10½*d.*, £15. 18*s.* 11½*d.*, £16. 0*s.* 0½*d.*, £17. 1*s.* 0½*d.*, £18. 0*s.* 1½*d.*

(3) £24. 4*s.* 4½*d.*, £48. 8*s.* 9*d.*, £16. 7*s.* 10½*d.*, £57. 19*s.* 0½*d.*, £89. 0*s.* 11½*d.*, £99. 19*s.* 11½*d.*

119. Next, to reduce a sum of money to farthings, reduce the sum to pence, multiply this number by 4, and to the product add the number of farthings; thus to reduce £6. 16*s.* 7¾*d.* to farthings.

$$\begin{array}{rrr}
\pounds & s. & d. \\
6 & 16 & 7\tfrac{3}{4} \\
\underline{20} & & \\
136 & & \\
\underline{12} & & \\
1639 & & \\
\underline{4} & & \\
6559 & &
\end{array}$$

∴ £6. 16*s.* 7¾*d.* contains 6559 farthings.

Exercise 48.

Reduce to farthings:

(1) £62. 14*s.* 6¾*d.*, £64. 16*s.* 5½*d.*, £75. 13*s.* 7¼*d.*, £86. 12*s.* 11½*d.*, £97. 15*s.* 4¾*d.*, £17. 17*s.* 7¼*d.*

(2) £76. 18*s.* 9½*d.*, £87. 19*s.* 10¾*d.*, £96. 18*s.* 11¼*d.*, £68. 11*s.* 10¼*d.*, £79. 5*s.* 5½*d.*, £19. 19*s.* 9¾*d.*

(3) £29. 19*s.* 11¼*d.*, £39. 0*s.* 0¼*d.*, £59. 0*s.* 0¾*d.*, £99. 11*s.* 0½*d.*, £100. 0*s.* 11¾*d.*, £77. 17*s.* 7¼*d.*

120. The converse operation, by which a *simple* quantity is expressed as a *compound* quantity, sometimes called Ascending Reduction, may be now explained. Reduce 100 shillings to pounds.

If we arrange the shillings in piles of 20 shillings each, we should have 5 piles exactly, and as each of these piles might be replaced by a sovereign we should have £5. In the same way if we had 107 shillings we should have 5 piles, or £5, and 7 shillings over, that is to say, we have as many pounds as 20 is contained in the number of shillings. ∴ to reduce shillings to pounds we proceed thus: divide the number of shillings by 20—

$$\begin{array}{r}
2{,}0)\underline{10{,}7} \\
5\text{—}7
\end{array}$$

∴ there are 107 shillings in £5. 7*s.*

Exercise 49.

Reduce to pounds the following number of shillings:

(1) 200, 280, 320, 170, 190, 150.
(2) 4900, 5140, 4980, 1400, 1960, 2880.
(3) 379, 316, 394, 2653, 2212, 2758.
(4) 1080, 1416, 1272, 3032, 2528, 3152.
(5) 1800, 2520, 2980, 1530, 1710, 1350.
(6) 4999, 5179, 5377, 7778, 8889, 9999.

121. Next, to reduce pence to pounds and shillings, arrange the pence in piles of 12 pence, replace these piles of pence by shillings, then arrange the shillings in piles of 20 shillings, replace them by pounds; in other words, divide the number of pence by 12, the quotient by 20; thus reduce 8372 pence to pounds, shillings, &c.:

12)8372
2,0)69,7—8
£34—17—8

∴ 8372 pence are equivalent to £37. 17*s.* 8*d.*

Exercise 50.

Reduce to pounds and shillings the following pence:

(1) 72, 104, 820, 3988, 5706, 8372.
(2) 96, 79, 1026, 3737, 5503, 8853.
(3) 108, 69, 1315, 4242, 6209, 9594.

122. In the same way, to reduce halfpence or farthings to pounds, &c., we should proceed thus:

Reduce 33593 half pence and 4275639 farthings to £ *s.* *d.*

2)33593	4)4275639
12)16796—½	12)1068909—¾
2,0)139,9—8	2,0)8907,5—9
£69—19—8½	£4453—15—9¾

∴ 33593 halfpence = £69. 19*s.* 8½*d.*: and 4275639 farthings = £4453. 15*s.* 9¾*d.*

Exercise 51.

Reduce to *£ s. d.* the following farthings:

(1) 416, 336, 276, 3280, 4104, 5260, 1552, 15148, 17968.

(2) 60216, 62224, 66032, 73854, 72694, 71894, 82442, 86370, 80374.

(3) 62230, 124262, 67186, 28682, 26670, 2778, 37438, 38106, 39383.

Reduce to *£ s. d.* the following halfpence:

(4) 30108, 36927, 41221, 31115, 14341, 18719, 64789, 32456.

(5) 31112, 366347, 43185, 32131, 13335, 19053, 38106, 76212.

(6) 33016, 35947, 40817, 33593, 13789, 18191, 36382, 39493.

123. Reduction, descending and ascending, are converse processes; if therefore we perform one process on a given quantity, and on the result the other process, we ought to get the original quantity. Thus, if by the descending process we find that £4453. 15*s.* 9¾*d.* is 4275639 farthings, we ought by the ascending process to find that 4275639 farthings is £4453. 15*s.* 9¾*d.*

Hence every example of reduction, with its proof, is an example of both processes, and serves as a test of the accuracy of the work.

124. The copper coins now in circulation in Great Britain are the farthing, the halfpenny, and the penny.

The silver coins in use are the crown (5*s.*), the half-crown (2*s.* 6*d.*), the florin (2*s.*), the shilling (12*d.*), the sixpence (or tester), the fourpenny piece (or groat), and the threepenny piece.

The gold coins in use are the sovereign (or pound), and the half-sovereign.

Copper coinage is not a legal tender for more than 12*d.*, nor silver coinage for more than 40*s.*

The following gold coins were formerly in circulation: the noble (6*s.* 8*d.*), the angel (10*s.*), the half-guinea (10*s.* 6*d.*), the mark (13*s.* 4*d.*), the guinea (21*s.*), the Carolus (23*s.*), the Jacobus (25*s.*), the moidore (27*s.*).

The subjoined list should be learnt by heart:

4 threepenny pieces	make	1 shilling
3 fourpenny pieces	„	1 shilling
5 sixpences . .	„	1 half-crown
40 sixpences . .	„	1 pound

8 half-crowns . make 1 pound
21 sixpences . . „ 1 half-guinea
42 sixpences . . „ 1 guinea.

The following examples worked out should be carefully noticed.

Ex. 1.—Reduce £684. 12*s.* to guineas.

$$\begin{array}{rr} \pounds & s. \\ 684 & 12 \\ 20 & \\ \hline 13692 & \end{array}$$

∴ £684. 12*s.* = 13692 shillings, and since there are 21 shillings in 1 guinea, there will be as many guineas in £684. 12*s.* as 21 is contained in 13692: dividing 13692 by 21 we obtain 652 as quotient; thus:—

$$21\begin{cases} 3\underline{)13692} \\ 7\underline{)4564} \end{cases}$$
$$652$$

∴ 652 guineas = £684 12*s.*

Ex. 2.—Reduce 384 guineas 15*s.* to pounds.

$$\begin{array}{rr} gns. & s. \\ 384 & 15 \\ 21 & \\ \hline 389 & \\ 769 & \\ \hline 2,0)807,9 & \\ \hline 403 & \text{—}19 \end{array}$$

i.e. since 1 guinea contains 21 shillings, 384 guineas 15*s.* contain 384 × 21 + 15 shillings, or 8079 shillings, which = £403. 19*s.*

Ex. 3.—Reduce 18942 half-crowns to half-guineas.

$$\begin{array}{r} 18942 \\ \underline{5} \end{array}$$
$$21\begin{cases} 3\underline{)94710} \\ 7\underline{)31570} \end{cases}$$
$$4510$$

∴ 18942 half-crowns = 4510 half-guineas; *i.e.* since 1 half-crown contains 5 sixpences, 18942 half-crowns will contain 18942 × 5 = 94710 sixpences; and as there are 21 sixpences in 1 half-guinea, we divide 94710 by 21 to obtain the number required.

EX. 4.—Reduce £61. 17*s.* 8*d.* to groats.

£	*s.*	*d.*
61	17	8
20		
1237		
3		
3713		

£61. 17*s.* = 1237 shillings, and 1 shilling = 3 groats, ∴ 1237 shillings = 1237 × 3 = 3711*s.* & 8*d.* = 2 groats, ∴ £61. 17*s.* 8*d.* = 3713 groats.

Exercise 52.

Reduce to farthings—

(1) 25623 guineas. **(2)** 45246 crowns. **(3)** 23456 half-crowns. **(4)** 16547 florins. **(5)** 38456 groats. **(6)** 46992 guineas. **(7)** 94728 crowns. **(8)** 50972 half-crowns. **(9)** 37484 florins. **(10)** 92508 groats. **(11)** 39 gns. 12*s.* 8¾*d.* **(12)** 37 gns. 14*s.* 6¼*d.*

(13) Reduce 123456789 farthings, and prove the accuracy of your results—

(i.) To halfpence. (ii.) To pence. (iii.) To threepenny pieces. (iv.) To groats. (v.) To sixpences. (vi.) To shillings. (vii.) To florins. (viii.) To half-crowns. (ix.) To crowns. (x.) To nobles. (xi.) To marks. (xii.) To pounds. (xiii.) To guineas. (xiv.) To Caroli. (xv.) To Jacobi. (xvi.) To moidores.

Exercise 53.

(1) Reduce £685. 13*s.* to guineas, 9281 threepences to crowns, 92640 farthings to guineas.

(2) 385 guineas 16*s.* to pounds, £720. 16*s.* 5¾*d.* to farthings, £817. 15*s.* to half-crowns and sixpences.

(3) 8458 crowns to florins, 17916 florins to guineas, 15120 florins to crowns and groats.

(4) 18032 groats to florins, 36064 farthings to crowns 371856 halfpence to half-crowns.

(5) 39746 half-crowns to half-sovereigns, 37212861 threepences to pounds, 12861 groats to threepences.

(6) 18654 half-sovereigns to farthings, 17856 halfpence to half-crowns, 85725 groats to guineas.

(7) 11786 guineas to sixpences, 16089 guineas to pounds, 9856 shillings to farthings.

(8) 6975 guineas to pounds, 17824 farthings to pounds, 92640 farthings to half-guineas.

(9) 12345 Jacobi to crowns, 45678 Caroli to pounds, 2579 moidores to farthings and pounds.

(10) 12345 testers to Jacobi, 45678 nobles to pounds, 13569 angels to guineas.

(11) 5463 marks to angels, 12357 moidores to nobles, 57258 Jacobi to moidores.

(12) Reduce to farthings a groat, a tester, a noble, an angel, a mark, a guinea, a Carolus, a Jacobus, and a moidore, add them all together and express the sum in £ *s.* *d.*

125. The following tables should be committed to memory:—

PENCE TABLE.

	s.	*d.*		*s.*	*d.*
12 pence are	1	0	84 pence are	7	0
18 "	1	6	90 "	7	6
20 "	1	8	96 "	8	0
24 "	2	0	100 "	8	4
30 "	2	6	108 "	9	0
36 "	3	0	110 "	9	2
40 "	3	4	120 "	10	0
48 "	4	0	130 "	10	10
50 "	4	2	132 "	11	0
60 "	5	0	140 "	11	8
70 "	5	10	144 "	12	0
72 "	6	0	150 "	12	6
80 "	6	8	156 "	13	0

SHILLINGS TABLE.

	£	*s.*		£	*s.*
20 shillings are	1	0	130 shillings are	6	10
30 "	1	10	140 "	7	0
40 "	2	0	150 "	7	10
50 "	2	10	160 "	8	0
60 "	3	0	170 "	8	10
70 "	3	10	180 "	9	0
80 "	4	0	190 "	9	10
90 "	4	10	200 "	10	0
100 "	5	0	300 "	15	0
110 "	5	10	400 "	20	0
120 "	6	0	500 "	25	0

ii. Addition.

126. In adding sums of money we arrange them so that the pounds stand under pounds in vertical columns, shillings under shillings, pence under pence, and farthings under farthings. Then, if we have to add together £1. 4*s.* 6¾*d.*, £9. 6*s.* 7½*d.*, £2. 6*s.* 8¼*d.*, £6. 8*s.* 4¼*d.*, £3. 6*s.* 9¼*d.*, £2. 7*s.*, we arrange them as below:—

£	*s.*	*d.*
1	4	6¾
9	6	7½
2	6	8¼
6	8	4¼
3	6	9¼
2	7	0
25	0	0

Add the farthings; 8 farthings are 2*d.*, which we carry to the pence, not writing anything under the farthings; add the pence; 36 pence are 3 shillings; write 0*d.* under the pence and carry 3 shillings to the shillings; add the shillings; 40 shillings are £2, write 0 under the shillings and carry the £2; add the pounds which now yield £25, making the total £25.

Exercise 54.

	£	*s.*	*d.*
(1)	3	6	8
	4	5	6
	5	7	8

	£	*s.*	*d.*
(2)	4	9	3
	5	6	8
	7	9	7

	£	*s.*	*d.*
(3)	6	5	7
	8	9	7
	9	9	6

	£	*s.*	*d.*
(4)	5	7	8
	8	5	7
	9	6	9

	£	*s.*	*d.*
(5)	6	8	7
	7	5	8
	9	7	11

	£	*s.*	*d.*
(6)	16	17	8¼
	15	15	5¾
	17	17	7½
	18	18	8¼

	£	*s.*	*d.*
(7)	23	19	11¾
	25	17	10½
	17	15	9½
	19	19	8¼

	£	*s.*	*d.*
(8)	25	17	6¼
	24	16	7½
	25	17	9¼
	37	14	10½

	£	*s.*	*d.*
(9)	37	15	4½
	39	17	6¾
	41	19	2½
	43	15	1¼

	£	*s.*	*d.*
(10)	65	17	7¾
	64	15	11½
	67	13	10¾
	69	12	9½

	£	*s.*	*d.*
(11)	78	16	4½
	79	18	6¼
	81	19	7¾
	83	1	0½
	85	11	7½

	£	*s.*	*d.*
(12)	87	15	0
	89	17	7¾
	91	14	11¼
	92	16	13½
	93	15	2½

	£	*s.*	*d.*
(13)	99	19	9¾
	97	17	7½
	95	15	5¼
	93	13	2½
	91	12	1¼

	£	s.	d.
(14)	93	12	8½
	87	9	0¼
	95	2	6½
	92	11	9¾
	97	13	5¼

	£	s.	d.
(15)	73	16	5½
	94	17	7¾
	85	19	9½
	75	17	7¼
	67	13	11½

	£	s.	d.
(16)	101	19	6½
	112	15	9¾
	120	0	0½
	102	17	0¾
	154	10	11¼

	£	s.	d.
(17)	150	15	7¾
	105	13	11½
	27	10	9¼
	270	15	6½
	200	10	10½

	£	s.	d.
(18)	500	15	5¼
	550	5	7¾
	50	9	11½
	705	15	9¼
	899	19	11¾

	£	s.	d.
(19)	777	17	7¼
	666	16	6½
	555	15	5¼
	444	14	4¾
	333	13	3½

	£	s.	d.
(20)	999	19	11¾
	888	18	8½
	777	17	7¾
	666	15	6½
	555	14	9½

	£	s.	d.
(21)	1111	11	11½
	2222	12	10¼
	3335	15	4½
	4444	14	5¼
	5555	15	6½
	6666	16	7¾

	£	s.	d.
(22)	8509	19	8¾
	2889	13	7½
	5087	15	8¾
	6739	10	0
	4769	15	8½
	8068	16	7¼

	£	s.	d.
(23)	3847	19	11¾
	5974	18	10½
	4795	17	9¼
	5974	16	8½
	4975	15	7¾
	9745	14	6¼

	£	s.	d.
(24)	5679	13	5½
	9765	12	6¼
	9657	11	7
	9576	10	8½
	6957	9	9¾
	9756	8	10

	£	s.	d.
(25)	6738	12	4¼
	8376	11	5½
	6378	9	6¾
	6837	10	7½
	6378	0	8¼
	8763	15	9¾

Exercise 55.

(1) £1609. 13*s*. 8¾*d*. + £3807. 15*s*. 4½*d*. + £5763. 19*s*. 5¼*d*. + £2857. 19*s*. 8¾*d*. + £4673. 15*s*. 8½*d*. + £5008. 15*s*. 4¾*d*.

(2) £1808. 15*s*. 7¾*d*. + £6739. 18*s*. 7½*d*. + £5409. 13*s*. 8¾*d*. + £2976. 18*s*. 5½*d*. + £4076. 13*s*. 8¾*d*. + £5769. 18*s*. 4½*d*.

(3) £9509. 18*s*. 6½*d*. + £8407. 13*s*. 7¾*d*. + £2957. 16*s*. 5¼*d*. + £8407. 19*s*. 8½*d*. + £8539. 13*s*. 7¾*d*. + £7398. 19*s*. 8½*d*.

(4) £8407. 19*s*. 6¾*d*. + £2958. 14*s*. 7¾*d*. + £3876. 15*s*. 2¾*d*. + £5739. 18*s*. 7½*d*. + £8407. 19*s*. 5¾*d*. + £8607. 17*s*. 8½*d*.

(5) £8509. 19*s.* 8¾*d.* + £2888. 13*s.* 7½*d.* + £5087. 15*s.* 8¾*d.* + £6739. 18*s.* 7½*d.* + £4769. 15*s.* 8½*d.* + £8068. 14*s.* 7¾*d.*

(6) £3847. 15*s.* 8¾*d.* + £2807. 13*s.* 9½*d.* + £5736. 18*s.* 4¾*d.* + £8047. 16*s.* 2½*d.* + £5739. 18*s.* 7¾*d.* + £8407. 12*s.* 8¾*d.*

(7) £8073. 19*s.* 5½*d.* + £5739. 14*s.* 8¾*d.* + £2504. 13*s.* 8½*d.* + £8607. 15*s.* 6¾*d.* + £7608. 19*s.* 7½*d.* + £8008. 19*s.* 6¼*d.*

(8) £5084. 16*s.* 8¾*d.* + £6009. 12*s.* 5½*d.* + £4873. 16*s.* 8½*d.* + £8173. 14*s.* 2¼*d.* + £5762. 19*s.* 8¾*d.* + £2987. 18*s.* 9¾*d.*

(9) £8076. 19*s.* 9½*d.* + £7045. 13*s.* 8¾*d.* + £8769. 19*s.* 5½*d.* + £7369. 15*s.* 4¾*d.* + £8476. 19*s.* 8¾*d.* + £5769. 15*s.* 4¼*d.*

(10) £3846. 15*s.* 8½*d.* + £8049. 16*s.* 7¾*d.* + £5762. 18*s.* 4¾*d.* + £9876. 15*s.* 8¾*d.* + £2052. 12*s.* 6*d.* + £9562. 18*s.* 3¾*d.*

(11) £3476. 14*s.* 5¼*d.* + £4587. 15*s.* 6*d.* + £5698. 16*s.* 7¾*d.* + £6709. 17*s.* 8*d.* + £7890. 18*s.* 9½*d.* + £8943. 19*s.* 0¼*d.*

(12) £4567. 18*s.* 9¼*d.* + £5678. 19*s.* 8¼*d.* + £6789. 14*s.* 4¾*d.* + £7890. 15*s.* 5¼*d.* + £8901. 16*s.* 6*d.* + £9094. 17*s.* 7¾*d.*

(13) £8076. 19*s.* 5¾*d.* + £2877. 15*s.* 9½*d.* + £3847. 18*s.* 7¾*d.* + £5804. 19*s.* 3¼*d.* + £2804. 13*s.* 7½*d.* + £5073. 19*s.* 6¼*d.*

(14) £8007. 13*s.* 6½*d.* + £2409. 14*s.* 8¾*d.* + £5857. 12*s.* 6¼*d.* + £3896. 13*s.* 8¾*d.* + £2807. 14*s.* 8¼*d.* + £5739. 15*s.* 7½*d.*

(15) £3754. 19*s.* 3¾*d.* + £4573. 12*s.* 8½*d.* + £3876. 12*s.* 9¾*d.* + £5784. 18*s.* 0½*d.* + £6457. 13*s.* 8¾*d.* + £8769. 12*s.* 5¼*d.*

(16) £6738. 12*s.* 6½*d.* + £3840. 9*s.* 7¼*d.* + £5739. 12*s.* 6*d.* + £4753. 18*s.* 3¾*d.* + £8040. 12*s.* 7½*d.* + £5797. 13*s.* 8¾*d.*

(17) £8576. 18*s.* 7¾*d.* + £2987. 13*s.* 7½*d.* + £5006. 14*s.* 2¾*d.* + £8007. 12*s.* 8¼*d.* + £5730. 19*s.* 6½*d.* + £5008. 12*s.* 7¾*d.*

(18) £9458. 14*s.* 9¼*d.* + £8567. 15*s.* 10½*d.* + £7676. 18*s.* 6¾*d.* + £6785. 17*s.* 7½*d.* + £5894. 16*s.* 6½*d.* + £4909. 15*s.* 5¾*d.*

(19) £8057. 19*s.* 3½*d.* + £8769. 13*s.* 6¾*d.* + £2987. 15*s.* 4½*d.* + £5847. 19*s.* 1¾*d.* + £6295. 12*s.* 8½*d.* + £8047. 19*s.* 6¾*d.*

(20) £5076. 19*s.* 11½*d.* + £2876. 15*s.* 3¾*d.* + £5847. 16*s.* 2½*d.* + £8057. 19*s.* 11¼*d.* + £7369. 18*s.* 2½*d.* + £8095. 17*s.* 11¾*d.*

NOTE 1.—These columns may be added up across, placing the totals down at once, and then copied down as Ex. 54, when the results may be compared.

NOTE 2.—And also may be used for a series of exercises of increasing difficulty. Thus, Ex. 1: add cols. 1 to 7; Ex. 2: add cols. 1 to 8, and so on. The teacher may further vary the exercises by prescribing cols. 5 to 12, 9 to 18, 13 to 20, &c.

Exercise 56.

(1) Add six hundred and eighty-seven pounds fifteen shillings and seven pence halfpenny; two hundred and thirty pounds ten shillings and fivepence farthing; four hundred and ninety-four pounds twelve shillings and twopence three-farthings; seventeen hundred and twenty-seven pounds seventeen shillings and seven pence.

(2) Find the sum of nineteen hundred pounds nineteen shillings and nine pence; eighteen hundred and eighty-five pounds eighteen shillings and eight pence halfpenny; one hundred and sixty-seven pounds sixteen shillings and six pence three farthings; twelve hundred and twenty-nine pounds twelve shillings and three pence farthing; and twelve shillings and three farthings.

(3) Add one hundred and twenty-three pounds fourteen shillings and five pence farthing; four hundred and fifty-six pounds seventeen shillings and eight pence; five hundred and sixty-seven pounds eighteen shillings and nine pence halfpenny; seven hundred and eighty-nine pounds nine shillings and three farthings.

(4) Find the sum of seven hundred pounds seventeen shillings and one halfpenny; eight hundred and forty-seven pounds nineteen shillings and nine pence; twelve hundred and three pounds fourteen shillings and four pence three farthings; sixteen hundred and sixty pounds sixteen shillings and six pence farthing; two thousand pounds two shillings and two pence halfpenny.

(5) Add three thousand three hundred and thirty-three pounds and three pence; nineteen hundred and ninety-nine pounds nineteen shillings and nine pence farthing; five pounds fifteen shillings and fivepence halfpenny; seven hundred and three pounds four shillings and eleven pence three farthings; sixteen thousand and sixteen pounds sixteen shillings and six pence farthing; one million and one pounds one shilling and one penny.

(6) Add two hundred and seventy-nine pounds ten shillings and five pence three farthings; nine hundred and eleven pounds seventeen shillings and three pence halfpenny; seven hundred and sixteen pounds fifteen shillings and ten pence farthing; five hundred and ninety-nine pounds and eleven pence halfpenny.

(7) Find the sum of two hundred and forty-three pounds fourteen shillings and nine pence halfpenny; fifty-seven pounds twelve shillings and eight pence; two hundred and eighty-six

pounds five shillings and six pence three farthings; twelve hundred and forty-seven pounds eighteen shillings and nine pence farthing; fifteen pounds four shillings and two pence three farthings; nine pounds thirteen shillings and six pence farthing; four hundred and twenty-seven pounds four shillings.

(**8**) Add together two thousand four hundred and seventy-six pounds eighteen shillings and six pence farthing; four thousand three hundred and twenty-nine pounds four shillings and eight pence halfpenny; four hundred and twenty-seven pounds two shillings and ninepence three farthings; one thousand and four pounds thirteen shillings and six pence three farthings; eleven pounds eleven shillings and eleven pence.

(**9**) Find the sum of seventeen pounds nine shillings and eleven pence halfpenny; four hundred and twenty-seven pounds eighteen shillings and ten pence three farthings; four thousand three hundred and six pounds seven shillings and nine pence farthing; seven hundred and forty-two pounds nineteen shillings and eight pence three farthings; four hundred and seven pounds twelve shillings and seven pence farthing; forty-six pounds and two pence three farthings; seventeen pounds four shillings and ten pence farthing.

(**10**) Add eighteen hundred and forty-six pounds twelve shillings and nine pence farthing; forty-seven pounds thirteen shillings and eight pence three farthings; two hundred and forty-seven pounds fifteen shillings and eleven pence farthing; three thousand and forty-six pounds ten shillings and two pence farthing; five thousand seven hundred and twenty-four pounds twelve shillings and nine pence halfpenny; forty-seven pounds nine shillings and ten pence three farthings.

(**11**) Find the sum of seven hundred and sixty-three pounds; six hundred and eighteen pounds and ninepence farthing; two hundred and thirty-two pounds ten shillings and five pence three farthings; fifty-five pounds and five pence three farthings; nineteen shillings and eleven pence halfpenny; nine hundred and fifty-four pounds seventeen shillings and ten pence three-farthings; twelve pounds twelve shillings and a halfpenny.

(**12**) Add seven hundred and eighty-nine pounds seventeen shillings and eight pence three farthings; six hundred and seventy-eight pounds and eleven pence three farthings; five hundred and sixty-seven pounds ten shillings and ten pence; two hundred and thirty-four pounds seventeen shillings and six pence farthing.

Exercise 57.

(1) A lady buys at a draper's shop cloth to value of £1. 17*s.* 9½*d.*, linen £2. 7*s.* 8½*d.*, cotton 15*s.* 11¾*d.*, silk £7. 19*s.* 3¼*d.*, and thread 10¾*d.* : what was the amount of her bill?

(2) A paid into the bank £7296. 14*s.* 8¾*d.*, B £417. 9*s.* 7½*d.*, C £3246. 17*s.* 4¼*d.*, D £29. 8*s.* 9½*d.*, E £1877. 3*s.* 11*d.*, F £824. 19*s.* 3¾*d.* : what sum does this come to?

(3) What sum must I have to give A £491. 17*s.* 7½*d.*, B £34. 5*s.* 9*d.*, C 7*s.* 4¾*d.*, D £1058. 18*s.* 10¼*d.*, E £97. 1*s.* 11¼*d.*?

(4) What is the amount of the following sums of money: £51. 13*s.* 7¾*d.*, 18*s.* 0¼*d.*, £270., £1235. 16*s.* 5*d.*, and £189. 0*s.* 9½*d.*?

(5) A merchant bought 4 lots of goods ; for the first he paid £2750. 3*s.* 11¾*d.*, for the second £106. 18*s.* 1½*d.*, for the third 3*s.* 4¼*d.*, and for the fourth £50. 7*s.* 1¾*d.* : what did he pay for the whole?

(6) A servant spent £1. 9*s.* 11½*d.* on tea, 15*s.* 6*d.* on sugar, 5*s.* 8*d.* on coffee, £3. 9*s.* 8*d.* on wine, £2. 19*s.* 0¾*d.* on other goods : what did she spend?

(7) A collector on Monday took £201. 3*s.* 5½*d.*, on Tuesday £18. 0*s.* 9¾*d.*, on Wednesday £178. 6*s.* 11¼*d.*, on Thursday £125. 17*s.* 1½*d.*, on Friday £574. 4*s.* 0*d.*, on Saturday £13. 19*s.* 7¼*d.* : what sum did he take during the week?

(8) A butcher sold in a market cows for £200. 13*s.* 2½*d.*, sheep for £23. 5*s.* 1¾*d.*, pigs for £195. 9*s.* 3½*d.*, calves for £127. 2*s.* 5*d.* : what did he receive altogether?

(9) A merchant paid the following bills in one week : to A £76. 15*s.* 10½*d.*, to B £856. 6*s.* 7*d.*, to C £18. 17*s.* 9¼*d.*, to D £7064. 15*s.*, to E £936. 5*s.* 6*d.*, to F £178. 19*s.* 8*d.*, to G £630. 9*s.* 6½*d.*, to H £504. 15*s.* 7½*d.* : how much did he pay altogether?

(10) A gentleman in his will left to his wife £6000., to his five children £350. each, to his three nephews £260. each, to a society 100 guineas, and to his two executors 30 guineas each : what was the total sum left?

(11) I spend £472. 11*s.* 4½*d.* the first year, £321. 7*s.* 8¼*d.* the second, and £541. 9*s.* 6¾*d.* the third : what do I spend in the three years?

(12) Reduce £147. 17*s.* 6½*d.* to farthings, add ten thousand

and forty-seven farthings to the result, and express the sum in £. *s.* *d.*

(**13**) A man owes his butcher £17. 8*s.* 6¼*d.*, his baker £25. 14*s.* 6¾*d.*, and his tailor £12. 19*s.* 11¾*d.* : what sum will pay these bills?

(**14**) Goods were bought for £560. 15*s.* 6*d.*; there was further paid for packing £1. 10*s.*, for carriage £12. 6*s.* 8½*d.*, and for import duty £8. 3*s.* 6¾*d.* : what must I sell the goods for to clear £10.?

(**15**) The several items of a bill amounted respectively to £18. 0*s.* 0*d.*, £3. 4*s.* 3*d.*, £26. 2*s.* 7*d.*, £7. 9*s.* 6*d.*, £44. 4*s.* 0*d.*, £4. 0*s.* 6*d.*, £2. 1*s.* 7*d.*, £6. 7*s.* 6*d.*, 13*s.* 8*d.*, and £43. 0*s.* 6*d.*: what is the gross amount?

(**16**) In furnishing a house a person expends £96. 10*s.* 3¼*d.* on one room, £78. 19*s.* 11¾*d.* on another, £56. 7*s.* 8½*d.* on a third, £35. 18*s.* 7½*d.* on a fourth, £109. 8*s.* 3¼*d.* on five others, and £268. 12*s.* 10¾*d.* for plate and other articles: what does the whole amount to?

(**17**) The following sums are given in aid of a charity: £17. 3*s.* 2*d.*, £12. 19*s.* 5½*d.*, £21. 17*s.* 11*d.*, and £24. 12*s.* 10*d.* : how much does this amount to?

(**18**) If A receive £12. 14*s.* 6¾*d.*, B £16. 17*s.* 8¼*d.*, C as much as A and B together, what will the receipts of A, B, and C be?

(**19**) An innkeeper buys three lots of spirits, for the first lot he gives £6. 14*s.* 7¾*d.*, for the second £3. 12*s.* 11½*d.*, for the third £5. 17*s.* 6*d.* : what must he sell the lot for to gain £3. 9*s.* 6¾*d.* by the outlay?

(**20**) Seven million two hundred thousand penny stamps, and nine million four hundred and six thousand halfpenny stamps were issued in a year: what is their value in £. *s.* *d.*?

(**21**) Reduce 87191268 farthings into pounds?

iii. Subtraction.

127. The process of subtracting one compound quantity from another is the same in principle as that of simple quantities already explained; the following example will make the method sufficiently clear. From £59. 13*s.* 7½*d.* take £43. 15*s.* 9¾*d.*

£ *s.* *d.*
59 13 7½—minuend.
43 15 9¾—subtrahend.
15 17 9¾—difference.

We arrange the columns as in Addition: and as 3 farthings cannot be taken from 2 farthings, add 4 farthings to the minuend and 1 penny to the subtrahend; now 3 farthings from 6 farthings leaves 3 farthings: 10 pence from 7 pence we cannot, so add 12 pence to the minuend and 1 shilling to the subtrahend; 10 from 19, 9 pence: 16 from 13 we cannot, so add 20 shillings to the minuend and £1. to the subtrahend, 16 from 33, 17 shillings, 4 from 9, 5: 4 ten pounds from 5 ten pounds leaves 1 ten pounds.

Exercise 58.

	£	s.	d.		£	s.	d.		£	s.	d.
(1)	69	14	5	(2)	107	18	3½	(3)	87	16	7
	23	8	1¾		29	4	5¼		23	8	3
(4)	99	19	6¾	(5)	100	11	6½	(6)	79	15	7½
	96	0	11½		99	12	7¾		64	17	9¾
(7)	89	14	9	(8)	100	0	0	(9)	134	15	7¼
	73	15	11¾		99	19	11¾		129	16	5½
(10)	653	14	5¼	(11)	100	19	11½	(12)	66	13	4
	596	17	8¾		98	15	4¾		59	15	8½
(13)	72	18	6¾	(14)	75	12	9½	(15)	356	14	7¾
	69	19	7		65	11	8¾		297	15	9
(16)	897	19	6¾	(17)	789	12	0¼	(18)	596	12	7¾
	875	14	5½		758	15	0¾		489	15	9½
(19)	327	12	6½	(20)	125	15	9¼				
	289	15	9¾		49	1	11¾				

Exercise 59.

Find the difference between—

	£	s.	d.		£	s.	d.
(1)	151	16	4½	and	149	17	5¾.
(2)	269	15	11¼	„	199	17	0¾.
(3)	390	0	0¼	„	299	19	11¾.
(4)	456	17	9½	„	387	19	10¾.
(5)	797	11	11¾	„	689	12	5¼.
(6)	347	9	9½	„	329	12	11¾.
(7)	298	12	7¼	„	139	19	0¾.
(8)	660	15	9½	„	303	9	11¾.

	£	s.	d.		£	s.	d.
(9)	770	0	11½	and	70	19	0¾.
(10)	880	11	7½	,,	80	19	0¾.
(11)	579	16	5¼	,,	389	17	6½.
(12)	795	17	6½	,,	637	18	7¾.
(13)	597	18	7¼	,,	25	15	9½.
(14)	788	19	6	,,	699	0	7¾.
(15)	999	15	7¼	,,	99	16	9½.
(16)	650	15	0	,,	65	19	11¾.
(17)	543	17	6½	,,	446	11	11.
(18)	565	15	0¾	,,	65	17	11½.
(19)	660	11	9	,,	560	12	11¾.
(20)	497	16	11¼	,,	99	17	0¾.

Exercise 60.

(1) Subtract two hundred and twenty-nine pounds nineteen shillings and nine pence three farthings from five hundred and forty-seven pounds sixteen shillings and a farthing.

(2) Take sixty-nine pounds sixteen shillings and eleven pence halfpenny from six hundred and eighteen pounds and three farthings.

(3) What is the difference between nine hundred and nineteen pounds nine pence three farthings and seven hundred and sixty-three pounds nineteen shillings and ten pence?

(4) If the minuend is ninety pounds and the subtrahend forty-six pounds nineteen shillings and eleven pence three farthings, what is the difference?

(5) Express in *£. s. d.* the difference between one million pence and one million farthings.

(6) From six hundred and twenty pounds fifteen shillings take five hundred and fifteen pounds and eleven pence halfpenny.

(7) Subtract six hundred and forty-six pounds nineteen shillings and three farthings from seven hundred and nineteen pounds eighteen shillings and one penny.

(8) What is the difference between eighteen shillings and seven pence halfpenny and ninety-eight pounds?

(9) If the subtrahend is four hundred and ninety pounds nineteen shillings and one farthing and the minuend six hundred and sixty-five pounds eleven shillings and tenpence, what is the difference?

(10) Express in £. *s.* *d.* the difference between one thousand groats and one thousand halfpence.

(11) Express in farthings four hundred and forty pounds seventeen shillings and sixpence, and ninety-eight pounds and tenpence. Subtract one from the other and reduce your answer to £. *s.* *d.*

(12) By how much does six hundred and sixty pounds eleven shillings and sevenpence halfpenny exceed five hundred and fifty-eight pounds and eleven pence three farthings?

(13) From sixty-nine pounds eight pence farthing take forty-seven pounds sixteen shillings and eleven pence three farthings.

(14) What sum must be added to ninety-eight pounds and eleven pence halfpenny to make five hundred and sixty-five pounds fifteen shillings?

(15) What sum must be subtracted from six hundred and sixty-six pounds sixteen shillings and six pence halfpenny to leave four hundred and eighty-nine pounds nineteen shillings and ten pence?

(16) I have one thousand groats and by mistake pass them as three-pences, what do I lose? Give your answer in farthings and £. *s.* *d.*

(17) Subtract ninety-four pounds fifteen shillings and six pence halfpenny from nine hundred and nineteen pounds nineteen shillings and three farthings.

(18) Take ninety halfcrowns from twenty pounds thirteen shillings and nine pence, and express the remainder in halfpence.

(19) Find the difference between six hundred and sixty-nine pounds nine shillings and one farthing and five hundred pounds fifteen shillings and five pence three farthings.

(20) Express in farthings the difference between one million guineas and one million pounds; and in £. *s.* *d.* the difference between the remainder and one million farthings.

Exercise 61.

Subtract each sum in the second column three times successively from the sum in the first column.

	£	s.	d.		£	s.	d.
(1)	12	18	9	−	4	6	3
(2)	24	12	8¼	−	8	4	2¾
(3)	27	17	2¼	−	9	5	8¾
(4)	23	10	4½	−	7	16	9½
(5)	16	8	8	−	4	2	2
(6)	36	16	9	−	9	4	2¼
(7)	32	19	3	−	8	4	9¾
(8)	39	10	2	−	9	17	6½
(9)	9	18	9	−	3	6	3
(10)	24	15	11¼	−	8	5	3¾
(11)	28	0	3¾	−	9	6	9¼
(12)	23	19	4½	−	7	19	9½
(13)	36	16	8	−	9	4	2
(14)	32	12	10	−	8	3	2½
(15)	28	19	11	−	7	4	11¾
(16)	39	14	1	−	9	18	8¾
(17)	24	15	9¾	−	8	5	3¼
(18)	21	12	8¼	−	7	4	2¾
(19)	15	19	2¼	−	5	6	4¾
(20)	26	3	11¼	−	8	14	7¾
(21)	28	12	8	−	7	3	2
(22)	36	14	9	−	9	3	8¼
(23)	40	19	11	−	10	4	11¾
(24)	55	19	11	−	13	19	11¾
(25)	2489	7	1½	−	829	15	8½
(26)	2948	6	8¼	−	982	15	6¾
(27)	2678	3	4½	−	892	14	5½
(28)	387	13	6½	−	129	4	6
(29)	985	12	7¾	−	328	10	10½
(30)	783	13	6½	−	261	4	6
(31)	598	12	7¾	−	199	10	10½
(32)	738	13	6½	−	246	4	6
(33)	958	12	7¾	−	319	10	10½
(34)	1299	15	6	−	292	15	0¼
(35)	1263	0	9	−	282	15	11¾
(36)	1215	12	0¾	−	276	15	9
(37)	1209	18	9	−	207	9	6¾
(38)	1246	18	0	−	274	9	9
(39)	1239	0	0	−	213	15	7¼
(40)	1233	0	9¼	−	225	15	0

Exercise 62.

(1) To pay a bill of 7¾*d.* I have one shilling: what change shall I get?

(2) If out of a sovereign I spend fifteen shillings and six pence halfpenny, what shall I have?

(3) A man leaves 1000 guineas to A, B, C—A gets £231. 9*s.* 2¼*d.*, B gets £457. 8*s.* 7½*d.*: what will C have?

(4) A has £25. 16*s.* 9½*d.*, B has £19. 12*s.* 7¾*d.*, C has £76. 8*s.* 6½*d.*, and D £37. 13*s.* 8¾*d.*; if E have as much as A and B together, and C the same as D and F together, which will have the most, E or F, and by how much?

(5) A man receives £46. 8*s.* 7¼*d.*, £235. 16*s.* 8¾*d.*, 45 guineas, and £127 14*s.* 6*d.*, and then pays £53. 12*s.* 2½*d.*, £87. 9*s.* 4¼*d.*, and £197. 15*s.* 8¾*d.*: how much will he have left?

(6) A saves 14*d.* a month and B 2*d.* a week: how much does A save more than B in a week?

(7) A cow and calf were sold for £25. 16*s.* 9½*d.*; the calf was sold for £6. 10*s.* 11¾*d.*: what did the cow fetch?

(8) If the wages of a man are £7. 2*s.* 7½*d.*, and that of a boy

£1. 0s. 4½d., how many boys would be required to earn as much as a man?

(9) The amount of a charity to be annually distributed is £73. 7s. 9d., the legal expenses amount to £12. 4s. 7½d., the sum each pauper receives. How many were there?

(10) If a bill of £23. 14s. 7¼d. be paid with a bank note of £100., what change should be received?

(11) A man having £1000. 3s. 4¾d. owes his butcher £17. 8s. 6¼d., his baker £25. 14s. 6¾d., his tailor £12. 19s. 11¾d. Find how much he has in hand after paying these bills.

(12) Goods were bought for £560. 15s. 6d.; there was further paid for packing £1. 10s., for carriage £12. 6s. 8½d., and for import duty £3. 8s. 6¾d. If the goods were sold for £655. 8s. 5d., what was the profit?

(13) If A have £456. 13s. 5¾d. and B £562. 10s. 3½d., how much has B more than A?

(14) If I give a cheque for £650. to pay a bill of £645. 17s. 9d., what ought I to receive in return?

(15) I have £100.; I give A £7. 19s. 6¾d., B £11. 0s. 10d., C £28. 3s. 4½d., D £16. 8s. 0¼d.: what have I left for myself? How many more farthings has B than A and C than D? and express the difference between these two results in £. s. d.

(16) What sum must be added to £7892. 14s. 7½d. to make £1047983. 17s. 6¾d.? Express your answer in farthings.

(17) What is the difference between £1. 0s. 1½d. + £21. 1s. 6d. + £6. 8s. 0¼d. + £1. 17s. 5½d. and £30. 7s. 0¼d.?

(18) If a boy receive £12. 4s. 7½d., a woman 5 times as much as a boy, and a man 3 times as much as a woman, what would be left out of £256. after paying the boy, woman, and man?

(19) Among how many persons may I distribute £5159. 1s. 3d. if each receive £1031. 16s. 3d.?

(20) How many articles each at £3. 7s. 10½d. can I buy with £16. 19s. 4½d.?

(21) If I have to pay 8 men each £137. 17s. 6d., and have only £1100. to do it with, how much shall I be short?

(22) If the child's fare from Oxford to Plymouth is 9s. 3½d., from London to Plymouth 8s. 3d., and from Oxford to London 2s. 3½d., what would be the fare for 2 men and 2 children from London to Oxford, thence to Plymouth, returning to London? What would be left out of £10?

(**23**) My income is £600. a year. My quarter's rent is £14. 3*s.* 6*d.*, taxes per quarter £3. 10*s.* 10½*d.*, half-year's insurance, £9. 3*s.* 4*d.*, 3 servants' wages, each per quarter £3. 16*s.* 8*d.*, £156. per annum for food, £7. 4*s.* for coals in winter, £1. 16*s.* in the summer, £33. 6*s.* 8*d.* three times a year for children's education. How much have I left for other purposes?

(**24**) My weekly expenses are, lodgings 15*s.* 9*d.*, £1. 1*s.* for board, 4*s.* 6*d.* for travelling, 8*s.* 4*d.* for sundries. How long will £10. last me, and what will be the condition of my funds in 5 weeks?

(**25**) If I have £100. in the bank, and put by each year £177. 7*s.* 6*d.*, how long shall I take to save £1164. 5*s.*?

(**26**) If a man owes £253. 0*s.* 5*d.*, £6. 2*s.* 3*d.*, £12. 10*s.* 0*d.*, £6. 0*s.* 0*d.*, £10. 0*s.* 0*d.*, £2. 7*s.* 6½*d.*, £2. 14*s.* 5¾*d.*, £3. 12*s.* 3*d.*, and can pay only £191. 3*s.* 4*d.*, how much will his creditors lose?

(**27**) A house, fixtures, and furniture cost £1160; the fixtures cost £27. 10*s.*, the furniture £335.: what is the cost of the house?

(**28**) If my daily expenses come to £1. 9*s.* 7*d.*, for how many weeks will £51. 15*s.* 5*d.* last me?

(**29**) The weekly wage of 1 man is £3. 7*s.* 2¾*d.*: how many men can I employ for a month with £53. 15*s.* 8*d.*?

(**30**) If I spend £4. 3*s.* 2*d.* a day, how many weeks will £87. 6*s.* 6*d.* last me?

iv. Multiplication.

128. To multiply a compound expression, as £7. 8*s.* 9¾*d.*, by a number, say 6, is the same thing as finding the sum of six expressions each = £7. 8*s.* 9¾*d.*; the operation being thus performed:—

$$\begin{array}{rrr} \pounds & s. & d. \\ 7 & 8 & 9\frac{3}{4} \\ & & 6 \\ \hline 44 & 12 & 10\frac{1}{2} \end{array}$$

6 times 3 farthings are 18 farthings; put down 2 farthings and carry 4 pence; 6 times 9 pence are 54 pence, which, with the 4 pence carried, make 58 pence; put down 10 pence and carry 4 shillings; 6 times 8 shillings are 48 shillings, which, with the 4 shillings carried, make 52 shillings; put down 12 shillings and carry 2 pounds; 6 times 7 pounds are 42 pounds, which, with the 2 pounds carried, make £44; total being £44. 12*s.* 10½*d.*

Exercise 63.

Multiply each of the following sums of money separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 :—

(1) £3. 6*s.* 5¼*d.* (2) £4. 7*s.* 2¾*d.* (3) £5. 8*s.* 3½*d.*
(4) £9. 17*s.* 7¾*d.* (5) £11. 12*s.* 11½*d.* (6) £13. 14*s.* 11½*d.*
(7) £15. 15*s.* 5¾*d.* (8) £17. 17*s.* 10¼*d.* (9) £19. 19*s.* 7¾*d.*
(10) £21. 13*s.* 7¼*d.* (11) £341. 15*s.* 6½*d.* (12) £456. 17*s.* 9½*d.*
(13) £697. 0*s.* 0¾*d.* (14) £891. 11*s.* 0¼*d.* (15) £198. 12*s.* 6½*d.*
(16) £879. 19*s.* 9¾*d.* (17) £989. 11*s.* 11¾*d.* (18) £909. 0*s.* 10½*d.*
(19) £673. 15*s.* 0½*d.* (20) £999. 19*s.* 11¾*d.*

129. To multiply by a number between 12 and 20 we proceed thus :—

Find the value of £7. 16*s.* 4¾*d.* × 17.

Now £7. 16*s.* 4¾*d.* × 17 means that £7. 16*s.* 4¾*d.* is to be repeated 17 times, which is the same as the sum of £7. 16*s.* 4¾*d.* repeated 10 times and £7. 16*s.* 4¾*d.* repeated 7 times—*i.e.*, £7. 16*s.* 4¾*d.* × 10 + £7. 16*s.* 4¾*d.* × 7 ; the work therefore may be arranged thus :—

£	*s.*	*d.*	
7	16	4¾	—A
		10	
78	3	11½	—B = 10 × A
54	14	9¼	—C = 7 × A
132	18	8¾	—D = 17 × A

Here line D = sum of lines B, C = 10A + 7A = 17A.

∴ £132. 18*s.* 8¾*d.* = £7. 16*s.* 4¾*d.* × 17.

Exercise 64.

Multiply each of the following sums of money separately by 13, 14, 15, 16, 17, 18, 19 :—

(1) £5. 16*s.* 4¾*d.* (2) £7. 19*s.* 11¾*d.* (3) £9. 11*s.* 9½*d.*
(4) £11. 12*s.* 10¼*d.* (5) £13. 13*s.* 3½*d.* (6) £23. 19*s.* 11¼*d.*
(7) £35. 11*s.* 10½*d.* (8) £77. 9*s.* 11¾*d.* (9) £99. 11*s.* 9¾*d.*
(10) £567. 14*s.* 10½*d.*

130. The method by which we multiply by a number between 20 and 100 will be easily understood from the following examples :—

N.B.—In Exercises 64-69, do the first sum in each group first, then the second, and so on.

Find the value of £11. 11*s.* 11¾*d.* × 89.

£	*s.*	*d.*	
11	11	11¾	—A
		10	
115	19	9½	—B = 10A
		8	
927	18	4	—C = 8B = 80A
104	7	9¾	—D = 9A
1032	6	1¾	—E = 89A

Line E = sum of the lines C, D = 8B + 9A = 80A + 9A = 89A.
∴ £1032. 6*s.* 1¾*d.* = £11. 11*s.* 11¾*d.* × 89.

Exercise 65.

	£	*s.*	*d.*	
(1)	34	16	2¾	× 23, 31, 39, 37, 43, 47.
(2)	73	15	6½	× 53, 57, 65, 69, 78, 79.
(3)	89	17	11¼	× 85, 89, 87, 93, 95, 97.
(4)	98	17	2¼	× 25, 33, 41, 48, 49, 34, 29.
(5)	84	18	7¾	× 56, 59, 51, 66, 63, 73, 71.
(6)	67	19	5½	× 82, 83, 89, 92, 94, 99.

131. The following example will explain the method of multiplying by numbers between 100 and 1000:—

Find the value of £13. 16*s.* 4¼*d.* × 897.

£	*s.*	*d.*	
13	16	4¼	—A
		10	
138	3	6½	—B = 10 × A
		10	
1381	15	5	—C = 10 × B = 100 × A
		8	
11054	3	4	—D = 8 × C = 80 × B = 800 × A
1243	11	10½	—E = 9 × B = 90 × A
96	14	5¾	—F = 7 × A
12394	9	8¼	—G = 897 × A

∴ £12394. 9*s.* 8¼*d.* = £13. 16*s.* 4¼*d.* × 897.

Exercise 66

	£	s.	d.
(1)	4	17	6½ × 165, 324, 435, 657.
(2)	38	16	7¾ × 678, 789, 895, 387.
(3)	387	19	11¼ × 748, 853, 605, 880.
(4)	456	12	11¾ × 763, 637, 367, 459, 986, 869.
(5)	798	19	10½ × 730, 859, 736, 706, 987, 789.
(6)	897	16	5¼ × 956, 965, 809, 999, 908, 777.

132. We will work one more example to illustrate how the multiplication by numbers higher than 1000 may be effected.

Find the value of £87. 13*s.* 8¾*d.* × 5729.

1st Method.

£	s.	d.			
87	13	8¾	—A		
		10			
876	17	3½	—B =		10 A
		10			
8768	12	11	—C = 10 × B	=	100 A
		10			
87686	9	2	—D = 10 × C	=	100 × B = 1000 × A
		5			
438432	5	10	—E = 5 D	=	5000 × A
61380	10	5	—F = 7 C	=	700 × A
1753	14	7	—G = 2 B	=	20 × A
789	3	6¾	—H	=	9 × A
502355	14	4¾	—K	=	5729 × A

2nd Method.

£87. 13*s.* 8¾*d.* × 5729 = £87. × 5729 + 13*s.* × 5729 + 8*d.* × 5729 + ¾*d.* × 5729.

	£	s.	d.
£87. × 5729 =	498423	0	0
13*s.* × 5729 =	3723	17	0
8*d.* × 5729 =	190	19	4
¾*d.* × 5729 =	17	18	0¾
∴ £87. 13*s.* 8¾*d.* × 5729 =	502355	14	4¾

This method is evidently the simplest in cases of large numbers; in any case, either method may be used to *prove* the other.

Exercise 67.

£ s. *d.*
(1) 47 13 6¼ × 3456, 4543, 5789, 4635.
(2) 74 18 9¾ × 9876, 5403, 7083, 8730.
(3) 38 19 5½ × 7836, 9279, 8376, 5926.
(4) 79 19 11¾ × 6953, 7004, 4700, 5073.
(5) 89 12 6½ × 6543, 5434, 8975, 6435.
(6) 99 13 4½ × 7038, 7058, 5008, 6706.
(7) 74 12 11¼ × 6534, 4365, 5643, 3456, 1234.
(8) 47 13 10½ × 6785, 7008, 7060, 9800, 4321.
(9) 83 12 6¼ × 4783, 5874, 7483, 9367, 5678.
(10) 97 19 11¾ × 9080, 6307, 9259, 8748, 8675.
(11) 98 11 7½ × 8009, 6037, 5299, 7848, 7658.
(12) 88 12 5¼ × 3596, 4007, 4700, 3705, 8643.

133. If the multiplier be the product of 2 or more factors, each of which is not greater than 12, we multiply the compound expression first by either of the factors, and then the product by another of the factors. Thus, to find the value of £56. 13*s.* 11¼*d.* × 72, there are three modes by which the result is obtained.

	£	*s.*	*d.*
(i.)	56	13	11¼
			10
	566	19	4½
			7
	3968	15	7½
	113	7	10½
	4082	3	6

	£	*s.*	*d.*
(ii.)	56	13	11¼
			8
	453	11	6
			9
	4082	3	6

	£	*s.*	*d.*
(iii.)	56	13	11¼
			9
	510	5	5¼
			8
	4082	3	6

It will be seen that the multiplication by factors (as in ii. and iii.) requires the fewest figures; it is therefore the more elegant, and enables us by altering the order of the factors to test the accuracy of the result.

nd three hundred and thirteen
nd three pence farthing by three

three hundred and fourteen thou-
irty pounds fourteen shillings and
by seventy-two.

ive thousand and eighteen pounds
ence farthing by eighteen.

usand and sixty-two pounds nineteen
farthing by two hundred and twenty.

t of eighty-two thousand six hundred
ine shillings and nine pence halfpenny
enty-four.

nty-six thousand three hundred and sixty-
shillings and five pence farthing by five

value of nine thousand eight hundred and
en shillings and eleven pence farthing
ndred and one?

irteen thousand eight hundred and forty-one
s and six pence halfpenny by two hundred

product of eighty-two thousand six hundred
s ten shillings and three farthings by three
-three.

Exercise 70.

the price of 32 yards of ribbon at 1*s.* 4¾*d.* per

on spend 15*s.* 3*d.* in one day, what will he spend

persons hire an omnibus for the day at £1. 17*s.*
h would it cost if they each paid 7½*d.* more?

an spends as much in four months as he gains in
ust be his income if he save £37. 10*s.* at the end

many crowns, halfcrowns, sixpences, and groats are

an's weekly wage amount to £2. 14*s.* 2¼*d.*, what
f 327 men be?

Exercise 68.

	£	s.	d.	
(1)	9	16	10½	× 28, 40, 35, 45, 55.
(2)	39	14	7¾	× 36, 42, 48, 77, 88.
(3)	75	19	6¾	× 64, 66, 72, 81, 96.
(4)	77	18	11½	× 55, 64, 80, 84, 100.
(5)	88	11	11¾	× 132, 144, 121, 110, 99.
(6)	99	16	2¼	× 108, 120, 240, 288, 242.
(7)	27	4	11¼	× 140, 160, 245, 225, 330.
(8)	117	10	6½	× 180, 294, 384, 385, 228.
(9)	150	18	1½	× 320, 462, 504, 648, 288.
(10)	155	6	11¼	× 165, 448, 560, 336, 672.
(11)	264	11	9¾	× 396, 432, 363, 330, 297.
(12)	396	16	2¼	× 540, 600, 1200, 2016, 1692.

Exercise 69.

(1) Multiply five million eight hundred and four thousand three hundred and seven pounds nine shillings and nine pence farthing by one hundred and two.

(2) What is the product of four hundred and fourteen thousand five hundred and seventeen pounds nine shillings and three pence halfpenny by seventy-seven?

(3) Multiply four hundred and thirty thousand and sixteen pounds ten shillings and ten pence three farthings by ten thousand and ten.

(4) Multiply sixteen thousand and sixty-nine pounds thirteen shillings and two pence farthing by two hundred and nine.

(5) Find the product of four hundred and fourteen thousand five hundred and seventeen pounds nine shillings and three pence halfpenny by seven hundred and seven.

(6) Multiply nine million nineteen thousand and nineteen pounds fourteen shillings and six pence by nineteen thousand and nineteen.

(7) What is the value of five hundred and three thousand eight hundred and twenty-one pounds eleven shillings and a penny multiplied by one hundred?

(**8**) Multiply three thousand three hundred and thirteen pounds fourteen shillings and three pence farthing by three hundred and thirteen.

(**9**) Find the product of three hundred and fourteen thousand nine hundred and thirty pounds fourteen shillings and seven pence three farthings by seventy-two.

(**10**) Multiply twenty-five thousand and eighteen pounds eleven shillings and five pence farthing by eighteen.

(**11**) Multiply forty thousand and sixty-two pounds nineteen shillings and four pence farthing by two hundred and twenty.

(**12**) Find the product of eighty-two thousand six hundred and forty-nine pounds nine shillings and nine pence halfpenny by two hundred and twenty-four.

(**13**) Multiply seventy-six thousand three hundred and sixty-nine pounds thirteen shillings and five pence farthing by five hundred and fifteen.

(**14**) What is the value of nine thousand eight hundred and fifty pounds nineteen shillings and eleven pence farthing multiplied by two hundred and one?

(**15**) Multiply thirteen thousand eight hundred and forty-one pounds five shillings and six pence halfpenny by two hundred and twenty-seven.

(**16**) Find the product of eighty-two thousand six hundred and seventy pounds ten shillings and three farthings by three hundred and thirty-three.

Exercise 70.

(**1**) What is the price of 32 yards of ribbon at 1*s.* 4¾*d.* per yard?

(**2**) If a person spend 15*s.* 3*d.* in one day, what will he spend in 365 days?

(**3**) A dozen persons hire an omnibus for the day at £1. 17*s.* 6*d.*: how much would it cost if they each paid 7½*d.* more?

(**4**) If a man spends as much in four months as he gains in three, what must be his income if he save £37. 10*s.* at the end of the year?

(**5**) How many crowns, halfcrowns, sixpences, and groats are there in £25?

(**6**) If a man's weekly wage amount to £2. 14*s.* 2¼*d.*, what will the wage of 327 men be?

(7) I bought 37 articles and gave 12*s.* 10¼*d.* each for them: what did they cost me?

(8) If I save 11*s.* 6¾*d.* a day, what shall I save in a week, and in 365 days?

(9) 2 quarters of barley cost as much as 3 quarters of oats, or as much as 1 quarter of wheat; if a merchant buys 425 quarters of each kind of grain and gives 14*s.* a quarter for the oats, what sum must he spend?

(10) If A have 115 halfcrowns and B 115 shillings, how much have they between them? and how much more has A than B?

(11) If one mina = £4. 1*s.* 3*d.*, reduce 140 minæ to £. *s.* *d.*

(12) 27 boys each have a purse containing 1 groat, 2 testers, 3 nobles, 4 angels, 5 marks, 6 guineas, 7 Caroli, 8 Jacobi, 9 moidores: find in £. *s.* *d.* the amount they have between them.

(13) A earns £2. 16*s.* 5¾*d.* and spends £1. 19*s.* 6½*d.* each day; B earns £9. 5*s.* 11*d.* × 7 a month and spends £14. 3*s.* 9½*d.* a week: find how much each earns and spends during the year.

(14) If every 1*d.* in the £. of income tax add £1,000,000 to the revenue, what is the total amount of income on which the tax is levied?

(15) The village A contains 8 cottages, each inhabited by 7 persons; the village B contains 7 cottages, each inhabited by 8 persons: if each person receive the same sum, but a cottage in B receive £16. 15*s.* 8¼*d.* more than a cottage in A, how much will each village receive?

(16) If I buy 112 lbs. of tea at 3*s.* 9*d.* per lb. what must I sell it for to gain £5. 12*s.* 6*d.*?

(17) Show that 75 yards of silk at 5*s.* 6*d.* per yard cost as much as 33 yards of cloth at 12*s.* 6*d.* a yard.

(18) A mixture is made of 25 gallons of spirit at 23*s.* a gallon, 32 gallons at 21*s.* 6*d.*, 42 gallons at 25*s.*, and 21 gallons at 27*s.*: if 2*s.* is gained on each gallon by selling it, what is the whole prime cost, selling price, and profit?

(19) If a farmer buy 30 oxen at £11. 5*s.* apiece, and, after losing 4, sell the remainder for £13. 15*s.* apiece, what does he gain?

(20) How much must I have to distribute among 8 men, 12 women, and 15 children, giving each man twice as much as each woman, each woman three times as much as each child, a child's share being 2*s.* 11*d.*?

(21) What would be the value of 465 bags if each contained

a sovereign, a half-sovereign, a crown, a florin, a shilling, a sixpence, and a threepence?

(**22**) If I give 35 yards of cloth at 12*s.* 5*d.* a yard in exchange for 135 lbs. of tea at 3*s.* 2½*d.* a lb., what shall I gain?

(**23**) How much shall I require to deduct from £197. 11*s.* 6*d.* to give 23 persons each £8. 11*s.* 9½*d.*?

(**24**) A tea merchant mixes 25 lbs. of tea at 1*s.* 9*d.* a lb., 40 lbs. at 2*s.* 5*d.*, and 27 lbs. at 3*s.* 2*d.*: he sells the mixture at 2*s.* 5½*d.* a lb.: what does he gain?

(**25**) A man has a yearly income of £486. 15*s.*: if he spend £8. 2*s.* 11¾*d.* every week, what would he save by the end of the year?

(**26**) A boy's wage is 6*s.* 3*d.*, a woman's twice as much, a man's as much as a boy's and woman's together: what will be the wages of 7 men, 9 women, and 11 boys?

(**27**) If a purse is worth 3*s.* 6*d.* and contains 10 times its value in money, what is the value of the purse and money together?

(**28**) What annual income would enable a person to spend 9*s.* 8*d.* a day and save £8. 11*s.* 10½*d.* every calendar month?

(**29**) A man exchanges 47 sheep at £2. 6*s.* 8*d.*, and 39 pigs at £3. 13*s.* 6*d.* for 15 oxen at 17½ guineas: how much does he gain or lose?

(**30**) In a mill 14 women are employed at 2*s.* 10*d.* a day, five times as many men at 5*s.* 6*d.* a day, and 6 times as many boys at 2*s.* 4*d.* a day: what do the weekly wages at the mill amount to?

v. Division.

134. The process by which we divide a compound expression will be easily understood from the principles already explained when treating of simple division. The following example will illustrate the method to be adopted:—

Find the value of £54. 14*s.* 9¼*d.* divided by 7.

$$\begin{array}{r@{\,}rrl} & \pounds & s. & d. \\ 7) & \underline{54} & \underline{14} & \underline{9\tfrac{1}{4}} \\ & 7 & 16 & 4\tfrac{3}{4}\text{—quotient} \end{array}$$

We reason thus:—7 in £54., 7 times, put down the £7. and carry the £5. or 100 shillings—we now have 114 shillings; 7 in 114 shillings 16 times, put down the 16 shillings and carry the 2 shillings or 24 pence—we now have 33 pence; 7 in 33 pence 4 times, put down the 4 pence and carry the 5 pence or 20 farthings—we now have 21 farthings; 7 in 21 farthings 3, put down the 3 farthings.

$$\therefore \pounds 54.\ 14s.\ 9\tfrac{1}{4}d. \div 7 = \pounds 7.\ 16s.\ 4\tfrac{3}{4}d.$$

Exercise 71.

Divide—

(1) £6. 12*s*. 10*d*., £14. 14*s*. 7½*d*., £16. 15*s*. 7½*d*., £19. 15*s*. 3*d*., £8. 14*s*. 4*d*., £16. 18*s*. 5*d*. by 2.

(2) £12. 18*s*. 9*d*., £24. 12*s*. 8¼*d*., £27. 17*s*. 2¼*d*., £23. 10*s*. 4½*d*., £9. 18*s*. 9*d*., £24. 15*s*. 11¼*d*. by 3.

(3) £32. 19*s*. 3*d*., £39. 10*s*. 2*d*., £28. 19*s*. 11*d*., £39. 14*s*. 11*d*., £40. 19*s*. 11*d*., £55. 19*s*. 11*d*. by 4.

(4) £34. 9*s*. 10¾*d*., £4148. 18*s*. 6½*d*., £28. 13*s*. 10¼*d*., £4913. 17*s*. 9¾*d*., £49. 19*s*. 10¾*d*., £4463. 12*s*. 3½*d*. by 5.

(5) £41. 7*s*. 10½*d*., £34. 8*s*. 7½*d*., £59. 19*s*. 10½*d*., £4582. 8*s*. 10½*d*., £2260. 13*s*. 7½*d*., £4041. 10*s*. 10½*d*. by 6.

(6) £48. 5*s*. 10¼*d*., £40. 3*s*. 4¾*d*., £69. 19*s*. 10¼*d*., £5808. 9*s*. 11½*d*., £6879. 8*s*. 11¼*d*., £6249. 1*s*. 2½*d*. by 7.

(7) £55. 3*s*. 10*d*., £45. 18*s*. 2*d*., £79. 19*s*. 10*d*., £6109. 18*s*. 6*d*., £3014. 4*s*. 10*d*., £5388. 14*s*. 6*d*. by 8.

(8) £348. 11*s*. 9¾*d*., £340. 12*s*. 7½*d*., £689. 14*s*. 2¼*d*., £7468. 1*s*. 4½*d*., £8845. 0*s*. 0¾*d*., £8034. 10*s*. 1½*d*. by 9.

(9) £387. 6*s*. 5½*d*., £378. 9*s*. 7*d*., £766. 6*s*. 10½*d*., £7637. 8*s*. 1½*d*., £3767. 16*s*. 0½*d*., £6735. 18*s*. 1½*d*. by 10.

(10) £424. 1*s*. 1¼*d*., £416. 6*s*. 6½*d*., £842. 19*s*. 6¾*d*., £848. 2*s*. 2½*d*., £832. 13*s*. 1*d*., £1685. 19*s*. 1½*d*. by 11.

(11) £464. 15*s*. 9*d*., £454. 3*s*. 6*d*., £919. 12*s*. 3*d*., £9957. 8*s*. 6*d*., £11793. 6*s*. 9*d*., £10712. 13*s*. 6*d*. by 12.

135. If the divisor be greater than 12 and can be split up into factors we divide first by one of the factors, then divide the quotient by one of the other factors without any regard to order, and proceed in this way until all the factors have been used as divisors.

For example, to divide £8004. 6*s*. 6*d*. by 56 the factors of which are 8 and 7, or 2, 4, and 7.

$$\begin{array}{rrrrr} & & \pounds & s. & d. \\ 56 & \{\ 8) & 8004 & 6 & 6 \\ \cline{3-5} & 7) & 1000 & 10 & 9\frac{3}{4} \\ \cline{3-5} & & 142 & 18 & 8\frac{1}{4} \end{array} \qquad \begin{array}{rrrrr} & & \pounds & s. & d. \\ & 2) & 8004 & 6 & 6 \\ \cline{3-5} 56 & \{\ 4) & 4002 & 3 & 3 \\ \cline{3-5} & 7) & 1000 & 10 & 9\frac{3}{4} \\ \cline{3-5} & & 142 & 18 & 8\frac{1}{4} \end{array}$$

$$\therefore \pounds 8004.\ 6s.\ 6d. \div 56 = \pounds 142.\ 18s.\ 8\tfrac{1}{4}d.$$

Exercise 72.

Divide—

(1) £421. 7*s.* 7¼*d.*, £475. 13*s.* 10¼*d.*, £367. 10*s.* 6½*d.* by 55

(2) £459. 13*s.* 9*d.*, £518. 18*s.* 9*d.*, £400. 18*s.* 9*d.* by 60.

(3) £490. 6*s.* 8*d.*, £553. 10*s.* 8*d.*, £427. 13*s.* 4*d.* by 64.

(4) £505. 13*s.* 1½*d.*, £570. 16*s.* 7½*d.*, £441. 0*s.* 7½*d.* by 66.

(5) £551. 12*s.* 6*d.*, £622. 14*s.* 6*d.*, £481. 2*s.* 6*d.* by 72.

(6) £2289. 6*s.* 1½*d.*, £1977. 1*s.* 1¼*d.*, £3751. 18*s.* 1¼*d.* by 77.

(7) £2378. 10*s.* 0*d.*, £2054. 1*s.* 8*d.*, £3898. 1*s.* 8*d.* by 80.

(8) £2408. 4*s.* 7½*d.*, £2079. 15*s.* 2¼*d.*, £3946. 16*s.* 2¼*d.* by 81.

(9) £2497. 8*s.* 6*d.*, £2156. 15*s.* 9*d.*, £4092. 19*s.* 9*d.* by 84.

(10) £2616. 7*s.* 0*d.*, £2259. 9*s.* 10*d.*, £4287. 17*s.* 10*d.* by 88.

(11) £6657. 16*s.* 10½*d.*, £3403. 0*s.* 7½*d.*, £6163. 8*s.* 1½*d.* by 90.

(12) £7101. 14*s.* 0*d.*, £3629. 18*s.* 0*d.*, £6574. 6*s.* 0*d.* by 96.

(13) £7323. 12*s.* 6¾*d.*, £3743. 6*s.* 8¼*d.*, £6779. 14*s.* 11¼*d.* by 99.

(14) £7397. 12*s.* 1*d.*, £3781. 2*s.* 11*d.*, £6848. 4*s.* 7*d.* by 100.

(15) £8137. 7*s.* 3½*d.*, £4159. 5*s.* 2½*d.*, £7533. 1*s.* 0½*d.* by 110.

(16) £6334. 1*s.* 9*d.*, £5905. 18*s.* 3*d.*, £9467. 6*s.* 6*d.*, by 108.

(17) £7037. 17*s.* 6*d.*, £6562. 2*s.* 6*d.*, £10519. 5*s.* 0*d.* by 120.

(18) £7096. 10*s.* 5¾*d.*, £6616. 16*s.* 2¼*d.*, £10606. 18*s.* 2½*d.* by 121.

(19) £7741. 13*s.* 3*d.*, £7218. 16*s.* 9*d.*, £11571. 3*s.* 6*d.* by 13

(20) £8445. 9*s.* 0*d.*, £7874. 11*s.* 0*d.*, £12623. 2*s.* 0*d* by 144.

N.B.--The results in these two sets of examples may be tested by multiplication, or, in the last set, by reversing the order of the factors.

136. If the divisor be a number greater than 12, and likewise a prime number, the operation of finding the quotient is commonly called 'Long Division,' the principles on which it is conducted being already explained in the case of Simple Division.

It will be profitable, however, to give a more detailed account of the method, which we will do in the subjoined example.

Divide £743405. 15*s.* 10½*d.* by 8478.

The beginner here meets at once with the practical difficulty that he only knows the multiplication table up to twelve; we will therefore make a multiplication table for him on this occasion—a plan he will find useful to adopt until some considerable facility has been acquired, when it may be omitted.

8478 × 1 = 8478
8478 × 2 = 16956
8478 × 3 = 25434
8478 × 4 = 33912
8478 × 5 = 42390
8478 × 6 = 50868
8478 × 7 = 59346
8478 × 8 = 67824
8478 × 9 = 76302

```
          £      s.   d.   £   s.  d.
     8478)743405 15   10½(87  13   8¾
          67824
          ─────
           65165
           59346
           ─────
            5819
              20
          ──────
          116395
            8478
          ──────
           31615
           25434
           ─────
            6181
              12
           ─────
           74182
           67824
           ─────
            6358
               4
           ─────
           25434
           25434
           ─────
           .....
```

Proceeding as in 'Simple Division,' we see, on reference to the table, that 8478 × 9 is greater, and 8478 × 8 is less than 74340; we therefore place the 8 in the quotient and write down the difference between 74340 and 67824, which is 6516; annexing to this the next figure in the dividend we have 65165, which in the same way contains 8478, 7 times, with a remainder £5819. Converting these into shillings we obtain, with the 15 shillings in the dividend, 116395 shillings, which contains 8478, 13 times, with a remainder 6181 shillings. Converting these into pence we obtain, with the 10 pence in the dividend, 74182 pence, which contains 8478, 8 times, with a remainder 6358 pence. Converting these into farthings we obtain, with the 2 farthings in the dividend, 25434 farthings, which contains 8478 exactly 3 times.

∴ £743405. 15*s.* 10½*d.* ÷ 8478 = £87. 13*s.* 8¾*d.*

Exercise 73.

Find the value of—

	£	s.	d.		£	s.	d.	
(1)	452	10	11¾	÷ 13	1919	8	1½	÷ 51
	3673	5	7½	÷ 75	8044	16	4¼	÷ 95
(2)	745	19	3¼	÷ 17	1499	8	10	÷ 52
	6454	17	1	÷ 76	9397	17	8½	÷ 97
(3)	1115	10	4¼	÷ 19	3602	11	3½	÷ 53
	7711	0	7½	÷ 78	6549	3	1½	÷ 98
(4)	1496	17	10¼	÷ 43	2672	2	3½	÷ 71
	4505	17	10	÷ 92	9484	8	4	÷ 112
(5)	2018	9	9½	÷ 46	2104	19	8½	÷ 73
	7898	14	0¾	÷ 93	10948	1	0½	÷ 113
(6)	2759	8	9¼	÷ 47	5029	19	11	÷ 74
	9292	15	7½	÷ 94	7618	8	1½	÷ 114
(7)	596	4	3	÷ 156	16836	5	10½	÷ 678
	24479	17	10½	÷ 629	328395	2	8¼	÷ 847
(8)	588	4	5¼	÷ 123	23652	6	3	÷ 678
	66868	4	1¾	÷ 763	231052	15	6½	÷ 506
(9)	693	3	1½	÷ 123	26881	19	10½	÷ 678
	57218	19	3	÷ 756	273480	1	0½	÷ 730
(10)	24966	0	0	÷ 3456	215150	17	4½	÷ 5689
	472152	6	5¼	÷ 6387	746743	2	3	÷ 3596
(11)	26650	16	0	÷ 3456	181502	13	9¾	÷ 3807
	622272	3	6¾	÷ 8307	261476	5	8¾	÷ 6709
(12)	37575	0	9	÷ 4356	140919	18	11	÷ 5876
	176791	12	1	÷ 3874	796193	0	10	÷ 9080

137. In all the above examples the division is exact—*i.e.*, there is no remainder.

Let us now consider the following example—£3796. 12*s.* 6¼*d* ÷ 457 :—

Ex. 1.

```
       £    s.  d.  £  s.  d.
457)3796   12  6¼(8   6  1¾
    3656
    ----
     140
      20
    ----
    2812
    2742
    ----
      70
      12
    ----
     846
     457
    ----
     389
       4
    ----
    1557
    1371
    ----
     186
```

Ex. 2.

```
       £    s.  d.  £  s.  d.
457)3796    8  7¾(8   6  1¾
    3656
    ----
     140
      20
    ----
    2808
    2742
    ----
      66
      12
    ----
     799
     457
    ----
     342
       4
    ----
    1371
    1371
    ----
    ....
```

i.e., if we distributed £3796. 12*s.* 6¼*d.* between 457 persons, each one would receive £8. 6*s.* 1¾*d.*, and we should have 186 farthings, or 3*s.* 10½*d.*, over: subtracting this sum from the dividend the division becomes exact, as we see from Ex. 2.

Exercise 74.

In the following examples find what sum must be subtracted from the dividend to make the division exact:—

	£	s.	d.	
(1)	38703	12	6¼	÷ 57, 238, 4567, 3857.
(2)	47613	13	7½	÷ 69, 844, 2763, 9854.
(3)	73901	15	4¾	÷ 78, 576, 3879, 5804.
(4)	83703	13	7¾	÷ 93, 546, 6457, 8537.
(5)	76385	13	4¼	÷ 47, 649, 3085, 7623.
(6)	85046	15	8¼	÷ 76, 943, 2804, 6732.
(7)	80016	3	7¾	÷ 79, 940, 2089, 5076.
(8)	90573	12	3½	÷ 57, 487, 3807, 6793.

Exercise 75.

(1) Divide seventy thousand eight hundred and fifty-six pounds fourteen shillings and ten pence farthing by twenty-nine. Write your answer in words.

(2) Divide eighty-six thousand and forty-three pounds ten shillings and ten pence farthing by thirty-four. Write your answer in words.

(3) Divide nineteen thousand nine hundred and eleven pounds four shillings and five pence three farthings by forty-seven. Prove your answer.

(4) Divide thirty-five thousand three hundred and sixteen pounds thirteen shillings and seven pence halfpenny by fifty-three.

(5) Divide sixty-seven thousand eight hundred and nine pounds twelve shillings and six pence farthing by sixty-seven.

(6) Divide eighteen thousand and nineteen pounds sixteen shillings and seven pence three farthings by seventy-nine.

(7) Divide nine hundred and eighty-seven pounds nineteen shillings and six pence three farthings by seventy-two.

(8) Divide six thousand five hundred and twenty-nine pounds ten shillings and eight pence farthing by seventy-seven.

(9) Divide seven hundred and forty-three pounds thirteen shillings and seven pence halfpenny by eighty-one.

(10) Divide three thousand four hundred and fifty-nine pounds six shillings and ten pence three farthings by eighty-nine.

(11) Divide three hundred and ninety-five pounds sixteen shillings and nine pence halfpenny by eighty-five.

(12) Divide four hundred and twenty-six pounds twelve shillings and four pence three farthings by sixty-nine.

(13) Divide nine hundred and forty-seven pounds fifteen shillings and nine pence three farthings by seventy-six.

(14) Divide six thousand five hundred and sixty-nine pounds eighteen shillings and eight pence farthing by eighty-five.

(15) Divide five thousand four hundred and seventy-eight pounds sixteen shillings and seven pence by three hundred and twenty-six.

(16) Divide nine thousand seven hundred and twenty-three

pounds fourteen shillings and three pence farthing by five hundred and eighty-seven.

(17) Divide eight thousand five hundred and thirty-eight pounds ten shillings and eleven pence farthing by eight hundred and twenty-five.

(18) Divide seven thousand four hundred and sixty-two pounds eleven shillings and six pence three farthings by seven hundred and sixty-nine.

(19) Divide fifty-one million six hundred and eighty-four thousand nine hundred and thirty-two pounds seventeen shillings and four pence halfpenny by three hundred and sixty-four.

(20) Divide seven thousand six hundred and ninety-two pounds fifteen shillings and five pence by six hundred and seventy-nine.

138. In ascertaining how many times one quantity is contained in another quantity we shall find the method of cumulative subtraction frequently so long as to be practically useless. A simpler method is to reduce each quantity to a common denomination and then proceed by Division. Thus :—

How many times is £4. 15*s*. 7¾*d*. contained in £588. 4*s*. 5¼*d*.?

This can be symbolised—£588. 4*s*. 5¼*d*. ÷ £4. 15*s*. 7¾*d*.

Reducing each sum to farthings, the lowest denomination, we may write this—

$$564693 \div 4591 = 123.$$

∴ £4. 15*s*. 7¾*d*. is contained 123 times in £588. 4*s*. 5¼*d*.

Exercise 76.

Find the value of

	£	*s*.	*d*.		£	*s*.	*d*.
(1)	596	4	3	÷	3	16	5¼.
(2)	16836	5	10½	÷	24	16	7¾.
(3)	24479	17	10½	÷	38	18	4½.
(4)	328395	2	8¼	÷	387	14	3¾.
(5)	588	4	5¼	÷	4	15	7¾.
(6)	23652	6	3	÷	34	17	8½.
(7)	66868	4	1¾	÷	87	12	9¼.
(8)	231052	15	6½	÷	456	12	6¼.
(9)	24966	0	0	÷	7	4	5¾.

	£	s.	d.		£	s.	d.
(10)	215150	17	4½	÷	37	16	4½.
(11)	472152	6	5¼	÷	73	18	5¾.
(12)	746743	2	3	÷	207	13	2¼.
(13)	26650	16	0	÷	7	14	2¾.
(14)	181502	13	9¾	÷	47	13	6¼.
(15)	622272	3	6¾	÷	74	18	2¼.
(16)	261476	5	8¾	÷	38	19	5¾.

Exercise 77.

(1) Divide £743405. 15*s.* 10½*d.* equally among 7036 men.

(2) I gave £33660. 10*s.* 3¾*d.* for 4365 articles: what was the price of each?

(3) If my income be £278. 6*s.* 3*d.*, how much less ought I to spend a day in a leap year than in an ordinary year?

(4) Divide £21. 10*s.* 0*d.* among 3 persons, so that one may have £3. 10*s.* 0*d.* more than each of the others.

(5) If a butcher sell 45 lambs at £2. 10*s.* 0*d.* each, how many could he buy with the money, of another kind which cost £2. 5*s.* 0*d.* each?

(6) If my income be £253. 16*s.* 3*d.*, and I spend 8*s.* 9*d.* a day, how much do I save in a month?

(7) How many reams of paper at 18*s.* 9¾*d.* per ream can I buy with £305. 14*s.* 0¾*d.*?

(8) If a man save £86. 6*s.* 10*d.* a year, what must he put by per week?

(9) I bought 9 dozen of wine for £28. 7*s.* 0*d.*: what did I give for a bottle, and what must I sell 16 for to gain 2*d.* on each bottle?

(10) How many dollars at 4*s.* 1½*d.* each must be given in exchange for 2475 thalers of 2*s.* 11½*d.* each?

(11) Divide £23. 10*s.* 9*d.* between a man, a woman, and a child, so that a man may receive twice as much as a woman, and a woman twice as much as a child.

(12) If 98 men earn £6549. 3*s.* 1½*d.*, find how much 103 men would earn, and 49 men?

(**13**) How many times must £3. 5*s.* 8½*d.* be added to £562. 12*s.* 11*d.* to make £3500.?

(**14**) A merchant buys 4 dozen of wine at 6*s.* a bottle, and 8 dozen of wine at 4*s.* 6*d.* a bottle. If he sell 6 dozen at 5*s.* 6*d.* a bottle, at what price per bottle must he sell the remainder?

(**15**) A boy earns £7. 13*s.* 2¾*d.*, a woman £29. 14*s.* 7½*d.*, and a man £73. 19*s.* 6¼*d.* The amount earned by 14 boys, 21 women, and a certain number of men was £2950. 17*s.* 11½*d.* How many men were there?

(**16**) A bought 7 articles at £5. 14*s.* 9¼*d.* each, B 11 articles at £37. 16*s.* 11½*d.* each, C 10 articles at £376. 15*s.* 7¼*d.* each, and D bought 12 articles; between them they spent £16017. 12*s.* 8¾*d.*: what did D give per article?

(**17**) Reduce £746915. 6*s.* 2½*d.* to farthings, take 698 farthings from the result, and divide the remainder by £915. 6*s.* 2½*d.*

(**18**) I sold 56 articles for £4080. 7*s.* 6*d.*, by which I made 4*d.* profit: what did I give for each?

(**19**) A receives 12 times as much as C, and B receives 10 times as much as C; if A, B, C together receive £800. 13*s.* 3¼*d.* what does C receive?

(**20**) How many dollars at 4*s.* 1½*d.* each must be given in exchange for 14850 thalers at 2*s.* 11½*d.* each?

(**21**) If I buy 112 articles for 5½ guineas and sell them at 1*s.* 4½*d.* each, what shall I gain?

(**22**) If 19 articles cost £1115. 10*s.* 4¼*d.* what will 29 cost; how much more than this last sum shall I require to buy 47 articles at the same price?

(**23**) A purse is worth half its contents, and 4 such purses, with their contents, are worth £82. 15*s.* 9*d.*: find the value of one purse.

(**24**) How many oxen at 16½ guineas each may I exchange for 45 sheep at £2. 5*s.* 9*d.* each and 37 pigs at £3. 13*s.* 6*d.* each?

(**25**) A man bought 150 apples at 2 a penny, and 150 more at 3 a penny, and by selling them he lost 5*d.*: how many did he sell for 2*d.*?

(**26**) A man's income is £1000.; what tax would he pay at 7*d.* in the £., and what income would he have if by the tax falling from 9*d.* to 4*d.* he saved £29. 15*s.* 10*d.*?

MEASURES OF TIME.

139. — second (s.)
60 seconds are 1 minute (m.)
60 minutes „ 1 hour (h.)
24 hours „ 1 day (d.)
7 days „ 1 week (w.)
4 weeks „ 1 month (mth.)
12 months } „ 1 year (yr.)
365 days }

The following lines will help the remembrance of the number of days in each month:—

Thirty days have September,
April, June, and November;
February hath twenty-eight alone,
And all the rest have thirty-one,
Except in leap year, at which time
February's days are twenty-nine.

If the number of the year be divisible by 4, it is a leap year, and contains 366 days; but, to correct an error in our present Calendar, the *centuries* which are not divisible by 400—as 1900, 2100—are to be taken as common years, and not as leap years.

The following examples worked out will need no explanation after our previous remarks:—

Reduce 1 yr. 5 h. 48 m. 51 s. to seconds, and *prove* it.

$$\begin{array}{rrrr} \text{y.} & \text{h.} & \text{m.} & \text{s.} \\ 1 & 5 & 48 & 51 \end{array}$$

$$\begin{array}{r} \underline{365} \\ 365 \\ \underline{24} \\ 1465 \\ \underline{730} \\ 8765 \\ \underline{60} \\ 525948 \\ \underline{60} \\ 31556931 \text{ seconds.} \end{array}$$

$$\begin{array}{ll} & \qquad\text{s.} \\ \text{Proof.} & 6{,}0\underline{)3155693{,}1} \\ & 6{,}0\underline{)52594{,}8}\text{—51 s.} \\ 24\left\{\begin{array}{l} 8\underline{)8765} \\ 3\underline{)1095} \end{array}\right. & \begin{array}{l} \text{—48 m.} \\ \text{—5h.} \end{array} \\ & 365 \text{ d. 5 h. 48 m. 51 s.} \end{array}$$

∴ 1 y. 5 h. 48 m. 51 s. = 31556931 seconds.

Exercise 78.

i. Reduction.

Reduce. In each case prove your result.

(1) 42 w. $8\frac{1}{4}$ h. $15\frac{1}{2}$ m. 5 s.; 35 d. 8 h. 48 m. 29 s.; 3 w. 4 d. 5 h. 54 m.; 7 w. 3 d. 21 h. 4 m. 17 s. to seconds.

(2) 6 h. 17 m. 25 s.; 17 h. 0 m. 43 s.; 3 y. 143 d. 16 h.; 1 y. 13 d. 4 m.; 3 y. 19 d.; 1 cent. 14 y. 7 d. 9 h., to seconds.

(3) 22 h. 52 m. 49 s.; 19 h. 14 m. 54 s.; 20 h. 9 m. 22 s.; 16 h. 48 m. 37 s.; 59 w. 3 d. 20 h. 39 m., to seconds.

In the following seconds how many weeks, days, &c.:—

(4) 1084722408.
(5) 1118333536.
(6) 1281944664.
(7) 1380555792.
(8) 1479166920.
(9) 1577778048.
(10) 1676389176.
(11) 1775000304.
(12) 1873611432.
(13) 1972222560.
(14) 4634723016.
(15) 5719445424.

ii. Addition.

(1)

h.	m.	s.
22	51	39
20	13	55
19	19	19
17	48	39
27	43	36
19	37	59

(2)

h.	m.	s.
34	35	36
20	36	29
19	37	59
29	43	36
29	19	19
17	48	39

(3)

h.	m.	s.
65	59	24
23	32	48
46	37	29
28	35	59
29	43	36
19	37	59

(4)

h.	m.	s.
24	21	37
17	13	23
29	47	43
22	27	50
54	43	36
49	59	11

(5)

h.	m.	s.
768	59	59
354	57	32
972	29	56
543	27	39
164	54	29
167	39	28

(6)

d.	h.	m.
23	15	59
22	13	12
20	0	10
25	23	22
27	22	23
28	16	17
19	13	0

(7)

d.	h.	m.
24	23	59
119	21	48
109	20	37
129	22	0
116	17	10
100	0	30
149	10	49

(8)

d.	h.	m.
27	23	59
157	22	49
143	19	48
144	18	8
259	7	17
395	17	35
487	6	26

	d.	h.	m.
(9)	59	23	59
	348	0	48
	483	10	37
	843	22	36
	100	19	45
	297	7	30
	563	6	24

	d.	h.	m.
(10)	60	23	59
	337	11	11
	22	10	10
	404	9	9
	550	20	40
	673	17	30
	376	15	25

	w.	d.	h.
(11)	4	3	16
	15	6	23
	25	5	22
	26	4	21
	37	3	20
	84	2	19
	95	1	18

	w.	d.	h.
(12)	5	4	17
	45	3	18
	54	2	18
	76	1	19
	67	5	20
	89	6	22
	98	0	23

	w.	d.	h.
(13)	7	6	23
	10	3	17
	11	5	16
	23	4	15
	24	2	14
	25	0	13
	16	0	12

	w.	d.	h.
(14)	8	6	23
	9	5	17
	10	3	19
	11	4	21
	12	5	20
	13	6	18
	14	5	15

	w.	d.	h.
(15)	9	6	23
	13	0	20
	15	0	12
	17	2	10
	19	4	15
	21	6	19
	23	1	21

	y.	d.	h.
(16)	42	364	23
	65	359	15
	74	326	6
	24	317	7
	23	308	12

	y.	d.	h.
(17)	42	364	23
	14	243	21
	32	137	13
	10	135	17
	16	56	12

	y.	d.	h.
(18)	42	364	23
	61	101	14
	47	210	22
	42	311	11
	32	19	18

	y.	d.	h.
(19)	57	364	23
	68	143	15
	53	236	12
	4	138	7
	19	352	10

	y.	d.	h.
(20)	17	364	23
	19	17	5
	28	212	9
	35	323	6
	60	248	7

	d.	h.	m.	s.
(21)	42	23	59	59
	65	14	21	37
	74	17	13	32
	24	9	47	43
	43	12	53	54
	34	22	17	50

	d.	h.	m.	s.
(22)	23	23	59	59
	57	15	16	13
	13	12	38	40
	24	17	43	53
	16	22	7	47
	29	5	58	8

	d.	h.	m.	s.
(23)	32	23	59	59
	75	16	30	31
	31	17	19	42
	42	9	42	15
	61	13	58	57
	92	19	29	48

	d.	h.	m.	s.
(24)	24	23	59	59
	56	22	58	53
	47	21	57	52
	42	20	56	51
	34	19	55	50
	43	18	54	49

iii. Subtraction.

	h.	m.	s.		h.	m.	s.		h.	m.	s.
(1)	27	43	36	(2)	29	19	19	(3)	29	43	36
	19	37	59		17	48	39		19	37	59

	h.	m.	s.		h.	m.	s.		d.	h.	m.
(4)	54	43	36	(5)	167	39	28	(6)	28	16	17
	49	59	11		164	54	29		27	22	23

	d.	h.	m.		d.	h.	m.		d.	h.	m.
(7)	149	0	30	(8)	487	6	26	(9)	563	6	24
	100	10	49		395	27	35		297	7	30

	d.	h.	m.		w.	d.	h.		w.	d.	h.
(10)	673	17	30	(11)	95	1	18	(12)	98	0	23
	376	15	25		84	2	19		89	6	22

	w.	d.	h.		w.	d.	h.		w.	d.	h.
(13)	25	0	12	(14)	14	5	15	(15)	23	1	21
	16	0	13		13	6	18		21	6	19

	y.	d.	h.		y.	d.	h.		y.	d.	h.
(16)	24	317	7	(17)	16	56	12	(18)	42	19	11
	23	318	12		10	135	17		32	311	18

	y.	d.	h.		y.	d.	h.
(19)	24	138	7	(20)	60	248	7
	19	352	10		35	323	9

	d.	h.	m.	s.		d.	h.	m.	s.
(21)	43	22	17	50	(22)	29	5	7	8
	34	12	53	54		16	22	58	47

	d.	h.	m.	s.		d.	h.	m.	s.
(23)	92	13	29	48	(24)	43	18	54	49
	61	19	58	57		34	19	55	50

iv. Multiplication.

(1) 36 h. 24 m. 36 s. × 5, 24, 36, 82, 543
(2) 30 h. 27 m. 59 s. × 7, 42, 63, 80, 345.
(3) 37 y. 124 d. 20 h. × 4, 64, 88, 114, 750.
(4) 45 y. 318 d. 19 h. × 8, 78, 153, 164, 350.
(5) 43 d. 22 h. 17 m. 50 s. × 9, 46, 35, 64, 215.
(6) 92 d. 13 h. 29 m. 48 s. × 4, 42, 63, 82, 365.

v. Division.

(1) 384 w. 5 d. 12 h. ÷ 9, 32, 45, 63, 384.

(2) 382 w. 4 d. 16 h. ÷ 8, 32, 46, 57, 452.

(3) 456 y. 3 m. 4 w. ÷ 8, 42, 56, 51, 457.

(4) 465 y. 2 m. 4 w. ÷ 7, 42, 63, 95, 418.

Exercise 79.

(1) Reduce 29 years 7 months 3 weeks 5 days 23 hours 57 minutes 45 seconds to seconds.

(2) How many years are there in 15760906 minutes?

(3) How many seconds are there in January, March, September, and December?

(4) How old is a man who has lived 1000 seconds, 1000 minutes, 1000 hours, 1000 days, and 1000 weeks ?

(5) How many seconds are there from March 1 to April 30 ?

(6) Find the number of seconds from the beginning of the year to Dec. 17, 1878, 9 A.M. ; express the result in words.

(7) A has lived 44 y. 3 m. 1 w. 5 d ; B 30 y. 8 m. 3 w. 6 d. : express in seconds how much A is older than B.

(8) A person walked 136 miles in 3 d. 4 h. 48 m. : how long did he take to walk a mile (a day consisting of 12 hours)?

(9) Find the value of 20 h. 37 m. 40 s. + 17 h. 20 m. 35 s. + 21 h. 16 m. 34 s. + 16 h. 27 m. 46 s. + 22 h. 19 m. 52 s. + 19 h. 22 m. 16 s. + 21 h. 31 m. 37 s.

(10) What number of seconds must be taken from 1861 y. 3 w. 2 d. 7 h. 51 m. 43 s. to leave 74 y. 28 w. 6 d. 14 h. 59 s.?

(11) If it take 111 d. 7 h. 24 m. 13 s. to write a book, how many of the same kind could be written in 504 y. 144 d. 5 h. 34 m. 22 s.?

(12) Find the sum of seventeen years ten weeks two days eleven hours + two hundred and one years twenty weeks four days sixteen hours + eighteen years fifteen weeks twenty-three hours + one hundred and eighty-nine years thirty-seven weeks six days twenty-three hours.

(13) Find the number of seconds in each of the following periods of time: Nineteen weeks two days twelve hours fifty minutes; one week fifty-nine minutes; four weeks one day twenty-three hours; thirteen hours forty-eight minutes. Add them together and reduce the result to weeks, days, &c.

(14) If A has lived 11 y. 12 w. 1 d.; B 14 y. 6 d.; C 13 y. 10 w. 4 d. 17 h.; and D 9 y. 40 w. 1 d. 20 h.: find the difference in the ages of A and C, B and D, and the number of hours they have lived altogether.

(15) How long would it take to count £100,000. in penny-pieces, at the rate of a piece in a second?

(16) If a boy waste 20 minutes a day, how much time would he waste in threescore years and ten?

(17) Wire rope can be spun at the rate of 10 miles a day: how long would it take to make a rope 24680 miles?

(18) If I walk 4 miles in 63 minutes, how many days should I be walking 1197 miles, supposing I walk 3 hours a day?

(19) A boy has 4 weeks 3 days holiday at Christmas, 5 weeks 2¼ days at Midsummer: how much holiday has he in 9 years?

(20) How long should I take to walk 7042 miles, walking 3 miles in 4 hours?

(21) Sound travels 1125 feet in a second: what distance am I from a cliff if the echo of a pistol-shot is heard 24 seconds after the report?

(22) In how many seconds shall I hear the report of a gun two miles off?

(23) If I hear the report of a gun three seconds after seeing the flash, what distance am I from the gun?

(24) How many days from May 1 till November 1? How many hours from May 20 till September 29?

(25) A letter-carrier delivers letters three times a day, except Sundays, walking 7½ miles each delivery: how far does he walk in 27 years, allowing 6 leap years?

(26) How many more seconds are there in July and August than in April and June?

(27) In 19358¼ days, how many years of 365 days 6 hours each?

(28) A and B set to work to find out how many seconds there were in the year 1860. A reckons 52 weeks to the year

and 7 days to the week; B reckons 365 days 6 hours in the year. How much is each out in his calculation; and what is the difference between their results?

(29) How many days, hours, and minutes are there in June, July, and August?

(30) The average age of children attending an infants' school was 2 y. 7 m. 3 w. 5 d. 20 h., and the total age was 838 y. 10 m. 3 w. 3 d. 12 h.: how many children were there?

(31) If a boy earn 2 pence an hour and work nine hours a day, how much will he earn in six weeks?

(32) How many seconds are there in the last 5 months of the year?

(33) How many days from January 20, 1857, to August 16, 1863?

(34) How long would it take to count a billion at the rate of 1000 an hour, 9 hours a day, and 6 days a week?

(35) How many seconds in February, 1876?

(36) Find the difference in the quotients of 5 y. 84 d. 6 h. by 8 and by 89.

(37) What are the quotients, when 9 cent. 5 y. 7 d. is divided by 38 and 56?

(38) Divide 750 y. 194 d. 9 h. 7 m. 5 s. by 374, 291, 85, 27, 51 and 22.

III. MEASURES OF LENGTH.

140. 12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
6 feet = 1 fathom (fth.)
11 half-yards = 5½ yards = 1 rod, pole, or perch (po.)
40 poles or 220 yards = 1 furlong (fur.)
8 furlongs or 1760 yards = 1 mile (m.)
3 miles = 1 league (lea.)

The following special measures are used:—12 lines = 1 inch; 3 barleycorns = 1 in.; 4 inches = 1 hand (used in measuring the height of horses), 100 links = 22 yards = 1 chain.

In measuring *cloth*:—2¼ in. = 1 nail; 4 nails = 1 quarter; 4 quarters = 1 yard; 5 quarters = 1 ell.

Ex. 1. Reduce 5 m. 3 fur. 17 po. 4 yds. 1 ft. 7 in. to inches.

m. fur. po. yds. ft. in.

5 3 17 4 1 7

8

43 furlongs

40

1737 poles

11

2)19107 half-yards

9553 yds. 1 ft. 6 in.

4 yds. 1 ft. 7 in

9558 yds. 0 ft. 1 in.

3

28674 feet

12

344089 inches

The only difficulty here is the reduction of the poles to yards. Multiplying 1737 by 11 gives the result in half-yards; we divide by 2 and obtain 9553 yards and one *half-yard*, or 1 ft. 6 in. over; to this we add the remaining 4 yds. 1 ft. 7 in. The subsequent steps are obvious. The proof of this will need close attention. Thus :—

Ex. 2. Reduce 344089 inches to miles, furlongs, poles, &c.

inches

12)344089

3)28674 ft. 1 in.

9558 yds. 0 ft. 1 in.

2

11)19116 half-yds. 0 ft. 1 in.

1737 po. 9 half-yds. 0 ft. 1 in.

or 4,0)173,7 po. 4 yds. 1 ft. 7 in.

8) 43 fur. 17 po. 4 yds. 1 ft. 7 in.

5 m. 3 fur. 17 po. 4 yds. 1 ft. 7 in.

Observe, after the division by 11 the remainder is 9 half-yds. or 4 yds. 1 ft. 6 in., but we have 0 ft. 1 in. to add to this, ∴ the sum is 4 yds. 1 ft. 7 in.

Exercise 80.

i. Reduction.

Reduce to inches, and in each case prove your result :—

(1) 7 m. 5 fur. 32 po. 4 yds. ; 25 ft. 39 po. 3 yds. 2 ft. 8 in. ; 75 yds. 3 qrs. 3 nls. 2 in.

(2) 8 m. 3 fur. 32 po. 3 yds. 0 ft. 2 in.; 3 m. 7 fur. 9 po. 1½ yd. 2 ft. 11 in.

(3) 29 m. 1 fur. 11 po. 2 yds. 2 ft. 4 in.; 2 m. 5 fur. 39 po. 1½ yd. 2 ft. 4 in.

(4) 5 m. 3 fur. 38 po. 2 yds. 0 ft. 8 in.; 49 m. 2 fur. 31 po. 3 yds. 1 ft. 11 in.

(5) 1371 m. 0 fur. 19 po. 1 yd. 2 ft. 11 in.; 1245 m. 3 fur. 39 po. 3½ yds. 0 ft. 6 in.

And reduce to miles, furlongs, poles, &c., the following inches :—

(6) 5248986624, 5377010688, 5505034752, 5633058816, 5761082880.

(7) 3352778352, 3451389480, 3550000608, 3648611736, 3747222864.

(8) 3845833992, 3944445120, 4043056248, 4141687376, 4240278504.

(9) 4338889632, 4437500760, 2070833688, 2169444816, 2268055944.

(10) 2366667072, 2465278200, 2563889328, 2662500456, 2761111584.

(11) 2859722712, 2958333840, 3056944968, 3155556096, 5719445424.

ii. Addition.

	m.	fur.	po.	yds.	ft.	in.		m.	fur.	po.	yds.	ft.	in.
(1)	218	7	36	3	1	9	(2)	245	3	39	3½	0	6
	17	0	19	1	2	11		371	5	13	2½	1	10
	143	2	5	1	0	5		586	6	26	2½	1	9
	336	4	7	2	0	6		673	7	32	1½	0	3
(3)	963	0	2	2½	2	6	(4)	384	3	13	2	1	6
	673	7	32	1½	0	3		179	4	25	3	2	8
	360	5	28	3	2	11		300	4	24	3	2	11
	188	6	29	4	1	7		256	6	35	3	2	10
(5)	570	2	13	2	1	4	(6)	300	4	24	3	2	11
	288	3	35	4	2	6		517	2	16	2	1	5
	408	3	24	3½	1	6		189	3	36	3½	2	7
	295	5	36	4	2	8		609	2	35	4½	2	4

N.B.—It is possible that some discrepancy may appear to exist between the sum set and the answer obtained by proof. Thus: 38 po. 5 yds. 2 ft. 3 in. = 39 po. 0 yds. 0 ft. 9 in.

	m.	fur.	po.	yds.	ft.	in.		m.	fur.	po.	yds.	ft.	in.
(7)	407	5	23	3	1	4	(8)	187	6	29	3	2	8
	385	0	28	4½	2	5		510	7	23	3	1	5
	703	3	14	5		6		189	7	37	5	2	8
	389	4	28	4½	2	9		573	4	16	2	1	3

iii. Subtraction.

	m.	fur.	po.	yds.	ft.	in.		m.	fur.	po.	yds.	ft.	in.
(1)	346	4	21	4	2	3	(2)	631	1	24	2	1	7
	218	7	35	3	1	9		174	2	16	1	3	4
(3)	631	7	35	5	2	9	(4)	212	2	9	2	2	7
	427	4	21	4½	2	10		163	3	17	2	1	2
(5)	614	3	18	4	3	9	(6)	526	6	39	5	1	11
	216	5	21	3	1	11		371	3	27	3	2	9
(7)	204	6	27	3½	1	10	(8)	281	6	27	2½	1	10
	171	6	38	4½	2	4		43	5	28	4½	2	11
(9)	327	6	19	3	1	10	(10)	385	5	26	2½	1	9
	112	5	27	4	2	11		185	0	4	3	2	11
(11)	178	5	27	2	1	10	(12)	320	7	25	2½	1	9
	137	7	28	3	2	11		313	6	27	4	2	10

iv. Multiplication.

	m.	fur.	po.	yds.	ft.	in.	
(1)	41	4	38	5	2	3	× 2, 3, 5, 7, 9.
(2)	26	0	4	1	2	4	× 4, 6, 8, 10, 11, 12.
(3)	84	3	23	5	2	5	× 15, 24, 33, 44, 56.
(4)	84	6	34	3	1	9	× 16, 25, 35, 45, 60, 77.
(5)	64	3	25	3	2	9	× 19, 34, 57, 58, 59.
(6)	72	5	29	4	2	11	× 36, 38, 69, 71, 73, 74.
(7)	39	7	29	4	2	8	× 123, 563, 635, 653.
(8)	72	2	28	2	1	7	× 468, 739, 937, 846.
(9)	84	7	18	4	1	11	× 512, 523, 689, 356.
(10)	48	6	35	4	2	11	× 864, 986, 325, 215.
(11)	96	7	16	4	2	10	× 411, 261, 432, 571.
(12)	86	3	27	5	2	3	× 234, 423, 285, 976.

v. Division.

	m.	fur.	po.	yds.	ft.	in.	
(1)	69	6	27	3	1	9	÷ 2, 3, 5, 7, 9.
(2)	97	7	34	1½	1	3	÷ 4, 6, 8, 10, 11, 12.

	m.	fur.	po.	yds.	ft.	in.	
(3)	458	0	39	4	1	11	÷ 18, 27, 36, 48.
(4)	63	6	35	4	2	6	÷ 32, 42, 55, 70, 88.
(5)	634	2	19	4	1	5	÷ 26, 38, 69, 71.
(6)	197	7	34	3½	1	3	÷ 34, 57, 58, 59, 61.
(7)	333	4	21	3	2	10	÷ 234, 432, 653, 473.
(8)	548	7	0	4	2	8	÷ 285, 582, 473, 743.
(9)	870	3	16	3	2	8	÷ 411, 261, 801, 571.
(10)	584	3	36	0	2	7	÷ 296, 357, 735, 573.
(11)	1321	6	21	2½	2	8	÷ 321, 432, 635, 715.
(12)	6862	3	31	0½	0	4	÷ 635, 563, 834, 937.

Exercise 81.

(1) How many inches are there in 5 m. 6 fur. 37 po. 4 yds. 2 ft. 7 in.?

(2) In 50005 in. how many miles?

(3) A boy steps one foot seven inches: how many steps will he take in half a mile?

(4) How much string must I have to give seven hundred and thirteen boys each twenty-five yards five inches?

(5) How many rounds, three-quarters of an inch thick, can be cut off from a tree which stands a pole and a half high?

(6) If in walking a mile 2112 steps be taken, what is the distance of each step?

(7) Find the difference in inches between the equatorial diameter of the earth, which is 7925½ miles, and the polar diameter, which is 7899 miles.

(8) If I can walk 3 m. 2 fur. 30 po. in 1 hour, how long will it take me to walk 30 m. 0 fur. 30 po.?

(9) How many times does a wheel 3½ ft. in circumference turn in 1 mile?

(10) A man had to walk 250 m. in 10 days: he walks in each of the first 5 days, 27 m. 3 fur. 27 po.; how far would he walk in each of the remaining days?

(11) The hind wheel of a carriage is 6 ft. 6 in., the front wheel 4 ft. 6 in. in circumference: how many times does the front wheel revolve more than the hind wheel in travelling over 5 miles?

(12) How many pieces of cloth 2 yds. 1 qr. can be cut from 23 yds. 2 qrs. 2 nls.?

(13) A road half a furlong has to be drained: 36 yds. of drain-pipe are left from a former job; how much more is required?

(14) A circular course is 14 m. 3 fur. 15 po. 3 yds. 2 ft. 9 in. in circumference: how many times must I go round it to complete a distance of 1254 m. 7 fur. 6 po. 5 yds. 9 in.?

(15) How often will a wheel 3¼ yds. in circumference revolve in 246 miles?

(16) What would be the circumference of a wheel which revolved 1848 times in 3½ m.?

(17) How many pieces of cloth 3 yds. 1 qr. 1 nl. long can be cut from 135 yds. 3 qrs. 1 nl.?

(18) If a horse trot 15½ m. in 3 hours, how far would he travel in 1 hour; and how far would he travel in 5½ hours at the same rate?

(19) A man can walk 5¼ m. in 3 hours: how far does he go in 1 hour; and in how many hours will he accomplish a journey of 28 m. 7 fur.?

(20) A train can travel 65 m. in 1 hour: how far will it go in 1 second, and how far will it go in 17 min. 8 sec.?

(21) A man bought 10 pieces of cloth: 3 measured 24 yds. 3 qrs. 2 nls. each; 5, 17 yds. 2 qrs. 1 nl. each; the remainder, 46 yds. 2 nls. each: what was the total length?

(22) If a man advance 2 ft. 10 in. in each step, takes 64 steps in 1 minute, and walk 4¾ hours, how far will he go?

(23) If a wheel of a carriage is 5 yds. 2 ft. 9 in. in circumference, and revolve 3 times in a second, how far will it go in an hour?

(24) I make 108 steps of 2 ft. 8 in. each in 1 minute: how long would it take me to step 36 miles?

(25) A knot (in navigation) is 6076 ft., and there are 540 knots in 9 degrees on the meridian: how many miles are there in 1 degree?

(26) What is the height in feet, inches, of a horse 16½ hands high?

(27) How many links are there in a mile?

(**28**) A cubit measures 18 in. : how many cubits is a man 6 ft. high?

(**29**) The height of Goliath is given as 6 cubits and a span—this is equivalent to 11 ft. 4 in. : what is the length of a span, a cubit being nearly 21 in.?

(**30**) The length of Noah's ark was 300 cubits, the breadth of it 50 cubits, and the height of it 30 cubits. Reduce each of these lengths to inches, feet, &c., allowing 1 cubit = 21 in. Multiply these results together in inches.

(**31**) Sound travels 1125 ft. in a second: how far is a thunder-cloud off when I hear the clap 9 seconds after seeing the flash?

(**32**) The fore wheel of a carriage is 2 yds. 2 ft. 6 in. in circumference, and in travelling 19 m. 2 fur. 120 yds. makes 4350 more revolutions than the hind wheel: what is the circumference of the hind wheel?

(**33**) Find the value of 3 m. 44 yds. ÷ 2 ft. 8 in. × 108; and 3 fur. ÷ 75 × $2\frac{3}{4}$ ft.

(**34**) 169 m. 3 yds. 2 ft. 10 in. + 120 m. 1 po. $4\frac{1}{2}$ yds. 1 ft. 9 in. + 17 m. $3\frac{1}{2}$ yds. 9 in. + 65 m. 4 yds. 2 ft. 7 in. + 367 m. 5 yds. 11 in. + 24 m. 2 yds. 1 ft. $11\frac{1}{2}$ in. + 230 m. $3\frac{1}{2}$ yds. $2\frac{1}{4}$ in. + 740 m. 7 fur. 16 po. $4\frac{1}{2}$ yds. 2 ft. $11\frac{1}{2}$ in. + 7832 m. 2 ft. 7 po. $3\frac{1}{2}$ yds. 10 in.

(**35**) What is the sum of 15 yds. 3 qrs. 2 nls. 1 in. + 37 yds. 1 qr. 1 nl. 2 in. + 140 yds. 2 qrs. 2 nls. 2 in. + 60 yds. 1 qr. 3 nls. 2 in. + 16 yds. 2 qrs. 1 nl. 3 in. + 12 yds. 1 qr. + 2 qrs. 2 nls. + 117 yds. 3 qrs. 2 in.?

IV. MEASURES OF SURFACE.

141. 144 square inches = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
121 quarter square yards, or $30\frac{1}{4}$ square yards = 1 square pole (sq. po.)
40 square poles = 1 rood (rd.)
4 roods = 1 acre (ac.)

1 acre = 4840 sq. yds; 640 acres = 1 sq. m.; 10 sq. chains = 1 acre.

Ex. 1. Reduce 3 ac. 2 rds. 27 po. 27 sq. yds. 7 sq. ft. 23 sq. in. to inches.

```
   ac.  rds.  pls.  sq. yds.  sq. ft.  sq. in.
    3    2     27      27        7       23
    4
   14  roods
    40
   587  poles
   121
   587
  7044
4)71027  qr. sq. yds.  yds.    ft.   in.
  17756¾  yards  =     17756    6    108
                          27    7     23
                       17784    4    131
                           9
                      160060  feet
                         144
                      640371
                     640240
                    160060
                    23048771  sq. inches
```

The difficulty here is the reduction of poles to yards. Multiplying the poles by 121 gives the result in quarter-yards, we divide by 4 and obtain 17756 yds. and three-quarters of a yard, or 6 ft. 108 in., over; to this we add 27 sq. yds. 7 ft. 23 in., and the sum is 17784 yds. 4 ft. 131 in. The subsequent steps are obvious.

The proof of this is as follows :—

Ex. 2. Reduce 23048771 sq. in. to acres, roods, &c.

```
          inches
     { 12)23048771
 144 {  12)1920730—11 }
           9)160060—10 } 131 in.
            17784—4 ft. 131 in.
                4
     { 11)71136 qr. yds.                      yds.  ft.  in.
 121 {  11)6466—10 }  109 qr. yds.   =         27    2    36
             587—9 }                                 4   131
                                               27    7    23
             po.          yds. ft. in.
         4,0)58,7          27   7   23
           4)14—27         27   7   23
             3 ac. 2 rds. 27 po. 27 yds. 7 ft. 23 in.
```

The remainder in dividing by 121 is 109 qr. yds., or 27 yds. 2 ft. 36 in., and to this we add the previous remainder, 4 ft. 131 in., giving the remainder, 27 yds. 7 ft. 23 in.

Exercise 82.

i. Reduction.

Reduce each of the following quantities to inches, giving the proof in each case :—

(1) 17 ac. 25 po. 25 yds. ; 3 rds. 17 po. 21 yds. 8 ft. 73 in.

(2) 8 ac. 2 rds. 34 po. 3 ft. 87 in. ; 53 ac. 21 po. 8 ft. 125 in.

(3) 19 ac. 2 rds. 27 po. ; 5 ac. 3 rds. 16 po. 27 yds. ; 2 rds. 27 po. ; 51 ac. 39 po. 30 yds.

(4) 500 ac. 1 rd. 23 po. 6 yds. 2 ft. 103 in. ; 419 ac. 29 po. 75 yds. 8 ft. 140 in.

(5) 580 ac. 3 rds. 11 po. 2 yds. 3 ft. 110 in. ; 943 ac. 20 po. 17 yds. 7 ft. 129 in.

(6) 27 m. 620 ac. 1 rd. 78 po. 27 yds. 6 ft. ; 90 m. 439 ac. 2 rds. 30 po. 30 yds. 84 in.

And reduce to miles, acres, &c., giving the proof in each case, the following inches :—

(7) 3660938127, 3993750684, 4326563241, 4659375798, 4992188355.

(8) 5325009012, 5657813469, 599062026, 6323438583, 6656251140.

(9) 15642190179, 19303128306, 10317189267, 10650001824, 14643752508.

(10) 11315626938, 11648439495, 11981252052, 12314064609, 12646877166.

ii. Addition.

	ac.	rds.	po.	yds.		ac.	rds.	po.	yds.
(1)	59	3	36	28	(2)	25	1	16	12
	34	3	39	26		19	2	39	16
	195	2	6	20		161	1	8	16
	38	3	9	29		55	2	13	19
(3)	26	3	23	13	(4)	96	3	12	15
	19	2	25	18		47	1	19	21
	121	3	23	3		24	31	8	19
	26	3	12	5		16	18	5	14

N.B.—It is possible that some discrepancy may appear to exist between the sum set and the answer obtained by proof. Thus : 38 po. 30 yds. 2 ft. 44 in. = 39 po. 0 yds. 0 ft. 9 in.

	po.	yds.	ft.	in.
(5)	132	12	3	65
	98	30	7	112
	121	2	2	93
	25	18	6	112

	po.	yds.	ft.	in.
(6)	16	12	4	113
	39	16	3	102
	8	16	5	108
	13	19	7	115

	po.	yds.	ft.	in.
(7)	16	20	3	116
	13	22	7	108
	15	13	6	17
	23	16	5	113

	po.	yds.	ft.	in.
(8)	9	2	5	139
	10	12	7	140
	1	12	7	124
	65	4	8	113

	ac.	rds.	po.	yds.	in.
(9)	39	3	36	26	130
	124	1	23	12	117
	136	2	19	9	115
	123	2	13	12	104
	251	3	35	16	140
	126	1	24	9	116
	134	3	29	16	108

	ac.	rds.	po.	yds.	in.
(10)	236	1	15	13	109
	163	3	12	9	99
	198	2	25	27	140
	254	3	32	11	16
	124	2	35	9	78
	163	1	19	24	134
	153	3	16	19	128

	ac.	rds.	po.	yds.	ft.
(11)	123	2	34	12	3
	21	1	10	6	7
	234	3	24	25	8
	122	3	5	29	4
	251	3	32	22	3
	135	2	9	13	6
	123	3	25	3	8

	ac.	rds.	po.	yds.	ft.
(12)	156	1	24	12	3
	239	3	25	24	3
	162	2	29	3	7
	154	3	58	19	8
	29	1	94	21	4
	94	3	23	13	7
	76	2	51	9	3

(13) 75 yds. 8 ft. 140 in. + 2 yds. 3 ft. 110 in. + 17 yds. 7 ft. 129 in. + 6 ft. 90 in. + 5 yds. 84 in. + 7 yds. 2 ft. 119 in.

(14) 78 po. 27 yds. 6 ft. + 900 po. 16 yds. 5 ft. + 90 po. 29 yds. 4 ft. + 74 po. 10 yds. 8 ft. + 60 po. 25 yds. + 26 yds. 7 ft.

(15) 17 rds. 30 po. 30 yds. + 91 rds. 27 po. 19 yds. + 54 rds. 23 po. 19 yds. + 764 rds. 38 po. 17 yds. + 89 rds. 29 yds. + 18 po. 14 yds.

(16) 300 ac. 3 rds. 39 po. + 91 ac. 2 rds. 26 po. + 751 ac. 2 rds. 18 po. + 54 ac. 38 po. + 2 rds. 36 po. + 497 ac. 3 rds. 15 po.

(17) 3 yds. 7 ft. 116 in. + 8 ft. 41 in. + 17 yds. 5 ft. + 13 yds. 4 ft. 88 in. + 62 yds. 8 ft. 139 in. + 2 ft. 99 in.

(18) 6 rds. 27 po. 14 yds. + 15 po. 18 yds. + 3 rds. 16 po. 27 yds. + 2 rds. 27 po. + 51 rds. 39 po. 30 yds.

(19) 19 ac. 2 rds. 27 po. 18 yds. + 5 ac. 3 rds. 16 po. 29 yds. + 2 rds. 28 po. 30 yds. + 12 ac. 36 po. 27 yds. + 29 yds.

(20) 419 ac. 29 po. 27 yds. + 580 ac. 3 rds. 11 po. 29 yds. + 943 ac. 20 po. 30 yds. + 17 ac. 18 po. + 3 rds. 25 po.

iii. Subtraction.

	ac.	rds.	po.	yds.		ac.	rds.	po.	yds.		ac.	rds.	po.	yds.
(1)	59	3	36	28	(2)	25	1	16	12	(3)	26	3	23	13
	34	3	39	29		19	2	39	16		19	2	25	18

	ac.	rds.	po.	yds.		po.	yds.	ft.	in.		po.	yds.	ft.	in.
(4)	96	1	12	15	(5)	132	12	3	65	(6)	39	12	4	113
	47	3	19	21		98	30	7	112		16	16	3	114

	po.	yds.	ft.	in.		po.	yds.	ft.	in.
(7)	16	20	3	108	(8)	10	2	5	139
	13	22	7	116		9	12	7	140

	ac.	rds.	po.	yds.	ft.		ac.	rds.	po.	yds.	ft.
(9)	123	1	10	6	7	(10)	234	3	10	6	7
	21	2	34	12	3		122	4	12	7	8
(11)	239	1 ,	24	12	3	(12)	162	2	29	3	7
	156	3	25	24	4		159	3	39	21	8

(13) From 7 ac. 3 rds. 27 po. take 2 ac. 2 rds. 34 po.

(14) 3 rds. 26 po. 17 yds. – 2 rds. 26 po. 19 yds.

(15) Take 4 ac. 3 rds. 27 po. 15 yds. 4 ft. from 5 ac.

(16) 115 ac. 3 rds. 37 po. 15 yds. – 110 ac. 3 rds. 39 po. 30 yds.

(17) Subtract 369 yds. 7 ft. 120 in. from 1000 yds.

(18) 5 ac. 3000 yds. 8 ft. 2 in. – 2 ac. 4000 yds. 8 ft. 10 in.

(19) 3 m. 329 ac. 3 rds. 20 po. – 1 m. 620 ac. 3 rds. 30 po.

(20) Subtract 90 ac. 3 rds. 39 po. 5 yds. from 91 ac.

(21) 17 po. 27 yds. 3 ft. 100 in. – 30 yds. 6 ft. 101 in.

(22) 78 ac. 2 rds. 20 po. 20 yds. – 8 ac. 2 rds. 21 po. 30 yds.

iv. Multiplication.

	ac.	rds.	po.	yds.	ft.	in.	
(1)	24	2	36	12	3	58	× 7, 28, 36, 37, 381.
(2)	62	2	19	13	5	37	× 6, 20, 36, 243.
(3)	36	4	25	18	7	39	× 4, 24, 32, 39, 268.
(4)	36	3	18	29	4	63	× 10, 25, 17, 638.
(5)	17	2	0	13	6	121	× 11, 30, 54, 29, 492.

	ac.	rds.	po.	yds.	ft.	in.	
(6)	45	1	16	20	6	52	× 12, 18, 49, 604.
(7)	36	4	25	8	3	29	× 9, 24, 72, 41, 583.
(8)	16	3	12	15	6	38	× 5, 22, 31, 432.
(9)	23	1	14	18	6	133	× 2, 27, 28, 51, 234.
(10)	19	3	12	24	5	100	× 11, 12, 50, 756.
(11)	8	3	26	23	5	48	× 13, 15, 19, 43, 765.
(12)	2	2	22	22	2	22	× 689, 251, 986.

v. Division.

	ac.	rds.	po.	yds.	ft.	in.	
(1)	3007	2	32	15	6	72	÷ 2, 3, 5, 7, 9.
(2)	4089	1	2	2½	1	18	÷ 4, 6, 8, 10, 11, 12.
(3)	655	2	4	0	2	132	÷ 18, 27, 36, 48.
(4)	336	2	10	10½	8	40	÷ 32, 42, 55, 88.
(5)	3049	2	38	21½	6	0	÷ 26, 38, 69, 71.
(6)	21069	3	7	16	0	81	÷ 34, 57, 59, 61.
(7)	7364	1	8	2	0	0	÷ 234, 432, 653, 473.
(8)	26418	3	12	11	0	0	÷ 285, 582, 473, 743.
(9)	15927	0	37	5¾	0	0	÷ 411, 261, 801, 571.
(10)	6008	3	7	12¼	7	126	÷ 296, 397, 735, 573.
(11)	2063	3	35	9¾	5	9	÷ 321, 432, 635, 715.
(12)	1142	3	28	6½	2	81	÷ 635, 563, 834, 937.
(13)	17279	3	36	24	2	84	÷ 453, 345, 308, 854.
(14)	9454	3	13	29¾	4	96	÷ 350, 263, 646, 743.
(15)	2327	0	35	9¾	3	72	÷ 354, 632, 485, 697.
(16)	14095	3	18	0½	8	124	÷ 512, 523, 689, 357

Exercise 83.

(1) Reduce 309034 yds. to acres.

(2) If to 10 times 40 ac. 3 rds. 16 po. I add 122210 yds. and divide the sum by 40, what is the quotient expressed in acres, roods, &c.?

(3) Express in inches, &c., the difference between one million square yards and 206 ac. 2 rds. 17 po. 26 yds.

(**4**) How many fields each 1 ac. 2 rds. 37 po. can be made out of an estate 361 ac. 3 rds. 13 po.?

(**5**) I reserve for myself 554 acres out of 2 square miles of land, and divide the remainder into farms of 121 acres each: how many farms are there?

(**6**) If a parish be 2728 ac. 3 rds. 6 po. 8¾ yds. in extent, and contain a certain number of estates each 209 ac. 3 rds. 25 po. 3 yds., how many estates were there?

(**7**) A square drill-yard, 3125 sq. ft. 142 sq. in., is paved with square slabs, each measuring 3 sq. ft. 86 sq. in.: how many slabs were there?

(**8**) How many solid yards of coal will 50 miners get in a year if each dig 3 yds. 19 ft. 112 in. daily?

(**9**) In an estate of 460 acres are 39 fields: find to the nearest square inch the average size of each?

(**10**) Multiply 4 miles long by twelve hundred yards broad, and express your answer in acres, &c.

(**11**) Express in square inches the difference between 46 ac. 2 rds. 22 po. and 2031530 sq. ft.

(**12**) Divide 16 ac. 2 rds. 26 po. 20 yds. 1 ft. 72 in. by 6, and multiply the quotient by 9.

(**13**) A field consists of 2 rds. 672 sq. yds. 2 sq. ft.; on it are built 16 houses each standing on 2000 sq. ft. of land; the remainder is occupied by a street and a back lane, of which the latter is half the size of the former: how much land does the lane occupy?

(**14**) If one square inch just holds one halfpenny, find the value in £ *s. d.* of a surface 3 ac. 3 rds. 38 po. 26 yds. 6 ft. 4 in. covered with these coins.

(**15**) If I exchange a field 4 ac. 2 rds. 34 po. 25 yds. in extent for a square plot of ground 453 feet long, how much do I lose?

(**16**) A surveyor in measuring a field reckons 30 instead of 30¼ yards to a pole: what will be the real size of a field which he returns as 25 acres?

(**17**) A man distributes 1600 ac. 2 rds. between his 2 sons and a certain number of daughters: if each son have 400 ac. 20 po., and each daughter 266 ac. 3 rds., how many daughters were there?

(**18**) If A have 37 ac. 3 rds. 25 po. 26 sq. yds., and B have 2 sq. ft. 36 sq. in. less, how many square yards has B?

(19) A man and his 6 sons own together 2728 ac. 3 rds. 6 po. 8¾ yds.: if the father's share be 7 times as much as each of his sons' share, which are equal, what is a son's share?

(20) A is 43 yrs. 119 days old, B is 42 yrs. 116 days old, C is 40 yrs. 106 days old; each is to receive as many square inches of ground as the number of seconds he has lived: what amount of land, expressed in acres, &c., will they receive—a year being = 365 days 6 hrs.?

(21) If a square inch of ground is worth 7*s.* 6*d.*, what is the value of an estate of 160 ac. 1 rd. 37 po.?

(22) If 5 ac. 3 rds. 37 po. 19 yds. 4 ft. 45 in. be sold for £625724. 11*s.* 0*d.*, what is the selling price of one square inch?

(23) A house and a garden are so built that one garden occupies as much ground as 8 houses: if a garden occupy 128 po. 5 yds. 2 ft. 24 in., how many pieces of ground, each 384 po. 15 yds. 6 ft. 72 in., will have to be sold to make 3 such gardens and 24 houses?

(24) Divide 3131 ac. 0 rds. 12 po. 13½ yds. 1 ft. 122 in. between 7 men, 9 women, and 11 boys, so that each man may receive 3 times as much as a boy, and each woman twice as much as a boy.

(25) A has 4 fields, B has 6 times as much, C has as much as twice A's and B's together, and has 791 ac. 1 rd. 5 po. 2¾ yds. 0 ft. 128 in.: find the size of a field.

(26) Express in acres the difference between half a square mile and half a mile square.

(27) How much ground at 4*s.* 8½*d.* a square perch can be bought for £523. 11*s.* 4*d.*?

V. MEASURES OF SOLIDITY.

142. 1728 cubic inches = 1 cubic foot (1 cub. ft.)
27 cubic feet = 1 cubic yard (1 cub. yd.)

A cube is a solid figure whose length, breadth, and depth are the same, and its volume or solidity is the continued product of these 3 dimensions. Thus, a cubic foot is a figure whose length is 12 in., breadth 12 in., depth 12 in.; ∴ its volume = 12 × 12 × 12 = 1728 cub. in. In the same way, a cubic yard is a figure whose length is 3 ft., breadth 3 ft., and depth 3 ft., ∴ its volume = 3 × 3 × 3 = 27 cub. ft.

Ex. 1. Reduce 25 cub. yds. 5 cub. ft. 143 cub. in. to cubic inches :—

$$
\begin{array}{rrr}
\text{cub. yds.} & \text{ft.} & \text{in.} \\
25 & 5 & 143 \\
\underline{27} & & \\
180 & & \\
\underline{50} & & \\
680 & & \\
\underline{1728} & & \\
138240 & & \\
10368 & & \\
\underline{143} & & \\
1175183 & &
\end{array}
$$

Remark.—The quickest way of multiplying by 1728 is thus:—

$$
\begin{array}{rl}
12345 \times 1728 & \\
12345 & \text{—A} \\
\underline{1728} & \\
12345 & \text{—B} \\
86415 & \text{—C} \\
\underline{345660} & \text{—D} \\
21332160 &
\end{array}
$$

B = 1000 A
C = 700 A
D = 4 C ÷ 100 = 28 A

Proof. **Ex. 2.** Reduce 1175183 cub. in. to yards, &c. :—

$$
\begin{array}{l}
1728 \left| 144 \left\{ \begin{array}{l} 12)\underline{1175183} \text{ cub. in. to yards, \&c.} \\ \left.\begin{array}{l} 12)\underline{97931}-11 \\ 12)\underline{8160}-11 \end{array}\right\} 143 \end{array}\right.\right. \\
\qquad 27 \left\{ \begin{array}{l} 9)\underline{680}-\ 0 \\ \left.\begin{array}{l} 3)\underline{75}-\ 5 \\ 25-\ 0 \end{array}\right\} \ 5 \end{array}\right.
\end{array}
$$

∴ final remainder = 0 × 144 + 143 = 143

∴ 1175183 cub. in. = 25 cub. yds. 5 cub. ft. 143 cub. in.

Exercise 84.

i. Reduction.

Reduce the following to inches, giving the proof in each case :—

(1) 384 yds. 23 ft. 1645 in. ; 573 yds. 20 ft. 645 in. ; 473 yds. 24 ft. 1604 in.

(2) 4186 yds. 19 ft. 487 in. ; 2895 yds. 24 ft. 1452 in. ; 6329 yds. 23 ft. 1719 in.

Reduce the following inches to yards, feet, giving the proof in each case :—

(3) 6955103232, 11923034112, 18381344256, 18878137344.

(4) 7451896320, 12419827200, 18878137344, 19374930432.

ii. Addition.

	yds.	ft.	in.		yds.	ft.	in.		yds.	ft.	in.
(1)	483	23	1645	(2)	573	17	1008	(3)	375	26	1726
	296	13	645		614	16	1655		753	16	1625
	614	22	1723		891	15	1603		703	13	1027
	384	20	891		486	12	609		307	12	116

	yds.	ft.	in.		yds.	ft.	in.
(4)	8564	21	1719	(5)	402	9	207
	6584	12	1197		681	16	1653
	6845	2	1179		645	20	827
	4658	22	1190		473	24	816

iii. Subtraction.

	yds.	ft.	in.		yds.	ft.	in.		yds.	ft.	in.
(1)	614	20	891	(2)	891	12	609	(3)	703	12	116
	384	22	1723		486	15	1603		307	13	1027

	yds.	ft.	in.		yds.	ft.	in.
(4)	6845	2	1190	(5)	645	20	816
	4658	22	1199		473	24	827

iv. Multiplication.

	yds.	ft.	in.	
(1)	87	8	924	× 9, 16, 36, 53, 345.
(2)	2746	24	563	× 7, 18, 42, 57, 355.
(3)	35	23	1688	× 5, 12, 27, 65, 285.
(4)	1795	13	1532	× 11, 17, 41, 59, 543.
(5)	90	23	1700	× 86, 74, 150, 169, 32, 85, 96.
(6)	7	24	5	× 726, 85, 74, 39, 84, 72, 81, 75.

v. Division.

	yds.	ft.	in.	
(1)	1987	16	1624	÷ 9, 16, 36, 53, 345.
(2)	6394	15	288	÷ 7, 18, 42, 57, 355.
(3)	2086	13	905	÷ 5, 12, 27, 65, 285.
(4)	3164	18	882	÷ 17, 43, 61, 71, 543.
(5)	4819	20	1604	÷ 354, 263, 584, 796.
(6)	8564	21	1719	÷ 350, 362, 646, 743, 697.
(7)	194	23	1600	÷ 127, 64, 31, 19, 58, 72, 81.
(8)	1764	5	4	÷ 712, 810, 376, 72, 187, 847, 31.

Exercise 85.

(1) Reduce 45 cub. yds. 23 cub. ft. 1216 cub. in. to inches.

(2) In one million cubic inches how many cubic yards?

(3) A stone is 11 in. long, 7 in. thick, and 9 in. wide: how many inches are there in 25 such stones?

(4) In 1300 cub. in. and 1300 cub. ft. how many cubic yards?

(5) Find the value of 9 yds. 25 ft. 1680 in. + 7 yds. 13 ft. 1417 in. + 23 ft. 1580 in. + 8 yds. 395 in. + 7 yds. 14 ft. 1340 in.

(6) Find the value of 8 yds. 14 ft. 930 in. + 6 yds. 16 ft. 867 in. + 4 yds. 1395 in. + 7 yds. 25 ft. 1630 in. + 3 yds. 23 ft. 1336 in. + 26 ft. 144 in.

(7) From 760 yds. 13 ft. 1620 in. take 90 yds. 21 ft. 1700 in.

(8) Take 20 ft. 151 in. from 100 yds.

(9) 468 yds. 13 ft. 728 in. − 32 yds. 26 ft. 1260 in.

(10) How many heaps, each 82 yds. 12 ft. 100 in., are there in 6678 yds. 4 ft. 1188 in.?

(11) If the content of 100 trucks be 300 yds. 19 ft. 432 in., how many trucks will be required to remove 33078 yds. 11 ft. 864 in. of earth?

(12) If a room is 20 ft. long, 18 ft. 6 in. wide, and 12 ft. high, how many cubic feet does it contain?

(13) If a sheet of water, of uniform depth, is 400 yds. 2 ft. 6 in. long and 300 yds. broad, and contains 3246750 ft. of water, what is its depth?

(14) What is the length of a pond which is 18 yds. broad, 5 ft. 6 in. deep, and which contains 17820 cub. ft. of water?

(15) If the side of a solid cube is 7 ft. 3 in., find its content in yards.

(16) The content of a cube is 2 ft. 1457 in.: what is the length of one of its sides?

(17) How many bricks will be required to build a wall 73 ft. long, 6 ft. high, and 18 in. thick, each brick being 9 in. long, 4½ in. wide, and 3 in. deep?

(18) A tank has a square bottom, and is 2 ft. deep, and contains 5 ft. 24 in. of water: what is the length of a side of the bottom?

(19) The length of a rectangular piece of timber is 47 ft., its breadth is 4 ft.: what is its thickness if it contain 564 cub. ft.?

(20) A room is 22 ft. 6 in. long, 18 ft. 3 in. broad, and contains 4106 yds. 6 ft. 1296 in. of air: what is its height?

(21) In making a road 110 yds. long and 18 ft. wide, I spend £37. 14*s*. 2*d*.; the cost of excavating it to a depth of 1 ft. is 1*s*. per cub. yd.; laying rubble to a depth of 8 in. costs 1*s*. per cub. yd.; gravel is then placed on the top to a depth of 9 in. Find (1) what is the price per cubic yard of the gravel; (2) the difference in outlay if the gravel had been 8 in. deep and the rubble 9 in.

(22) If I require 1716 cub. in. of wood to make a box, how many can I make out of 5 cub. yds. 1 cub. ft. 84 cub. in. of wood?

(23) A reservoir contains 2334 cub. yds. 2 ft. 1693 in. of water: how many tanks containing 3 cub. yds. 16 ft. 181 in. can I fill with the water from it?

(24) The content of a tank is 5 times the metal of which it is composed: if the united volume is 32 cub. yds. 7 ft. 498 in., what is the volume of the metal?

(25) A rectangular sheet of water is 270 yds. broad, 7 ft. deep, and contains 7314300 cub. ft. of water: how long is it?

(26) Determine the edge of a cube, whose volume is 16 yds. 18 ft. 1088 in.

(27) If 100 cub. in. of stone weigh 14 lbs., how many lbs. will a block 4 ft. long, 2½ ft. broad, and 1¼ ft. thick weigh?

(28) What must be the thickness of an embankment 210 yds. long, 14 ft. deep, which has the same cubical content as one 90 yds. 12 ft. deep, and 8 ft. thick?

(29) Find the value of a block of wood 8 ft. long, 4 ft. 2 in. wide, and 3 in. thick, if a cubic foot cost 5*s*.

(30) How high should a room be made when its length and breadth are 31 ft. 3 in. and 24 ft., in order that it may contain 10000 cub. ft. of air?—and how much less air would it contain than a room 6 in. higher?

VI. MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

143.
16 drams = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
14 pounds = 1 stone (st.)

28 pounds = 1 quarter (qr.)
4 qrs. = 112 lbs. = 1 hundredweight (cwt.)
20 hundredweights = 1 ton (ton)

Ex. 1. Reduce 2 tons 15 cwts. 3 qrs. 14 lbs. 13 ozs. 12 drs. to drams:—

tons	cwts.	qrs.	lbs.	ozs.	drs.
2	15	3	14	13	12

$$\begin{array}{r} \underline{20}\\ 55\\ \underline{4}\\ 223\\ \underline{28}\\ 1798\\ \underline{446}\\ 6258\\ \underline{16}\\ 37561\\ \underline{6258}\\ 100141\\ \underline{16}\\ 600858\\ \underline{100141}\\ 1602268 \end{array}$$

∴ 2 tons 15 cwts. 3 qrs. 14 lbs. 13 ozs. 12 drs. = 1602268 drs.

Proof. Ex. 2. Reduce 1602268 drs. to tons, cwts., &c.

$$\begin{array}{rrl}
 & \text{drams} & \\
16\,\{ & 4)\underline{1602268} & \\
 & 4)\underline{400567}-0 & \} -12 \text{ drams.}\\
16\,\{ & 4)\underline{100141}-3 & \\
 & 4)\underline{25035}-1 & \} -13 \text{ ounces.}\\
28\,\{ & 4)\underline{6258}-3 & \\
 & 7)\underline{1564}-2 & \} -14 \text{ pounds.}\\
 & 4)\underline{223}-3 & \\
 & 2,0)\underline{5,5}-3 & \text{quarters.}\\
 & 2 \text{ tons } 15 \text{ cwts.} &
\end{array}$$

∴ 1602268 drs. = 2 tons 15 cwts. 3 qrs. 14 lbs. 13 ozs. 12 drs.

Exercise 86.

i. Reduction.

Reduce each of the following compound quantities to **drams**, giving the proof in each case:—

(1) 53 tons 16 cwts. 2 qrs. 24 lbs.; 16 cwts. 3 qrs. 19 lbs. 13 ozs.; 19 cwts. 3 qrs. 14 lbs. 11 ozs. 7 drs.; 13 cwts. 1 qr. 26 lbs.

(2) 124 tons 13 cwts. 2 qrs. 19 lbs.; 19 cwts. 1 qr. 21 lbs. 7 ozs.; 12 cwts. 3 qrs. 26 lbs. 14 ozs. 13 drs.; 17 cwts. 2 qrs. 16 lbs.

(3) 113 tons 14 cwts. 3 qrs. 21 lbs.; 18 cwts. 3 qrs. 26 lbs. 13 ozs.; 15 cwts. 3 qrs. 26 lbs. 14 ozs. 11 drs.; 18 cwts. 3 qrs. 18 lbs.

(4) 200 tons 13 cwts. 3 qrs. 23 lbs.; 16 cwts. 1 qr. 13 lbs. 14 ozs.; 19 cwts. 2 qrs. 24 lbs. 13 ozs. 4 drs.; 14 cwts. 2 qrs. 24 lbs.

(5) 161 tons 15 cwts. 3 qrs. 27 lbs.; 19 cwts. 2 qrs. 21 lbs. 13 ozs.; 14 cwts. 3 qrs. 27 lbs. 13 ozs. 9 drs.; 14 cwts. 3 qrs. 26 lbs.

(6) 134 tons 13 cwts. 3 qrs. 26 lbs.; 17 cwts. 3 qrs. 16 lbs. 14 ozs.; 11 cwts. 1 qr. 16 lbs. 11 ozs. 13 drs.; 12 cwts. 1 qr. 17 lbs.

And reduce each of the following numbers of drams to tons, &c., giving the proof in each case:—

(7) 5657813469, 8985939039, 13312502280, 9349795840.
(8) 5990626026, 9318751596, 13645314837, 9583540736.
(9) 6323438583, 9651564153, 13978127394, 9817285632.
(10) 6656251140, 9984376710, 14310939951, 10051030528.
(11) 15642190179, 10317189267, 14643752508, 10284775424.
(12) 19303128306, 10650001824, 14976565065, 10518520320.

ii. Addition.

	tons	cwts.	qrs.	lbs.		tons	cwts.	qrs.	lbs.
(1)	134	11	3	18	(2)	12	19	3	27
	21	19	2	14		19	14	3	21
	186	18	2	7		18	10	2	15
	97	14	1	13		14	9	1	11
	141	13	3	26		17	16	1	18
	118	11	2	18		16	17	2	26
	154	13	3	11		9	14	3	13
	87	17	2	15		10	18	3	14

	tons	cwts.	qrs.	lbs.
(3)	53	16	2	14
	68	12	3	26
	97	9	1	16
	86	10	2	18
	47	15	1	24
	124	19	2	26
	173	15	1	17
	197	12	3	10

	tons	cwts.	qrs.	lbs.	ozs.	drs.
(4)	95	14	2	7	12	15
	63	12	3	18	8	9
	114	3	1	14	9	4
	197	10	2	22	14	12
	200	19	3	16	11	13
	57	15	3	25	4	3
	163	12	2	15	11	7
	68	4	2	16	10	10

(5) Find the value of 794 lbs. 15 ozs. 15 drs. + 392 lbs. 12 ozs. 10 drs. + 741 lbs. 11 ozs. 9 drs. + 654 lbs. 13 ozs. 12 drs. + 781 lbs. 5 ozs. 6 drs. + 854 lbs. 14 ozs. 11 drs.

(6) 794 qrs. 27 lbs. 15 ozs. + 391 qrs. 25 lbs. 14 ozs. + 584 qrs. 23 lbs. 12 ozs. + 968 qrs. 14 lbs. 9 ozs. + 357 qrs. 11 lbs. 10 ozs. + 567 qrs. 18 lbs. 11 ozs.

(7) Add together 964 cwts. 3 qrs. 27 lbs.; 351 cwts. 2 qrs. 19 lbs.; 598 cwts. 1 qr. 18 lbs.; 659 cwts. 3 qrs. 25 lbs.; 958 cwts. 1 qr. 24 lbs.; 647 cwts. 2 qrs. 26 lbs.

(8) Find the sum of 749 tons 19 cwts. 3 qrs.; 654 tons 18 cwts. 1 qr.; 992 tons 13 cwts. 3 qrs.; 757 tons 12 cwts. 0 qrs.; 592 tons 11 cwts. 3 qrs.; 888 tons 18 cwts. 2 qrs.

(9) A butcher sold 5 cwts. 2 qrs. 11 lbs. of beef; 8 cwts. 3 qrs. 13 lbs. of mutton; 12 cwts. 2 qrs. 4 lbs. of veal; and 8 cwts. 2 qrs. of lamb: required the weight of meat he sold.

(10) On Monday I sold 5 tons 3 cwts. 2 qrs. of coal; on Tuesday 4 tons 15 cwts. 17 lbs.; on Wednesday 14 tons 5 cwts.; and on Thursday 4 tons 3 qrs. 21 lbs.: how many pounds did I sell?

(11) Required the weight of 4 bags weighing respectively 3 qrs. 17 lbs.; 15 lbs. 10 ozs.; 2 qrs. 13 lbs. 7 ozs.; 5 lbs. 13 ozs.

(12) What is the total weight of the following: 27 tons 15 cwts. 1 qr. 8 ozs.; 13 cwts. 14 lbs.; 3 qrs.; 2 qrs. 19 lbs. 11 drs.; 27 lbs. 13 ozs. 13 drs.; 15 ozs. 15 drs.; 200 tons 17 cwts. 3 qrs. 14 lbs. 9 ozs. 11 drs.?

iii. Subtraction.

	tons	cwts.	qrs.	lbs.
(1)	131	11	2	18
	29	19	3	19
(3)	68	12	2	14
	53	16	3	26

	tons	cwts.	qrs.	lbs.
(2)	18	10	2	15
	17	14	3	21
(4)	186	14	1	7
	97	18	2	15

	tons	cwts.	qrs.	lbs.
(5)	141	13	1	13
	118	14	3	26

	tons	cwts.	qrs.	lbs.
(6)	154	11	2	11
	118	13	3	27

	tons	cwts.	qrs.	lbs.	ozs.	drs.
(7)	95	12	1	7	8	9
	67	14	3	18	12	15

	tons	cwts.	qrs.	lbs.	ozs.	drs.
(8)	154	13	2	11	12	12
	147	19	3	27	15	15

(9) From 15 tons 3 cwts. 3 qrs. take 13 tons 11 cwts. 1 qr. From 13 cwts. 3 qrs. 14 lbs. take 11 cwts. 2 qrs. 27 lbs.

(10) From 3 qrs. 17 lbs. 14 ozs. take 1 qr. 24 lbs. 15 ozs. From 2 qrs. 13 lbs. 11 ozs. take 26 lbs. 7 ozs. 15 drs.

(11) 800 tons 13 cwts. 1 qr. – 69 tons 3 qrs. 17 lbs.; 7 cwts. 2 qrs. 15 ozs. – 1 qr. 13 lbs. 13 drs.

(12) The great bell at Oxford weighs 7 tons 11 cwts. 2 qrs. 8 lbs., and with that of St. Paul's weighs 12 tons 13 cwts. 2 qrs. 26 lbs.: what does the bell of St. Paul's weigh?

(13) A hogshead of sugar weighs 3 cwts. 2 qrs. 12 ozs.: how many together would weigh 1 ton 1 cwt. 4 lbs. 8 ozs.?

(14) In a butcher's shop I purchased 25 lbs. 9 ozs. of meat—viz. 3 lbs. 7 ozs. of mutton, 7 lbs. 9 ozs. of beef, 4 lbs. 14 ozs. of lamb: how much pork did I buy?

(15) How many chests of tea each weighing 3 qrs. 23 lbs. would amount to 8 cwts. 2 qrs. 11 lbs.?

(16) How many pounds must I add to 3 cwts. 2 qrs. + 18 tons 17 lbs. + 8 cwts. 3 qrs. + 12 cwts. 19 lbs. to make the amount 19 tons 6 cwts. 1 qr. 5 lbs.?

iv. Multiplication.

(1) 70 tons 5 cwts. 1 qr. 8 lbs. 3 ozs. × 29, 36, 57, 81; 154 tons 19 cwts. 2 qrs. 15 lbs. 9 ozs. × 72, 84, 96, 120, 37.

(2) 18 tons 14 cwts. 3 qrs. 12 lbs. × 56, 78, 91, 82, 74; 22 lbs. 15 ozs. 13 drs. × 81, 74, 826, 864, 107.

(3) 15 tons 3 cwts. 2 qrs. 5 lbs. 8 ozs. × 87, 39, 150, 67, 82; 5 tons 8 cwts. 3 qrs. 4 lbs. 8 ozs. 3 drs. × 84, 71, 126.

(4) 13 tons 15 cwts. 13 lbs. × 9, 32, 72, 71, 463; 15 tons 19 cwts. 15 lbs. × 534, 596, 237, 428, 364.

v. Division.

(1) 456 tons 18 cwts. 1 qr. 14 lbs. 15 ozs. 12 drs. ÷ 84; 685 tons 7 cwts. 2 qrs. 8 lbs. 7 ozs. 10 drs. ÷ 126.

(2) 5761 tons 16 cwts. 1 qr. 27 lbs. 6 ozs. ÷ 82; 5199 tons 13 cwts. 3 qrs. 17 lbs. 14 ozs. ÷ 74.

(3) 6317 tons 5 cwts. 2 qrs. 4 lbs. ÷ 72; 3246 tons 7 cwts. 2 qrs. 7 lbs. 8 ozs. ÷ 37.

(4) 1320 tons 8 cwts. 3 qrs. 2 lbs. 8 ozs. ÷ 87; 925 tons 16 cwts. 1 qr. 27 lbs. 8 ozs. ÷ 61.

(5) 1271 tons 18 cwts. 3 qrs. 17 lbs. ÷ 89; 1157 tons 12 cwts. 1 qr. 1 lb. ÷ 81.

(6) Find the value of 5480 tons 15 cwts. 22 lbs. 10 ozs. ÷ 70 tons 5 cwts. 1 qr. 8 lbs. 3 ozs.

(7) Divide 10528 tons 15 cwts. 3 qrs. 16 lbs. by 87 tons 14 cwts. 3 qrs. 5 lbs. 8 ozs.

(8) How many times is 15 tons 3 cwts. 2 qrs. 15 lbs. 8 ozs. contained in 516 tons 2 qrs. 19 lbs.?

(9) What is the quotient of 414 tons 9 cwts. 0 qrs. 9 lbs. by 14 tons 5 cwts. 3 qrs. 9 lbs.?

What weight must be subtracted from each of the following dividends to make the division exact?—

(10) 845 tons 3 qrs. 16 lbs. ÷ 9, 36, 54, 73, 348.

(11) 793 tons 19 cwts. 22 lbs. ÷ 9, 48, 63, 85, 257.

(12) 925 tons 15 cwts. 2 qrs. ÷ 9, 54, 56, 75, 785.

(13) 84 tons 9 cwts. 3 qrs. ÷ 74, 67; 97 tons 3 qrs. 2 lbs. ÷ 86, 57.

(14) 150 tons 4 cwts. 7 lbs. 5 ozs. 3 drs. ÷ 120, 94, 61, 33.

Exercise 87.

(1) How many casks of sugar each containing 10 cwts. 3 qrs. 7 lbs. must I sell at a profit of one penny per pound to gain £40. 7*s.* 4*d.*?

(2) What is the difference between 9016 lbs. and 4 tons?

(3) If 248 tons 15 cwts. 2 qrs. 14 lbs. be divided between 5 men, 5 women, and 5 boys, so that a woman has twice as much as a boy, and a man has twice as much as a woman, how much does a boy get?

(4) How many packages each 2 lbs. 3 ozs. 8 drs. can I make out of one thousand and ten pounds?

(5) If a cubic foot of water weigh 1000 ozs., what weight of water is there in a reservoir 15 yards long, 12 yards broad, and 4 ft. 6 in. deep?

(6) What is the weight of water in a cubic vessel whose side is 5½ ft.?

(7) 2520 lbs. of a liquid just fill a rectangular cistern 9 ft. long, 5 ft. 4 in. wide, and 2 ft. 3 in. deep: what weight of the same liquid will fill another cistern 8 ft. long, 5 ft. 6 in. wide, and 3 ft. 9 in. deep?

(8) If 7 cwt. 1 qr. cost £26. 10*s.* 4*d.*, what is the value of 1 qr., and of 43 cwt. 1 qr.?

(9) A merchant during the week receives 685 tons 4 cwts. 3 qrs. 14 lbs. 7 ozs. 2 drs. of goods, in this manner: Each day he receives 6 loads, a load weighing 18 tons 19 cwts. 1 qr. 23 lbs.; 9 chests, each chest containing 2 qrs. 16 lbs. 9 ozs. 3 drs., and 8 parcels each of the same weight: what was the weight of a parcel?

(10) In order to remove 5199 tons 13 cwts. 3 qrs. 17 lbs. 14 ozs. of coal, a train which carries 70 tons 5 cwts. 1 qr. 8 lbs. 3 ozs. takes twice as many journeys as another train which can carry 87 tons 14 cwts. 3 qrs. 5 lbs. 8 ozs. requires to remove a weight of stone: what quantity of stone was there?

(11) 3 reservoirs, each containing 414 tons 9 cwts. 9 lbs. weight of water, fill as many tanks, each holding 14 tons 5 cwts. 3 qrs. 9 lbs. of water, as one larger reservoir which holds 1320 tons 8 cwts. 3 qrs. 2 lbs. 8 ozs. fills cisterns: what weight of water does a cistern contain?

(12) If a train in 72 journeys remove 1028 tons 19 cwts. 3 qrs. 4 lbs., how much does it remove in one journey, and how much will the same train remove in 98 journeys?

(13) A besieged garrison has 1157 tons 12 cwts. 1 qr. 1 lb. of provisions, the daily consumption being 14 tons 5 cwts. 3 qrs. 9 lbs.; by what weight of provisions must the store be increased to enable the garrison to hold out one day longer if the daily consumption be raised 17 cwt. 3 qrs. 6 lbs. 8 ozs.?

(14) Prove that the quotient of 6317 tons 5 cwts. 2 qrs. 4 lbs. by 1028 tons 19 cwts. 3 qrs. 4 lbs. is the same as the quotient of 87 tons 14 cwts. 3 qrs. 5 lbs. 8 ozs. by 14 tons 5 cwts. 3 qrs. 9 lbs.

(15) A cubic foot of wood weighs 20 lbs.: find the weight of 10 planks, each 30 ft. long, 1 ft. broad, and 1 in. thick.

(16) A grocer bought £24. 10*s.* worth of rice, and by selling

it at 4½*d.* a pound cleared £7.: how many hundredweights did he buy?

(**17**) A block of stone 5 ft. 3 in. long, 2 ft. 4 in. wide, and 1 ft. 2 in. thick weighs 1 cwt.: what will be the weight of another block of the same material, the dimensions of which are 7 ft. long, 1 ft. 9 in. broad, and 10 in. thick?

(**18**) How many trucks each containing 5 tons 18 cwts. 3 qrs. 23 lbs. will be required for 136 tons 15 cwts. 3 qrs. 25 lbs. weight of coal?

(**19**) A grocer bought a certain number of chests of tea: if the average weight of each chest was 2 cwts. 3 qrs. 7 lbs., and the weight of the whole was 5 tons 9 cwts. 2 qrs. 21 lbs., how many chests did he buy?

(**20**) Divide 14 tons 17 cwts. 3 qrs. 9 lbs. 8 ozs. 4 drs. of goods equally among 63 people. If each person's share be worth £35. 5*s.* 11¾*d.*, how much is that per ounce?

(**21**) Divide 12 tons 3 cwts. 1 qr. 25 lbs. among 2 men, 3 women, and 4 boys, so that each man may receive four times as much as a woman, and each woman three times as much as a boy.

(**22**) If 1 ton 2 cwt. 1 qr. 8 lbs. of copper coin are worth £500., what is the value of 1 oz.?

(**23**) How many hogsheads of sugar, each weighing 5 cwts. 3 qrs. 24 lbs., will weigh 17 tons 11 cwts. 3 qrs. 16 lbs?

(**24**) A loaded truck weighs 7 tons 6 cwts. 3 qrs. 17 lbs.; the goods are eight times the weight of the empty truck: find the weight of the truck.

(**25**) I have 8 trucks and a number of barrels, each of which will hold 3 cwts. 2 qrs. 12 lbs. 5 ozs.: how many barrels must I put into each truck so that the whole weight of goods may be 30 tons 6 cwts. 1 qr. 24 lbs.?

TROY WEIGHT.

144. 24 grains = 1 pennyweight (dwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

Gold, silver, and jewels are chiefly weighed by these measures.

7000 grs. Troy = 1 lb. Avoirdupois
5760 grs. Troy = 1 lb. Troy

Exercise 88.

i. Reduction.

Reduce each of the following quantities, giving the proof in each case :—

(1) 754 dwts. 23 grs. to grains; 64 ozs. 19 dwts. to pennyweights; 467 lbs. 3 ozs. to pennyweights; 3 qrs. 27 lbs. 9 ozs. to ounces; 5 lbs. 3 ozs. 16 dwts. 17 grs. to grains; 8475 grs. to pennyweights; 3264 dwts. to ounces; 7721 ozs. to pounds.

(2) Bring 55464 grs. to ounces; 84750 grs. to pounds; 12 lbs. 2 dwts. to grains; 25644 dwts. to pounds; 15 lbs. 7 ozs. to pennyweights; 1 cwt. 2 qrs. 13 lbs. to ounces; 20501 grs. to pounds?

ii. Addition.

(1) 15 lbs. 11 ozs. 13 dwts. + 17 lbs. 4 ozs. + 9 ozs. 7 dwts. + 15 lbs. 14 dwts. + 16 ozs. 15 dwts. 14 grs.

(2) 17 dwts. 21 grs. + 15 ozs. 14 dwts. 21 grs. + 5 lbs. 4 ozs. 17 dwts. + 15 lbs. 11 ozs. + 24 lbs. 7 ozs. 15 dwts. + 216 lbs.

(3) 17 lbs. 5 ozs. 16 dwts. + 11 ozs. 17 dwts. + 16 lbs. 23 grs. + 7 ozs. 15 dwts. + 3 lbs. 19 dwts. 13 grs.

(4) 14 dwts. 21 grs. + 11 ozs. 12 grs. + 19 lbs. 14 dwts. + 36 lbs. 4 ozs. + 11 ozs. 13 dwts. + 15 dwts. 12 grs.

iii. Subtraction.

(1) From 24 lbs. 5 ozs. 17 dwts. take 16 lbs. 14 dwts. 7 grs.; from 16 ozs. 7 dwts. 14 grs. take 5 ozs. 19 dwts. 21 grs.

(2) 7 lbs. 9 ozs. 5 grs. – 6 lbs. 9 dwts. 19 grs.; 200 lbs. 19 dwts. – 50 lbs. 3 ozs. 21 dwts. 18 grs.

(3) Take 1 lb. 11 ozs. 19 dwts. 23 grs. from 1 cwt.; take 3 ozs. 13 dwts. from 8 ozs. 15 dwts. 21 grs.

(4) Subtract 25 lbs. 3 ozs. 17 dwts. from 38 lbs. 14 dwts.; 181 lbs. 11 ozs. 13 dwts. – 79 lbs. 13 dwts. 21 grs.

iv. Multiplication.

(1) 24 lbs. 6 ozs. 17 dwts. × 7, 9, 6, 12, 8; 17 ozs. 4 dwts. 15 grs. × 28, 42, 50, 64, 72.

(2) 6 lbs. 2 dwts. 15 grs. × 43, 57, 67, 79, 83; 7 lbs. 3 ozs. 21 dwts. 15 grs. × 94, 87, 63, 59, 98.

(3) 8 lbs. 7 ozs. 19 dwts. 23 grs. × 74, 81, 82, 68, 98, 120; 34 lbs. 17 dwts. 14 grs. × 99, 84, 36, 72.

v. Division.

(1) 9 ozs. 8 dwts. 5 grs. ÷ 27, 34; 7 lbs. 9 ozs. 3 dwts. ÷ 17, 43; 1 cwt. 19 lbs. 10 ozs. ÷ 88, 66, 87.

(2) 178 lbs. 9 ozs. 19 dwts. 23 grs. ÷ 120, 39, 87, 64, 72, 154; 34 lbs. 17 dwts. 14 grs. ÷ 99, 84, 36, 72.

145. APOTHECARIES' WEIGHT.

20 grains = 1 scruple (sc. or ℈)
3 scruples = 1 drachm (dr. or ʒ)
8 drachms = 1 ounce (oz. or ℥)
12 ounces = 1 pound (lb. or ℔.)

The grain, ounce, and pound are the same as in Troy weight.

APOTHECARIES' FLUID MEASURE.

60 minims = 1 drachm (fl. dr.)
8 drachms = 1 ounce (fl. oz.)
20 ounces = 1 pint (O.)—contraction for Octavus.
8 pints = 1 gallon (C.)—contraction for Congius.

The connection between the measures of capacity to those of weight in these tables is best remembered by the old lines:—

'A pint of water
Weighs a pound and a quarter.'

Exercise 89.

(1) Reduce to grains: 6 lbs. 4 ozs. 14 grs.; 1 lb. 9 ozs.; 4 lbs.; 17 ozs. 7 grs.; 8 lbs. 5 ozs.; 98 lbs. 2 ozs. 12 grs.

(2) Reduce 79864 grs. to pounds; 68739 grs. to ounces; 401325 grs. to pounds; 75832 grs. to ounces; 908325 grs. to pounds; 901746 grs. to ounces; 7156382 grs. to pounds. Give proof in each case.

(3) 69 lbs. 9 ozs. 16 grs. + 7 ozs. 18 grs. + 5 lbs. 4 ozs. 9 grs.

+ 5 lbs. 2 ozs. 14 grs.; 74 lbs. 5 ozs. 14 grs. + 5 lbs. 1 oz. 7 grs. + 26 lbs. 11 ozs. 17 grs. + 9 ozs. 17 grs.; 6 lbs. 12 ozs. 14 grs. + 7 lbs. 11 ozs. 19 grs. + 11 ozs. 12 grs. + 99 lbs. 7 ozs. 5 grs.

(4) 6 lbs. 2 ozs. 12 grs. − 9 ozs. 18 grs.; 94 lbs. 10 ozs. 19 grs. − 26 lbs. 13 ozs. 13 grs.; 97 lbs. 5 ozs. 7 grs. − 13 lbs. 9 ozs. 15 grs.

(5) 24 lbs. 7 ozs. 9 grs. × 8, 19, 57, 99, 136; 13 lbs. 5 ozs. 14 grs. × 15, 28, 39, 81, 107.

(6) 189 lbs. 2 ozs. 7 grs. ÷ 17, 25, 31, 86, 257; 740 lbs. 11 ozs. 13 grs. ÷ 13, 28, 41, 79, 361.

(7) Express the difference between 19 C., 3 O., 14 ʒ, 36 ♏, and 10 C., in ♏, and multiply result by 67, 92, 169.

(8) Bring 3 C., 4 O., 8 ʒ, to ♏, and divide the result by 6, 19, 34, 75, and 214.

(9) Reduce 36 C., 6 O., 17 ʒ, to ♏, and multiply the result by 8, 17, 35, 69, 253.

VII. MEASURES OF CAPACITY.

LIQUID AND DRY.

146. For grain and dry goods, as fruit, roots, &c.:—

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
2 quarts = 1 pottle (pot.)
4 quarts = 1 gallon (gal.)
2 gallons = 1 peck (pk.)

4 pecks = 1 bushel (bus.)
4 bushels = 1 coomb
8 bushels = 1 quarter (qr.)
5 quarters = 1 load
2 loads = 1 last

WINE AND BEER MEASURE.

WINE.

63 gallons = 1 hogshead (hhd.)
84 " = 1 puncheon (pun.)
126 " = 1 pipe (pi.)
252 " = 1 tun

BEER.

9 gallons = 1 firkin (fir.)
18 " = 1 kilderkin (kil.)
36 " = 1 barrel (bar.)
54 " = 1 hogshead (hhd.)

The gallon for liquors and corn contains 277·274 cub. in., and holds 10 lbs. adv. of distilled water.

Exercise 90.

i. Reduction.

Reduce each of the following quantities, giving the proof in each case :—

(1) 704064 gals. to pints, 34050 bars. to quarts, 74086 hhds. of ale to gallons, 6040 kils. to half-pints, 27169 hhds. of wine to quarts, 84 qrs. 3 bus. 2 pks. to gallons.

(2) 70460 pts. to hogsheads of ale, 7045 gals. to hogsheads of ale, 21 kils. 1 fir. 3 gals. to hogsheads of ale, 9567 gills to gallons, 4956 qts. to quarters, 7864 pks. to quarters, 3218 pts. to bushels.

(3) 5 bus. 3 pks. 4 gals. to pints, 5 pks. 1 gal. 3 qts. to quarts, 15 qrs. 3 bus. 2 pks. to gallons, 3 pks. 3 qts. to pints, 13 bus. to pints, 30004 pks. to quarters, 84061 gals. to bushels, 756 qrs. to gallons, 740604 pts. to pecks.

ii. Addition.

	qrs.	bus.	pks.	gals.		qrs.	bus.	pks.	gals.		qrs.	bus.	pks.	gals.
(1)	95	2	3	1	(2)	62	7	3	1	(3)	65	7	1	1
	84	7	2	0		95	4	2	1		84	5	2	0
	62	5	3	1		58	5	2	1		63	6	3	0
	54	6	1	1		18	6	3	1		72	4	3	1

	qrs.	bus.	pks.	gals.		gals.	qts.	pts.	gills		gals.	qts.	pts.	gills
(4)	48	7	3	1	(5)	63	1	0	3	(6)	76	3	1	1
	94	5	3	0		18	2	1	2		87	3	0	2
	76	4	2	1		75	3	0	1		98	3	1	3
	81	3	2	0		28	3	1	3		47	2	0	3

	gals.	qts.	pts.	gills		gals.	qts.	pts.	gills.
(7)	36	2	0	3	(8)	79	3	1	3
	67	3	0	2		99	2	1	3
	78	3	1	1		88	3	1	3
	89	3	1	3		77	1	1	3

iii. Subtraction.

	qrs.	bus.	pks.	gals.		qrs.	bus.	pks.	gals.
(1)	95	2	3	0	(2)	58	5	2	0
	84	7	2	1		19	6	3	1

	qrs.	bus.	pks.	gals.		qrs.	bus.	pks.	gals
(3)	85	3	2	0	(4)	99	2	1	1
	66	4	3	1		97	7	3	0
	gals.	qts.	pts.	gills		gals.	qts.	pts.	gills
(5)	63	1	0	2	(6)	87	2	0	2
	18	2	1	3		76	3	1	3
(7)	98	1	1	1	(8)	97	1	1	2
	76	3	1	3		88	3	0	3

iv. Multiplication.

(1) Multiply 73 gals. 3 qts. 1 pt. 3 gills by 6, 24, 36, 53; 76 gals. 3 qts. 1 pt. 2 gills by 8, 26, 38, 55, 245.

(2) Multiply 84 qrs. 7 bus. 2 pks. 1 gal. by 7, 32, 45, 246; 63 qrs. 6 bus. 3 pks. 1 gal. by 8, 33, 48, 68, 256.

v. Division.

(1) Divide 87 gals. 2 qts. 2 gills by 8, 36, 52, 68, 346; 98 gals. 1 qt. 1 pt. 1 gill by 9, 34, 51, 69, 272.

(2) Divide 75 qrs. 5 bus. 3 pks. 1 gal. by 8, 35, 72, 67, 542; 94 qrs. 5 bus. 3 pks. by 9, 36, 73, 68, 543.

147. Quarterly Terms are Lady-day, March 25; Midsummer, June 24; Michaelmas, September 29; Christmas, December 25.

The following facts should be remembered :—

In Number.

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

In Paper.

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
60 skins of parchment = 1 roll

The Angular Measures are—

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle
4 right angles, or 360° = 1 circle

Ex. 1. (1) Multiply 4 circ. 17° 35′ 42″ by 25, 50, 37, 84, 265; and divide results by 13, 27, 41, 65, 139.

(2) Multiply 21 circ. 16° 22′ 39″ by 42, 17, 89, 93, 248; and divide results by 17, 32, 53, 79, 352.

(3) Multiply 17 right angles 42′ 57″ by 9, 16, 58, 72, 175; and divide results by 8, 24, 48, 85, 418.

XIII. CONCRETE FRACTIONAL QUANTITIES.

I. FRACTIONAL MEASURES.

i. Reduction.

148. From (**36**) we see that if £1. be taken as unit, 13 shillings would be represented by the fraction $\frac{13}{20}$; conversely £$\frac{13}{20}$., or $\frac{13}{20}$ of £1., is equal to 13 shillings, which again would be represented by 13 if 1 shilling be taken as unit. So 15 shillings, £1. being the unit, would be represented by $\frac{3}{4}$, or by 15 if 1 shilling be the unit. Now $13 = 20 \times \frac{13}{20}$, and $15 = 20 \times \frac{3}{4}$: *i.e.*, in either case, when we change the unit, the numerical value of the fraction is multiplied or divided by the number of times the larger unit is contained in the smaller.

Ex. 1. Thus, reduce £$\frac{15}{64}$. to shillings.

In the given expression one pound is the unit, and we have to find what the fraction becomes when the unit is changed to 1 shilling. We proceed thus : £$\frac{15}{64} = \frac{15}{64} \times \frac{20}{1}s. = \frac{75}{16}s. = 4\frac{11}{16}s.$

Ex. 2. Reduce $\frac{9}{700}$ cwt. to the fraction of 1 lb.

Here the unit must be changed from 1 cwt. to 1 lb.

$$\therefore \frac{9}{700} \text{ of 1 cwt.} = \frac{9}{700} \times \frac{112}{1}\text{lb.} = \frac{36}{25}\text{ lb.} = 1\frac{11}{25}\text{ lb.}$$

Ex. 3. Reduce $10\frac{1}{2}d.$ to the fraction of $1s.$

Here the unit is to be changed from $1d.$ to $1s.$

$$\therefore 10\tfrac{1}{2}d. = 10\tfrac{1}{2}d. \div 12s. = \frac{21}{2} \times \frac{1}{12}s. = \frac{7}{8}s.$$

Ex. 4. Reduce $4\frac{8}{9}$ ft. to the fraction of a furlong.

Here the unit is to be changed from feet to furlongs.

$$\therefore 4\tfrac{8}{9}\text{ ft.} = 4\tfrac{8}{9} \div 660 \text{ fur.} = \frac{44}{9} \times \frac{1}{660}\text{ fur.} = \frac{1}{135}\text{ fur.}$$

149. Ex. 1. Express 1 guinea as the fraction of £1.

One guinea = $21s.$

= £$\frac{21}{20}$., the unit being changed to £1.

Hence, Ex. 2. Reduce $\frac{15}{14}$ of a guinea to the fraction of £1.

$$\frac{15}{14}\text{ guinea} = \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}} \times \frac{\overset{3}{\cancel{21}}}{\underset{4}{\cancel{20}}} = £\frac{9}{8}.$$

And, Ex. 3. Reduce £$\frac{9}{8}$. to the fraction of a guinea.

$$£\tfrac{9}{8}.=\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{5}{\cancel{20}}}{\underset{7}{\cancel{21}}}\text{gs.}=\tfrac{15}{14}\text{ gs.}$$

Ex. 4. Reduce 1 lb. troy to the fraction of 1 lb. avoirdupois.

1 lb. troy = 5760 grains ; 1 lb. avoirdupois = 7000 grains.
∴ 1 lb. troy = $\frac{5760}{7000}$ lb. avoirdupois = $\frac{144}{175}$ lb. avoirdupois.

Ex. 5. Reduce $\frac{7}{8}$ lb. avoirdupois to the fraction of 2 lbs. troy.

$\frac{7}{8}$ lb. avoirdupois = $\frac{7}{8}\times\frac{175}{144}\times\frac{1}{2}=\frac{1225}{2304}$.

150. In the above examples we have reduced a fraction of one denomination to a lower denomination, and conversely ; we now can proceed to reduce a fraction of one denomination to a compound quantity of lower denomination, and its converse.

Ex. 1. Find the value of $\frac{15}{8}$ acre.

$\frac{15}{8}$ ac. = $1\frac{7}{8}$ ac. = 1 ac. $\frac{7}{8}\times 4$ rds. = 1 ac. $3\frac{1}{2}$ rds. = 1 ac. 3 rds. 20 po.

This might also have been worked by compound division.

Ex. 2. Express 9 ozs. 17 dwts. 8 grs. as pounds troy.

1st method. 8 grs. = $\frac{1}{3}$ dwt.

$$\therefore\ 17\text{ dwts. }8\text{ grs.}=17\tfrac{1}{3}\text{ dwts.}=\frac{\overset{13}{\cancel{52}}}{3}\times\frac{1}{\underset{5}{\cancel{20}}}=\tfrac{13}{15}\text{ oz.}$$

$$\therefore\ 9\text{ ozs. }17\text{ dwts. }8\text{ grs.}=9\tfrac{13}{15}\text{ oz.}=\frac{\overset{37}{\cancel{148}}}{15}\times\frac{1}{\underset{3}{\cancel{12}}}=\tfrac{37}{45}\text{ lb.}$$

2nd method. Reduce both quantities to the lowest denomination—grains.

9 ozs. 17 dwts. 8 grs. = 4736 grains
1 lb. troy = 5760 grains

$$\therefore\ \text{fraction required}=\tfrac{4736}{5760}=\frac{8\times16\times37}{128\times5\times9}=\tfrac{37}{45}.$$

Ex. 3. Express 4 yds. 1 ft. 10½ in. as a fraction of a pole.

1st method. 10½ in. = $\frac{21}{2}\times\frac{1}{12}$ ft. = $\frac{7}{8}$ ft.
∴ 1 ft. 10½ in. = $\frac{15}{8}$ ft. = $\frac{15}{8}\times\frac{1}{3}$ yd. = $\frac{5}{8}$ yd.
∴ 4 yds. 1 ft. 10½ in. = $4\frac{5}{8}$ yds. = $\frac{37}{8}\times\frac{2}{11}$ po. = $\frac{37}{44}$ po.

2nd method. 4 yds. 1 ft. 10½ in. = 333 half-inches.
1 po. = 396 half inches.
∴ fraction required = $\frac{333}{396}=\frac{37}{44}$.

From these last three examples, it will be seen, the 1st method is preferable when the denominator of the fraction and the multiplier employed have a common factor.

Exercise 91.

Reduce—

(1) $7\frac{3}{4}d.$ to the fraction of a shilling; $17\frac{5}{8}s.$ to the fraction of a pound.

(2) $\frac{5}{6}$ of a farthing to the fraction of 1*s.*; $\frac{7}{18}$ of a shilling to the fraction of £1.

(3) $\frac{3}{5}$ of 1 crown to the fraction of £1.; $\frac{5}{6}$ of a guinea to the fraction of £1.

(4) $\frac{5}{6}$ of a moidore, $\frac{2}{3}$ of a crown, $\frac{3}{8}$ of a shilling, and $\frac{7}{8}$ of a florin to the fraction of £1.

(5) $\frac{5}{8}$ of a yard, $\frac{3}{4}$ of a pole, $\frac{7}{8}$ of a furlong and 4 inches to the fraction of a mile.

(6) $\frac{5}{8}$ of a foot, $\frac{3}{11}$ of a rod, $\frac{2}{3}$ of a rood, $\frac{17}{18}$ of a chain to the fraction of an acre.

(7) $\frac{12}{19}$ of an inch, $\frac{15}{17}$ of a foot to the fraction of a cubic yard.

(8) $\frac{7}{13}$ of a pint, $\frac{50}{63}$ of a gallon, $\frac{15}{16}$ of a peck, $\frac{9}{19}$ of a bushel $\frac{6}{7}$ of a quart to the fraction of a sack.

Express—

(9) $\frac{3}{8}$ of £1., $\frac{7}{9}$ of 1 guinea, $\frac{3}{4}$ of 1 halfcrown, $\frac{31}{40}$ of £1., $\frac{313}{380}$ of £1., $\frac{24}{85}$ of £1., $\frac{5}{12}$ of 128*s.* as farthings.

(10) $\frac{4}{24}$ of 1 mile, $\frac{3}{17}$ of a mile, $\frac{1}{4}$ of a pole, $\frac{11}{30}$ of a furlong, $\frac{16}{17}$ of a league as inches.

(11) $\frac{7}{8}$ of an acre, $\frac{7}{8}$ of a square mile, $\frac{13}{30}$ of a yard, $\frac{1}{6}$ of a rood, $\frac{9}{16}$ of a pole as square inches.

(12) $\frac{2}{5}$ of a ton, $\frac{61}{282}$ of a ton, $\frac{15}{36}$ of a cwt., $\frac{24}{45}$ of a quarter, $3\frac{1}{3}$ of a ton, $\frac{14}{45}$ of 1 cwt., and $1\frac{2}{7}$ of a stone as pounds.

(13) $\frac{25}{56}$ of 1 lb. avoirdupois as pennyweights, $\frac{3}{4}$ of 1 lb. troy as drams, $1\frac{1}{4}$ cwt. as drams, $3\frac{17}{1120}$ of a ton as ounces.

Reduce—

(14) 7*s.* 8*d.*, 6*s.* 2*d.*, 5*s.* $9\frac{1}{2}d.$, $4\frac{1}{2}$ guineas, £2. 5*s.* $7\frac{1}{2}d.$, 2*s.* $0\frac{1}{4}d.$, £1. 10*s.* $6\frac{3}{4}d.$, $8\frac{3}{4}d.$ to the fraction of £1.

(15) 1 qr. 23 lbs. 8 ozs., 3 cwts. 0 qrs. 16 lbs., $1\frac{1}{4}$ cwt., 1 ton 6 lbs. 6 cwts. 3 qrs. 17 lbs. to the fraction of 1 ton.

(16) 1 fur. $6\frac{1}{2}$ po., 2 po. 3 yds., 3 fur. 15 po., 63 yds. 1 ft. 3 in., 3 yds. 1 ft. 2 in. to the fraction of 1 mile.

(17) 5 sq. in., 8 sq. ft. 13 in., 15 sq. po., 7 sq. yds. 6 sq. ft., 30 po. 2 yds. 1 ft. to the fraction of 1 acre.

(18) 1 qr. 23 lbs. 8 ozs., 3 qrs. 16 lbs., $1\frac{1}{4}$ cwt. 3 st. $10\frac{1}{2}$ lbs., 3 lbs. 3 ozs. 18 dwts. to the fraction of 1 ton.

(19) $\frac{4}{5}$ of a guinea, $\frac{5}{9}$ of 1*s.* 6*d.*, $4\frac{1}{4}$ of 3*s.* 6*d.*, $\frac{5}{7}$ of 2*s.*, $\frac{1}{4}$ of £37. 6*s.* 8*d.*, $2\frac{1}{2}$ of £3. 13*s.* 6*d.* to the fraction of £1.

(20) $2\frac{2}{3}$ of £1. 12*s.* 9*d.*, $1\frac{7}{9}$ of 10*s.* 6*d.*, $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1., $\frac{49}{8}$ of £1. 7*s.*, $\frac{3}{11}$ of 16*s.* $0\frac{1}{2}$*d.* to the fraction of £1.

(21) $\dfrac{\;\dfrac{3\frac{1\frac{3}{14}}{4}}{20}\;}{}$ of 1 ton 1 cwt. 2 qrs. 13 lbs., $1\frac{23}{110}$ of 7 cwts. 3 qrs. 12 lbs., $\dfrac{1\frac{2}{3}}{1\frac{8}{11}}$ of 7 cwts. 2 qrs. 24 lbs. to pounds.

(22) $\dfrac{4\frac{2}{7}}{3\frac{1}{3}}$ of $\dfrac{7\frac{1}{2}}{10\frac{5}{7}}$ of 6 cwts. 3 qrs. 17 lbs., $\frac{6}{7}$ of $1\frac{3}{4}$ of 1 ton 3 cwts., $\frac{2}{3}$ of $1\frac{2}{7}$ of $1\frac{3}{4}$ lb. to pounds.

(23) $1\frac{2}{3}$ of 1 yd., $2\frac{4}{5}$ of 3 fur. 12 po. 4 yds., $7\frac{3}{5}$ of 9 fur. 38 po. 1 yd., $6\frac{5}{12}$ of 2 fur. 7 po. $1\frac{1}{2}$ yds. to the fraction of 1 mile.

Reduce to highest denomination mentioned—

(24) 4 m. 3 fur. 37 po. $4\frac{1}{5}$ yds. × $5\frac{5}{7}$; 2 cwts. 3 qrs. × $\frac{4}{11}$; £$4\frac{1}{6}$. + $11\frac{1}{3}$*s.* + $7\frac{9}{12}$*d.*

(25) $\frac{3}{4}$ of 21*s.* + $\frac{3}{8}$ of 5*s.* + $\frac{3}{5}$ of 7*s.* 6*d.* − $\frac{3}{4}$ of 2*s.*, $\frac{2}{3}$ of 10*s.* 6*d.* + $\frac{7}{8}$ of 27*s.* − $\frac{6}{11}$ of 6*s.* 8*d.*

(26) $\frac{3}{4}$ of $10\frac{1}{2}$*d.* + $1\frac{2}{3}$ of $\frac{1}{9}$ of 5*s.* + $\frac{3}{5}$ of $1\frac{1}{4}$ of £1., $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1. + $\frac{1}{5}$ of £140. 10*s.* 6*d.* + $\frac{3}{5}$ of 21*s.*

(27) $1\frac{3}{4}$ of $3\frac{1}{2}$ of £1. 7*s.* + $\frac{5}{8}$ of 13*s.* 4*d.* − $\frac{2}{7}$ of $1\frac{5}{9}$ of $6\frac{2}{3}$*s.*

ii. Multiplication and Division.

151. If we divide £5. 11*s.* 8*d.* by 4, the quotient is £1. 7*s.* 11*d.*, and we say that £1. 7*s.* 11*d.* is one-fourth of £5. 11*s.* 8*d.*; or, using symbols, we express the fact thus:—$\frac{1}{4}$ of £5. 11*s.* 8*d.* = £1. 7*s.* 11*d.*, and therefore three-fourths of the same sum, or $\frac{3}{4}$ of £5. 11*s.* 8*d.* = £1. 7*s.* 11*d.* × 3 = £4. 3*s.* 9*d.* Now, £5. 11*s.* 8*d.* × 3 = £16. 15*s.* 0*d.*, and $\frac{1}{4}$ of £5. 11*s.* 8*d.* × 3 = $\frac{1}{4}$ of £16. 15*s.* 0*d.* = £4. 3*s.* 9*d.*, which is the same result as we obtained before; ∴ we conclude that $\frac{3}{4}$ of £5. 11*s.* 8*d.* = $\frac{1}{4}$ of £5. 11*s.* 8*d.* × 3.

Rule 1. To find the product of a compound quantity by a proper fraction, multiply it by the numerator of the fraction, and then divide the product by the denominator.

Example. Find value of $\frac{3}{7}$ of £5. 18*s.* 5*d.*

$$\begin{array}{rrrr} & £ & s. & d. \\ & 5 & 18 & 5 \\ & & & 3 \\ \hline 7) & 17 & 15 & 3 \\ \hline & 2 & 10 & 9 \end{array} \quad \text{or} \quad \begin{array}{rrrr} & £ & s. & d. \\ 7) & 5 & 18 & 5 \\ \hline & & 16 & 11 \\ & & & 3 \\ \hline & 2 & 10 & 9 \end{array} \quad \textit{Ans.}\ £2.\ 10s.\ 9d.$$

Rule 2. To multiply a quantity by a mixed number: (1) Multiply separately by the integer and by the fraction, and add the results together; or (2) reduce the mixed number to an improper fraction, and proceed as in Rule 1.

Example. Find value of 15 tons 13 cwts. 2 qrs. 20 lbs. × $2\frac{2}{3}$.

$$\begin{array}{rrrrrl} & \text{tons} & \text{cwts.} & \text{qrs.} & \text{lbs.} & \\ & 15 & 13 & 2 & 20 & \\ & & & & 2 & \\ \hline 3) & 31 & 7 & 1 & 12 & \\ \hline & 10 & 9 & 0 & 13\frac{1}{3} & \text{product by } \frac{2}{3} \\ & 31 & 7 & 1 & 12 & \quad ,, \quad ,, \ 2 \\ \hline & 41 & 16 & 1 & 25\frac{1}{3} & \quad ,, \quad ,, \ 2\frac{2}{3} \end{array} \quad \text{or} \quad \begin{array}{rrrrr} & \text{tons} & \text{cwts.} & \text{qrs.} & \text{lbs.} \\ & 15 & 13 & 2 & 20 \times \frac{8}{3} \\ & & & & 8 \\ \hline 3) & 125 & 9 & 1 & 20 \\ \hline & 41 & 16 & 1 & 25\frac{1}{3} \end{array}$$

Rule 3. To divide a quantity by a fraction, multiply by the denominator, and divide the product by the numerator.

Example. £12. 16*s.* 8*d.* ÷ $\frac{5}{12}$.

$$\begin{array}{rrrr} & £ & s. & d. \\ & 12 & 16 & 8 \\ & & & 12 \\ \hline 5) & 154 & 0 & 0 \\ \hline & 30 & 16 & 0 \end{array}$$

Rule 4. To divide a quantity by a mixed number, reduce the mixed number to a fraction, and proceed by Rule 3.

Example. Divide £25. 14*s.* $9\frac{3}{4}$*d.* by $1\frac{4}{7}$.

$$\begin{array}{rrrr} & £ & s. & d. \\ & 25 & 14 & 9\frac{3}{4} \\ & & & 7 \\ \hline 11) & 180 & 3 & 8\frac{1}{4} \\ \hline & 16 & 7 & 7\frac{13}{44} \end{array}$$

152. Either of the two methods given in Rules 1 and 2 may be used as a test of the accuracy of the results obtained by the other: whereas the proof of the results by Rules 3, 4, can be shown by multiplication.

Exercise 92.

Find the value of—

(1) $9\frac{3}{4}$ guineas; $\frac{7}{16}$ of £9. 17s. 4d.; £5. 11s. 0d. × $2\frac{1}{8}$; £4. 5s. 0d. × $2\frac{5}{16}$; £3. 19s. 9d. × $\frac{2}{29}$.

(2) $\frac{3}{5}$ of £1. 2s. 0d.; $1\frac{1}{15}$ of a halfcrown; $\frac{7}{16}$ of £1.; $\frac{1}{6}$ of a guinea; $\frac{1}{15}$ of £1. 5s. 0d.

(3) $\frac{3}{4}$ of £1. 10s. 0d.; $\frac{3}{4}$ of $1\frac{1}{2}$ of $\frac{4}{5}$ of £1. 5s. 0d.; $\frac{2}{5}$ of £33. 5s. 10d.; $\frac{4}{11}$ of £31. 2s. 5d.

(4) $2\frac{4}{5}$ of $5\frac{3}{4} \div 27\frac{1}{2}$ of £1. 2s. 6d.; $5\frac{1}{12} \div (2\frac{1}{3} + 3\frac{3}{4})$ of 12s. 6d.; $\frac{3}{4}$ of £4. 17s. 4d.

(5) $\frac{5}{7}$ of $58\frac{1}{3}$ lbs.; $\frac{1}{3}$ of a qr.; $\frac{5}{112}$ cwt.; $\frac{1}{4}$ of a qr.; $\frac{1}{16}$ of 1 cwt.; $\frac{3}{4}$ of $\frac{7}{11}$ of $18\frac{2}{7}$ lbs.

(6) $\frac{4}{9}$ cwt.; $\frac{3}{7}$ of a qr.; $4\frac{3}{8}$ lbs.; $3\frac{5}{8}$ of 3 qrs. 12 lbs.; 13 cwts. 3 qrs. 26 lbs. $15\frac{5}{8}$ ozs. ÷ $7\frac{1}{7}$.

(7) 4 lbs. 9 ozs. 14 dwts. $15\frac{5}{9}$ grs. ÷ $\frac{15}{16}$; $\frac{2}{5}$ of $\frac{1}{10}$ of 3 tons 3 cwts. 3 qrs.; $\frac{3}{5}$ of $\frac{4}{9}$ of 5 cwt. 3 qrs. 17 lbs.

(8) 5 cwts. 2 qrs. 14 lbs. × $\frac{3}{5}$; 2 cwts. 3 qrs. ÷ $1\frac{7}{11}$; $(2\frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} - 1) \div (2\frac{1}{2} \times 2\frac{1}{2} - 1)$ of 5 tons.

(9) ([illegible] ÷ $\frac{923}{[illegible]}$) × ($\frac{774}{413} \div \frac{1926}{545}$) of 3 m. 7 fur. 110 yds.; 4 ÷ $\frac{2}{5}$ of 5 yds. 1 ft. 11 in.

(10) 6 m. 7 fur. 15 po. $0\frac{16}{13}$ yds. × $\frac{13}{41}$; 5 yds. 2 ft. $5\frac{3}{5}$ in. × $7\frac{3}{5}$; 9 yds. 2 ft. 7 $\frac{3}{13}$ in. × $3\frac{4}{7}$.

(11) 15 yds. 1 ft. $9\frac{3}{5}$ in. ÷ $3\frac{2}{9}$; $\frac{5}{7}$ of $3\frac{4}{5}$ po.; 3 fur. 4 po. 2 yds. 1 ft. 4 in. × $2\frac{4}{7}$.

(12) ($1\frac{1}{4}$ of $\frac{3}{7}$ of $6\frac{1}{10}$) × ($\frac{14}{13}$ of $2\frac{4}{11}$) × $\frac{32}{49}$ of 2 fur. 97 yds. $2\frac{1}{3}$ ft.; 25 m. 5 fur. 15 po. $4\frac{5}{14}$ yds. ÷ $\frac{7}{40}$.

(13) $4\frac{3}{5}$ of an acre; 3 ac. 3 rds. 3 po. × $8\frac{3}{4}$; 1 cwt. 2 qrs. 13 lbs. × $3\frac{1}{14}$; $\frac{4}{7} \div 4\frac{2}{7}$ of $14\frac{3}{8}$ sq. yds.

(14) 3 ac. 1 rd. 20 po. × $5\frac{3}{7}$; 3 ac. 1 rd. 22 po. ÷ $5\frac{4}{5}$; $\frac{4}{5}$ ac. + $\frac{1}{36}$ rd. + $\frac{5}{9}$ po.; $14\frac{1875}{1000}$ of 1 ac.

(15) 2 ac. 3 rds. 5 po. ÷ $4\frac{7}{13}$; $(4\frac{1}{7} - 2\frac{1}{4}) \div (6\frac{1}{3} - 2\frac{1}{7})$ of 4 ac. 5 rds. 27 po.; $7\frac{3}{5}$ of 3 ac. 2 rds. 5 po.

(16) $7\frac{3}{5}$ of 3 ac. 1 rd. 4 po. 3 yds.; $1\frac{1}{4}$ of $3\frac{1}{3}$ rds.; $(4\frac{1}{3} \times 4\frac{1}{3} \times 4\frac{1}{3} - 4) \div (4\frac{1}{3} \times 4\frac{1}{3} - 4) \times 7$ ac.

(17) 2 wks. 3 dys. $\times \frac{4}{15}$; $(1\frac{1}{4}$ of $\frac{2}{15}) \div (\frac{2}{17} + \frac{11}{51})$ of $365\frac{1}{4}$ dys.; $(10 \times \frac{1}{7}) \div (3 + \frac{9}{7})$ of 2 wks.

(18) $23 \div (2\frac{2}{3} + \frac{2}{5})$ of 3 wks. 4 dys.; $(2\frac{1}{4} - 1\frac{3}{5}) \div (8\frac{1}{2} + 1\frac{9}{10})$ of 6 dys. 1 hr. 3 m.

(19) $7\frac{2}{5}$ of $365\frac{1}{4}$ dys. + $3\frac{9}{10}$ of $\frac{5}{6}$ wk. + $\frac{3}{4}$ of $5\frac{5}{9}$ hrs.; 4 dys. 12 hrs. 48 m. $\times \frac{4}{15}$.

(20) $1 \div [2 + 1 \div (2 + \frac{1}{2})]$ of 5 yrs. 6 wks. 3 dys.; $2 \div [3 + 4 \div (5 + \frac{6}{7})]$ of 7 hrs. 6 m.

(21) $\frac{11}{8}$ of £1. + $\frac{7}{12}$ of a guinea + $3\frac{3}{4}$ of £1.; $2\frac{1}{2}$ of half-a-guinea + $\frac{3}{5}$ of £3. 11*s.* 3*d.*

(22) $\frac{1}{3}$ of $3\frac{1}{4}$ of $\frac{12}{13}$ of £1. + $2\frac{1}{2}$ of $5\frac{1}{9}$ of £15. 19*s.* 6*d.*; $\frac{5}{16}$ of 16*s.* $4\frac{4}{5}$*d.* − $\frac{2}{5}$ of 4*s.* 10*d.*

(23) $2\frac{4}{9} \div 17\frac{3}{5}$ of a yard + $\frac{2}{15}$ of a furlong + $\frac{29}{33}$ of a mile + $\frac{2}{3}$ of $\frac{6}{8}$ of $3\frac{1}{2}$ miles.

(24) $3\frac{3}{8}$ of $2\frac{1}{9}$ square yards + $\frac{5}{8}$ of an acre; $\frac{1}{7} \div [4\frac{1}{3} + 3\frac{2}{3}] \times \frac{1}{2}$ of 1 cwt.

(25) $2 \div [3 + 4 \div (5 + \frac{6}{7})]$ of 12*s.* 7*d.*; $1 \div \{2 + 1 \div [3 + 1 \div (4 + \frac{1}{5})]\}$ of 13*s.* 1*d.*

(26) $7\frac{3}{13}$ of $1 \div [10 + 1 \div (3 + \frac{1}{30})]$ of 1 ton + $12\frac{4}{5} \times$ 1 cwt. 2 qrs. 7 lbs.

(27) Express in yards, feet, inches, the difference between $\frac{3}{8}$ of $\frac{2}{9}$ of a mile and $\frac{2}{9}$ of $\frac{1}{20}$ of $2\frac{6}{7}$ furlongs.

(28) Find the sum of $\frac{31}{37}$ of £2. 6*s.* 3*d.*, $\frac{31}{74}$ of £1. 12*s.* $4\frac{1}{2}$*d.*, $\frac{7}{24}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of 3 guineas.

(29) Express in avoirdupois weight the quotient of 1 lb. 11 ozs. 10 dwts. by $\frac{2256}{3675}$.

(30) Find the value of £2. 6*s.* 8*d.* $\div 1\frac{3}{4}$ of $\frac{2}{7}$ and of $\frac{5}{8}$ of 15*s.* + $\frac{3}{8}$ of 4*s.* 6*d.* − $\frac{3}{4}$ of 4*s.* 11*d.*

iii. Miscellaneous Operations.

153. Since 13*s.* 4*d.* is $\frac{2}{3}$ of £1., we may say 13*s.* 4*d.* ÷ £1. = $\frac{2}{3}$; or, $\frac{13s.\ 4d.}{£1.} = \frac{2}{3}$—*i.e.* 13*s.* 4*d.* may be considered as the dividend, £1. as the divisor, and $\frac{2}{3}$ as the quotient; and the remarks made in (**21**) will apply in this case.

This question is usually set in this form :—Reduce 13*s.* 4*d.* to the fraction of £1.; but it may also appear in these forms, which should be noticed :—

(1) If 13*s*. 4*d*. is the dividend, £1. the divisor, what is the quotient?

(2) If £1. is the unit, what is the measure of 13*s*. 4*d*.?

(3) What fraction of £1. is 13*s*. 4*d*.?

(4) Express 13*s*. 4*d*. as the fraction of £1.

(5) What part of £1. is 13*s*. 4*d*.?

(6) What is the ratio of 13*s*. 4*d*. to £1.?

154. The unit, or the *quantity to be divided*, or the quantity that comes immediately after the words 'fraction of,' is the *divisor*, in all these cases.

N.B.—The concrete quantities may be reduced to their highest or lowest denominations. The following examples will explain how the operation is performed :—

Ex. 1. Express 12*s*. 3*d*. as the fraction of one guinea.

The fraction required $= \dfrac{12s.\ 3d.}{21s.} = \frac{147}{252} = \dfrac{7 \times 21}{12 \times 21} = \frac{7}{12}.$

Ex. 2. What part of an acre is 2 rds. 20 po.?

The part required $= \dfrac{2 \text{ rds. } 20 \text{ po.}}{1 \text{ ac.}} = \frac{100}{160} = \frac{5}{8}.$

Ex. 3. If 1 fur. is the unit, what is the measure of 5 po. 1 yd. 2 ft. 6 in.?

The measure required

$$= \frac{5 \text{ po. } 1 \text{ yd. } 2 \text{ ft. } 6 \text{ in.}}{1 \text{ fur.}} = \frac{29 \text{ yds. } 1 \text{ ft.}}{220 \text{ yds.}} = \frac{88 \text{ ft.}}{660 \text{ ft.}} = \frac{2}{15}.$$

Ex. 4. From what fraction of 5 guineas must £2. 18*s*. 4*d*. be subtracted to be equal to $\frac{3}{4}$ of $\frac{7}{9}$ of £1.?

This is the same operation as that of adding £2. 18*s*. 4*d*. to $\frac{3}{4}$ of $\frac{7}{9}$ of £1. and expressing the result as a fraction of 5 guineas.

Now $\frac{3}{4}$ of $\frac{7}{9}$ of £1. $= \dfrac{\cancel{3}}{\cancel{4}} \times \dfrac{7}{\underset{3}{\cancel{9}}} \times \dfrac{\overset{5}{\cancel{20}}}{1} = \dfrac{35}{3} = 11s.\ 8d.$: this added to £2. 18*s*. 4*d*. = £3. 10*s*., ∴ the fraction required

$$= \frac{£3.\ 10s.\ 0d.}{5 \text{ guineas}} = \frac{70}{105} = \frac{10}{15} = \frac{2}{3}. \quad \textit{Ans.}$$

Ex. 5. Reduce $\frac{3\frac{1}{9}}{1\frac{1}{13}}$ of ($\frac{19}{120}$ of £1. − $\frac{17}{48}$ of 1*s.*) to the fraction of a moidore.

$\frac{19}{120}$ of £1. = 38*d.* $\frac{17}{48}$ of 1*s.* = $4\frac{1}{4}$*d.* ∴ $\frac{19}{120}$ of £1. − $\frac{17}{48}$ of 1*s.* = $33\frac{3}{4}$*d.*

∴ the dividend is $\frac{3\frac{1}{9}}{1\frac{1}{13}}$ of $33\frac{3}{4} = \frac{28}{9} \times \frac{13}{14} \times \frac{135}{4}$*d.*

and the divisor is 27 × 12*d.*

∴ the fraction required = the quotient

$$= \frac{\overset{2}{\cancel{28}}}{9} \times \frac{13}{\cancel{14}} \times \frac{\overset{5}{\cancel{135}}}{4} \times \frac{1}{\cancel{27}} \times \frac{1}{12} = \frac{65}{216}.$$

Ex. 6. What fraction of $\frac{7\frac{4}{5}}{4\frac{2}{3}}$ of £30. 13*s.* $2\frac{3}{4}$*d.* is $(8\frac{4}{7} - 3\frac{3}{5})$ of £5. 9*s.* $11\frac{1}{2}$*d.*? Reducing the concrete quantities to farthings we arrange the work thus :—

$$\frac{(8\frac{4}{7} - 3\frac{3}{5})}{\frac{7\frac{4}{5}}{4\frac{2}{3}}} \text{ of } \frac{£5.\ 9s.\ 11\frac{1}{2}d.}{£30.\ 13s.\ 2\frac{3}{4}d.} = \frac{4\frac{34}{35} \times 5278}{\frac{39}{5} \times \frac{3}{14} \times 29435}$$

$$= \frac{\overset{\overset{2}{\cancel{58}}}{\cancel{174}}}{\underset{\cancel{7}}{\cancel{35}}} \times \frac{\overset{2}{\cancel{14}}}{3} \times \frac{\cancel{5}}{\underset{\cancel{13}}{\cancel{39}}} \times \frac{\overset{\overset{2}{\cancel{406}}}{\cancel{5278}}}{\underset{\underset{5}{\cancel{1015}}}{\cancel{29435}}} = \frac{8}{15}. \quad Ans.$$

Exercise 93.

(1) Add together $1\frac{1}{15}$ of half-a-crown, $\frac{7}{16}$ of £1., $\frac{25}{100}$ of £1. 12*s.* 4*d.*

(2) Subtract $\frac{2}{3}$ of £38. 12*s.* 6*d.* from $\frac{4}{5}$ of £47. 16*s.* 3*d.*, and reduce the remainder to the fraction of £100.

(3) Add together $1\frac{1}{15}$ of half-a-crown, $\frac{7}{16}$ of £1., $\frac{1}{4}$ of £1. 12*s.* 4*d.*, and $\frac{1}{30}$ of 15*s.*

(4) $\frac{7}{8}$ po. + $\frac{3}{8}$ yd. + $\frac{3}{10}$ ft. (lineal measure).

(5) Find the value of $\frac{3}{14}$ of a florin + $\frac{3}{5}$ of half-a-crown − $\frac{1}{10}$ of five shillings − $\frac{3}{4}$ of a pound.

(6) Find the amount of land in $\frac{4}{5}$ ac. + $\frac{1}{36}$ rd. + $\frac{8}{9}$ po.

(7) Find the value of $\frac{3}{8}$ of £1. 10*s.* 0*d.* + $\frac{2}{3}$ of $1\frac{1}{2}$ of $\frac{4}{5}$ of £1. 5*s.* 0*d.* − $\frac{45}{80}$ of £1.

(8) Find the value of $\frac{1}{3}$ $(\frac{1}{6} + \frac{5}{2})$ + $\frac{1}{5}$ $(\frac{1}{2} + \frac{2}{3})$ − $\frac{1}{2}$ $(\frac{1}{3} + \frac{2}{5})$ of £1.

(9) Simplify $\frac{1}{4} \times \frac{(4\frac{7}{9} \text{ of } 6\frac{2}{3})}{7\frac{26}{27}} \times \frac{3\frac{2}{5} - 3\frac{1}{6}}{3\frac{3}{5} + 2\frac{1}{6}}$ of £182. 7*s.* 5*d.*

(10) Add together $\frac{1}{8}$ of a guinea, $\frac{5}{8}$ of a pound, $\frac{7}{8}$ of a shilling; and reduce a farthing to the fraction of a guinea.

(11) Reduce $3\frac{5}{9}$ of £1. 0*s.* $9\frac{3}{4}$*d.* to the fraction of £1. 10*s.* 10*d.*

(12) Simplify $\frac{6}{7} + \frac{7\frac{3}{4} - 6\frac{1}{2}}{2\frac{1}{3} \times 9}$; and divide 33 yds. 0 ft. 8 in. by $13\frac{1}{7}$.

(13) What difference is there between the values of $\frac{2\frac{4}{9} \text{ of } 5\frac{1}{8}}{27\frac{1}{2}}$ of £1. 2*s.* 6*d.* and $\frac{5\frac{1}{12}}{2\frac{1}{2} + 3\frac{3}{4}}$ of 12*s.* 6*d.*?

(14) Simplify $\left(\frac{2\frac{1}{2}}{54} + \frac{1}{8} - \frac{1}{216} + \frac{1}{12}\right) \div \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right)$ of £2. 13*s.* 4*d.*

(15) Find the value of $\frac{2}{5}$ of £33. 5*s.* 10*d.* $-\frac{4}{11}$ of £31. 2*s.* 5*d.*

(16) Find the value of $\frac{2}{15}$ of a guinea + $5\frac{5}{9}$ of 8*s.* 9*d.* − $\frac{3}{7}$ of half-a-crown.

(17) Find the value of $\frac{3}{14}$ of 3*s.* 6*d.* + $\frac{2}{7}$ of £11. 7*s.* 6*d.* − $\frac{5}{8}$ of £4. 17*s.* 4*d.*

(18) Find the difference between $\frac{5}{3}$ of £1. 13*s.* 9*d.* and $\frac{3}{7}$ of £5. 18*s.* 5*d.*, and reduce the result to the fraction of a guinea.

(19) Add together $\frac{3}{4}$ of £1., $\frac{1}{6}$ of a florin, $\frac{2}{3}$ of a shilling, and $\frac{1}{2}$ of $\frac{1}{5}$ of 13*s.* 4*d.*, and subtract the result from $\frac{4}{25}$ of £10.

(20) If $2\frac{3}{4} - 1\frac{5}{6}$ of an estate cost £440., what will $2\frac{1}{2}$ of $6\frac{5}{12}$ of it cost?

(21) Find the value of $\frac{4}{9}$ of a cwt. + $\frac{3}{7}$ of a qr. + $4\frac{3}{8}$ of a lb.

(22) What fraction of $9\frac{1}{4}$ guineas is $\frac{7}{16}$ of £9. 17*s.* 4*d.*?

(23) A shopkeeper, having $\frac{6}{7}$ of $58\frac{1}{3}$ lbs. of tea, sold $\frac{1}{8}$ qr., $\frac{5}{112}$ cwt., $\frac{1}{4}$ qr., $\frac{1}{16}$ cwt., $\frac{3}{8}$ qr., and $\frac{3}{4}$ of $\frac{7}{11}$ of $18\frac{6}{7}$ lbs.: what quantity had he left?

(24) What portion of $3\frac{3}{4}$ tons is $33\frac{1}{4}$ of 3 qrs.?

(25) Add together $\frac{1}{4}$ of $\frac{3}{5}$ of 5*s.* 4*d.*, $\frac{2}{3}$ of $\frac{5}{7}$ of 6*s.*, and $\frac{3}{8}$ of $\frac{2}{9}$ of 2*s.*

(26) Find the value of $\frac{3}{4}$ of a guinea + $\frac{1}{5}$ of a £. + $\frac{3}{4}$ of 1*s.* $-\frac{1}{4}$ of sixpence.

(27) Reduce $\frac{3}{7}$ of a pole to the fraction of a league.

(28) Reduce 1 mile to the fraction of 8 po. 6 in.

(29) If $\frac{1}{18}$ of an article cost $5\frac{2}{3}$ of a £., what will $\frac{1}{19}$ cost?

(30) $\frac{7}{8}$ of my money is £5. : how much have I ?

(31) What sum of money multiplied by $2\frac{2}{5}$ will equal $3\frac{5}{9}$ of £1. 0*s*. $9\frac{1}{4}$*d.* ?

(32) Take $\frac{5}{9}$ of $2\frac{7}{10}$ of 1*s*. from $\frac{3}{5}$ of $\frac{5}{16}$ of £1.

(33) What fraction of a £. is 14*s*. $9\frac{1}{2}$*d.* ?

(34) $\frac{14}{17}$ of a certain sum of money is a guinea : find the sum.

(35) Find in square feet the value of $\frac{5}{6}$ of $3\frac{3}{11}$ of an acre.

(36) I bought $\frac{1}{3}$ of $\frac{9}{10}$ of a ship for a thousand pounds : what was the entire ship worth ?

(37) What is the $\frac{9}{115}$ part of 7*s*. $1\frac{1}{2}$*d.* ?

(38) If $13\frac{1}{2}$ lbs. cost £$1\frac{5}{8}$., what will 3 parcels each containing $5\frac{5}{8}$ lbs. cost ?

(39) Find the value of $\frac{1}{3}$ of a florin + $\frac{5}{6}$ of a crown + ·125 of 3 halfcrowns − $\cdot\dot{3}$ of a shilling.

(40) Express $3\frac{5}{9}$ of £1. 0*s*. $9\frac{3}{4}$*d.* as the fraction of £1. 10*s*. 10*d.*

(41) Reduce $\frac{1}{16}$ of 10*s*. + $\frac{4}{9}$ of 5*s*. 3*d.* + $\frac{8}{15}$ of half-a-crown to the fraction of £1. 5*s*. 9*d.*

(42) How often is $\frac{4}{5}$ of $\frac{2}{3}$ of 1*s*. contained in £2. 16*s*. ?

(43) Reduce $\frac{5}{7}$ of 7*s*. 6*d.* to the fraction of half-a-guinea.

(44) If $\frac{3}{8}$ of a yard cost $\frac{4}{5}$ of a £., what will $\frac{3}{4}$ of a yard cost ?

(45) What fraction of £3. is $\frac{5}{6}$ of a guinea ?

(46) What part of £3. 17*s*. 6*d.* is £2. 14*s*. $3\frac{3}{17}$*d.* ?

(47) Find the difference between $\frac{1}{5}$ of a £. − $\frac{1}{4}$*s*. and $\frac{1}{14}$ of a guinea + $\frac{1}{2}$*s*.

(48) If $\frac{3}{8}$ of $4\frac{4}{15}$ of a railway share is worth £$4\frac{1}{5}$., what is the value of $\frac{6}{7}$ of it ?

(49) If a man has $24\frac{7}{8}$ bushels of corn, and he sells $\frac{2}{3}$ of it, how much has he left ?

(50) Reduce 5 cwts. 2 qrs. to the fraction of a ton.

(51) Find the value of $\dfrac{(11\frac{2}{5}+2\frac{7}{10})\div 9\frac{2}{5}}{6\frac{1}{2}+3\frac{5}{6}-\frac{1}{3}}$ of £1.

(52) A man gave away $\frac{3}{8}$ of his money to his son, and then $\frac{2}{5}$ of the remainder to his daughter : what fraction of the original sum had he for himself ?

(53) If a person, travelling $13\frac{3}{4}$ hrs. a day, perform a journey

in $27\frac{5}{8}$ days, in what time will he perform the same if he travel $10\frac{2}{7}$ hrs. a day?

(54) If $5\frac{7}{35}$ articles are worth £728., what will $2\frac{3}{5}$ cost?

(55) A man has $\frac{2}{3}$ of $\frac{5}{8}$ of $3\frac{1}{4}$ of 3 qrs. 12 lbs. of tea; he sells $\frac{1}{5}$ of $\frac{2}{3}$ of $1\frac{1}{5}$ of $\frac{5}{8}$ of the parcel to his brother: how many pounds has he left for himself?

(56) Reduce $\frac{5}{8}$ of £1. to the fraction of 27*s.*

(57) In a school of 4 classes, $\frac{1}{15}$ of the boys are in Class I., $\frac{1}{3}$ in Class II., $\frac{1}{5}$ in Class III., and there are 60 boys in Class IV.: how many boys attend the school?

(58) If $\frac{4}{5}$ of a ship are worth £975., find the value of $\frac{1}{3}$ of her.

(59) What sum is the same fraction of 8*s.* 9*d.* that 7*s.* $3\frac{1}{5}$*d.* is of 7*s.*?

(60) Express $\frac{2}{5}$ of $3\frac{1}{3}$ of £1. 11*s.* 6*d.* as the fraction of £3.

(61) What is the value of $\frac{2}{3}$ of $\frac{7}{9}$ of $\frac{27}{14}$ of $\frac{5}{8}$ of a house, when $\frac{2}{15}$ of $\frac{3}{8}$ of $\frac{25}{16}$ cost £42.?

(62) Express 14*s.* $7\frac{1}{2}$*d.* as the fraction of £3.

(63) What is the value of $\dfrac{\dfrac{7\frac{8}{9}}{10\frac{1}{7}}+1\frac{2}{9}}{(3\frac{3}{4}\div\frac{5}{8})+\dfrac{2\frac{3}{5}}{6\frac{1}{2}}}$ of 1*s.* 4*d.*?

(64) Find the value of a prize cup weighing $\frac{3}{4}$ lb. at the rate of $3\frac{1}{4}$*d.* per grain.

(65) How long is $\frac{7}{15}$ of a day?

(66) Value of $7\frac{11}{2240}$ of 1 qr. 7 lbs.; and of $16\frac{7}{45}$ of £31. 12*s.* 6*d.*

(67) Reduce $\frac{5}{12}-\frac{1}{15}+\frac{2}{3}-\frac{7}{20}$ to a simple fraction.

(68) £5. 12*s.* $9\frac{3}{4}$*d.* $\div 1\frac{7}{8}$.

(69) Find the value of $\dfrac{2\frac{3}{4}+5\frac{6}{7}+8\frac{9}{10}}{\frac{19}{28}\times 25\frac{4}{5}}$.

(70) A ship which was lost at sea was worth £5060., and was insured for 60 per cent. of the full value; $\frac{1}{10}$ belonged to A, $\frac{7}{12}$ to B, and the rest to C: what will they each receive from the insurance company?

(71) Reduce 18*s.* 4*d.* to the fraction of half-a-guinea.

(72) In an election $\frac{2}{3}$ of the total number of voters polled for A and $\frac{3}{10}$ for B; the remainder (360) abstained from voting: by how many votes was the successful candidate returned?

II. DECIMAL MEASURES.

i. Reduction.

155. To reduce a decimal of one denomination to a lower denomination, we proceed as in (**148**), as will be seen by the following examples :—

Ex. 1. Reduce ·1875 of £1. to shillings and pence.

$$\begin{array}{r} \cdot1875 \\ 20 \\ \hline 3\cdot75\cancel{0}\cancel{0} \\ 12 \\ \hline 9\cdot00 \end{array}$$

∴ ·1875 of £1. = 3*s.* 9*d.*

Ex. 2. Find the value of 5·00625 of £1.

$$\begin{array}{r} £ \\ 5\cdot00625 \\ 20 \\ \hline 10\cdot125\cancel{0}\cancel{0} \\ 12 \\ \hline 1\cdot5\cancel{0}\cancel{0} \\ 4 \\ \hline 2\cdot0 \end{array}$$

∴ 5·00625 of £1. = £5. 10*s.* 1½*d.*

Ex. 3. Find the value of ·35 of 2 qrs. 17 lbs.

2 qrs. 17 lbs. = 73 lbs.

∴ ·35 of 2 qrs. 17 lbs. = ·35 of 73 lbs.

$$\begin{array}{r} \cdot35 \\ 73 \\ \hline 105 \\ 245 \\ \hline 25\cdot55 \\ 16 \\ \hline 8\cdot8\cancel{0} \\ 16 \\ \hline 12\cdot8 \end{array}$$

∴ ·35 of 2 qrs. 17 lbs. = 25 lbs. 8 ozs. 12⅘ drs.

Ex. 4. Multiply 3 cwts. 1 qr. 21 lbs. by 4·106.

3 cwts. 1 qr. 21 lbs. = 385 lbs.

$$\begin{array}{r} 4{\cdot}1606 \\ \underline{385} \\ 20530 \\ 32848 \\ \underline{12318} \\ 1580{\cdot}810 \\ \underline{16} \\ 12{\cdot}96 \end{array} \qquad \begin{array}{l} \quad\ \text{lbs.} \quad \text{ozs.} \\ 28\left\{\begin{array}{l} 4\underline{)1580} \quad 12{\cdot}96 \\ 7\underline{)395}\text{—}0 \end{array}\right. \left.\begin{array}{l} \\ \end{array}\right\} 12 \text{ lbs.} \\ \quad\ \ 4\underline{)56}\text{—}3 \\ \quad\ \ \ 14\text{—}0\text{—}12\text{—}12{\cdot}96 \end{array}$$

∴ 3 cwts. 1 qr. 21 lbs. × 4·106 = 14 cwts. 0 qrs. 12 lbs. 12·96 ozs.

Ex. 5. Find the value of $\cdot 83\dot{4}2857\dot{1}$ of £8. 15*s*.

$$\begin{array}{r} \cdot 83428571 \\ \underline{175} \\ 417142857 \\ 5839999999 \\ \underline{8342857142} \\ 145{\cdot}99999999 \end{array}$$

Ans. = 146*s*., or £7. 6*s*.

Exercise 94.

Find the value of—

(1) ·775 of £1., ·824 of a shilling, ·305 of a crown, 25·875 of a shilling, £3·646875.

(2) ·009875 of £1., ·09013 of a shilling, ·0983 of a guinea, ·00891 of a crown, ·63904 of £5.

(3) 7·2875 of £2. 7*s*. 4*d*., 1·396 of 7*s*. 9¾*d*., ·4158 of £4. 6*s*. 8*d*., ·2011 of £9. 11*s*. 6*d*.

(4) ·80975 of 1 lb. avoir., ·09875 of 1 lb. troy, ·9375 of 2 cwts. 3 qrs., 2·125 of 5 tons 2 cwts.

(5) ·00375 of 2 tons 5 cwts., 7·569 of 10 tons 3 cwts., 3·245 of 3 lbs. 4 ozs. troy.

(6) ·175 of a ton + ·195 of a cwt. + ·145 of a qr., $\cdot 4\dot{6}\dot{9}$ of 1 lb. avoir., 3 qrs. 25 lbs. 15 drs. ÷ 9·21.

(7) $2{\cdot}2758680\dot{5}$ of 1 lb. troy, $\cdot 2565814\dot{3}\dot{9}$ of 1 mile, $\cdot 7053\dot{5}7142\dot{8}$ of 1 guinea.

(8) $\cdot 14\dot{6}\dot{9}$ of 1 furlong, ·675 of 1 acre, ·00390625 of 1 mile, $12{\cdot}\dot{3}$ of 1 dwt. 12 grs.

(9) $\cdot 982666015625$ lb. troy, in avoirdupois, $\cdot 2628897\dot{1}4452\dot{2}$ of $2\frac{3}{5}$ miles.

(10) $\cdot 5\dot{9}0476\dot{1}$ of $4\frac{1}{2}$ guineas, $\cdot 0931\dot{8} \div \cdot 56\dot{8}\dot{1}$ of $2\frac{1}{12}$ of $2\cdot 5$ days, $4\cdot\dot{9}\dot{6}$ of 4 dys. 3 hrs.

(11) $2\cdot\dot{8}\dot{1}$ of $365\frac{1}{4}$ days, $\cdot 34375$ of a lunar month, $2\cdot 5384375$ of a day.

(12) $\cdot 875$ of $\frac{1}{2}$ league, $\cdot 777587890625$ of 2 cwts., $\cdot 875$ of 1 acre, $\cdot 00390625$ of 1 mile.

156. Next, to reduce a compound quantity to a decimal of a higher denomination, we proceed thus :—

Ex. 1. Reduce 7*s* $3\frac{3}{4}$*d.* to the decimal of a pound.

4)3—farthings
12)3·75—pence
2,0)7·3125—shillings
·365625—pounds

∴ 7*s.* $3\frac{3}{4}$*d.* = ·365625 of a pound.

Ex. 2. Reduce 2 lbs. 3 ozs. 6 dwts. 5 grs. to the decimal of a pound troy.

$$24\left\{\begin{array}{l} 4)\underline{5}\text{—grains} \\ 6)\underline{1\cdot 25} \end{array}\right.$$
$2{,}0)\underline{6\cdot 208\dot{3}}$—pennyweights
$12)\underline{3\cdot 31041\dot{6}}$—ounces
$\cdot 2758680\dot{5}$—pound

∴ 2 lbs. 3 ozs. 6 dwts. 5 grs. = $2\cdot 2758680\dot{5}$ of a pound troy.

Ex. 3. Reduce 2 fur. 11 yds. 1 ft. 9 in. to the decimal of a mile.

$12)\underline{9}$—inches
$3)\underline{1\cdot 75}$—feet
$$220\left\{\begin{array}{l} 11{,}0)\underline{1{,}1\cdot 58\dot{3}}\text{—yards} \\ 2)\ \underline{\cdot 1053\dot{6}\dot{3}} \end{array}\right.$$
$8)\underline{2\cdot 052651\dot{5}\dot{1}}$—furlongs
$\cdot 2565814\dot{3}\dot{9}$—mile

∴ 2 fur. 11 yds. 1 ft. 9 in. = $\cdot 2565814\dot{3}\dot{9}$ of a mile.

Ex. 4. Reduce 14*s*. 9¾*d*. to the decimal of a guinea.

$$\begin{array}{rl} & 4)\underline{3\cdot\ }\text{—farthings} \\ & 12)\underline{9\cdot75}\text{—pence} \\ 21\left\{\begin{array}{l} 3)\underline{14\cdot8125}\text{—shillings} \\ \ \ 7)\underline{4\cdot9375\qquad} \end{array}\right. & \\ & \qquad\cdot7053\dot{5}7142\dot{8}\text{—guineas} \end{array}$$

∴ 14*s*. 9¾*d*. = $\cdot7053\dot{5}7142\dot{8}$ guineas.

Ex. 5. Express 5 tons 6 cwts. 3 qrs. 23 lbs. 6 ozs. 8·448 drs. in tons only.

$$\begin{array}{l} 16\left\{\begin{array}{l} 8)\underline{8\cdot448}\text{—drams} \\ 2)\underline{1\cdot056} \end{array}\right. \\ 16\left\{\begin{array}{l} 8)\underline{6\cdot528}\text{—ounces} \\ 2)\underline{\ \cdot816\ } \end{array}\right. \\ 28\left\{\begin{array}{l} 4)\underline{23\cdot408}\text{—pounds} \\ \ \ 7)\underline{5\cdot852} \end{array}\right. \\ \qquad 4)\underline{3\cdot836}\text{—quarters} \\ \quad 2,0)\underline{6\cdot959}\text{—hundredweights} \\ \qquad\quad \cdot34795\text{—tons} \end{array}$$

∴ 5 tons 6 cwts. 3 qrs. 23 lbs. 6 ozs. 8·448 drs. = 5·34795 tons.

Exercise 95.

In the following examples reduce each expression to the highest denomination, and then divide.

(1) 447 tons 3 cwts. 2 qrs. 15 lbs. 13 ozs. 15 drs. ÷ 49 tons 13 cwts. 2 qrs. 26 lbs. 10 ozs. 7 drs.

(2) 404 lbs. 6 ozs. 8 dwts. ÷ 12 lbs. 7 ozs. 14 dwts.

(3) 606 fur. 18 po. 3 yds. ÷ 16 fur. 33 po. 4 yds. 2 ft.

(4) 3214 ac. 2 rds. 6 per. 28¼ sq. yds. ÷ 44 ac. 2 rds. 23 po. 13 yds.

(5) 44 hhds. 13 gals. 3 qts. 1 pt. ÷ 3537 hhds. 39 gals. (wine).

(6) 82 cub. yds. 12 ft. 100 in. ÷ 6678 cub. yds. 4 ft. 1188 in.

(7) 956 ells 3 qrs. 1 nl. ¾ in. ÷ 9 ells 2 qrs. 2 nls. 3 in.

(8) 33078 cub. yds. 11 ft. 864 in. ÷ 300 cub. yds. 19 ft. 432 in.

(9) £13654. 0*s.* 11*d.* ÷ £401. 11*s.* 9½*d.*

(10) 844 cwts. 61 lbs. 11 ozs. 8 drs. ÷ 13 cwts. 69 lbs. 10 ozs. 4 drs.

(11) 14812 m. 35 po. 4½ yds. ÷ 180 m. 5 fur. 3 po. 2 yds.

(12) 1553 lbs. 10 ozs. 15 dwts. 12 grs. ÷ 12 lbs. 2 ozs. 16 dwts. 12 grs.

(13) 2244 ozs. 3 drs. ÷ 10 ozs. 5 drs. 1 scr. 10 grs.

(14) 504 yrs. 144 dys. 5 hrs. 34 min. 22 sec. ÷ 111 dys. 7 hrs. 24 min. 13 sec.

157. To find what decimal one concrete quantity is of another concrete quantity, we proceed as in (**154**), and express the result as a decimal.

N.B.—We may reduce to the highest or the lowest denomination mentioned.

Ex. 1. Express 2 tons 17 cwts. 1 qr. 27 lbs. 7 ozs. 4 drs. as decimal of 2 tons 18 cwts. 3 qrs. 14 lbs.

$$\frac{\begin{matrix}\text{tons} & \text{cwts.} & \text{qrs.} & \text{lbs.} & \text{ozs.} & \text{drs.}\\ 2 & 17 & 1 & 27 & 7 & 4\end{matrix}}{\begin{matrix}2 & 18 & 3 & 14 & 0 & 0\end{matrix}} = \frac{1648500}{3774 \times 16 \times 16} = \cdot 9765625.$$

Ex. 2. Reduce 3 lbs. 6 ozs. troy to the decimal of 10 pounds avoirdupois.

$$\frac{3 \text{ lbs. } 6 \text{ ozs. troy}}{10 \text{ lbs. avoirdupois}} = \frac{3\cdot 5 \times 5760}{10 \times 7000} = \frac{\overset{5}{\cancel{35}} \times \overset{288}{\cancel{576}}}{\underset{2}{\cancel{10}} \times \underset{1000}{\cancel{7000}}} = \cdot 288.$$

Ex. 3. Reduce $9\frac{6}{11}$ of $\frac{11}{15}$ pecks to the decimal of $3\frac{1}{3}$ quarters.

Since 32 pecks = 1 quarter, we arrange the work thus :—

$$\frac{9\frac{6}{11} \text{ of } \frac{11}{15} \text{ pecks}}{3\frac{1}{3} \text{ quarters}} = \frac{\overset{7}{\cancel{105}}}{\cancel{11}} \times \frac{\cancel{11}}{\cancel{15}} \times \frac{3}{10} \times \frac{1}{32} = \cdot 065625.$$

Ex. 4. If 12*s.* 4*d.* be taken as the unit, how is 6*s.* 8·2*d.* represented?

$$\frac{6s.\ 8\cdot 2d.}{12s.\ 4d.} = \frac{80\cdot 2}{148} = \frac{40\cdot 1}{74} = \cdot 54\dot{1}8\dot{9}.$$

Ex. 5. Reduce $3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}}$ to a fraction, and find the number of square inches it expresses when the unit is 1 sq. ft. 3 in.

$$3\cdot\dot{3}\times\frac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}} = 3\tfrac{3}{9}\times\frac{4\frac{4}{9}}{\frac{735}{999}} = \frac{\overset{10}{\cancel{30}}}{\underset{3}{\cancel{9}}}\times\frac{\overset{8}{\cancel{40}}}{\cancel{9}}\times\frac{\overset{37}{\cancel{\overset{111}{\cancel{999}}}}}{\underset{49}{\cancel{\underset{245}{\cancel{735}}}}} = \frac{2960}{147}$$

∴ if 1 sq. ft. 3 in. or 147 in. be the unit,

$3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}}$ will be represented by $\frac{2960}{147}\times 147 = 2960$ in.

Exercise 96.

(1) Reduce 12*s*. 9*d*. to the decimal of 15*s*. 6*d*.; $\frac{4}{7}$ of a guinea to the decimal of 23*s*. 4*d*.

(2) What decimal is $\frac{7}{2}$ of $\frac{5}{3}$ of $\frac{1}{5}$ of £15. of 2*s*. 8*d*.? Express $\frac{5}{8}$ of 1 lb. troy as decimal of 1 lb. avoirdupois?

(3) Reduce 8 cwts. 2 qrs. 2 lbs. to decimal of 18 cwts. 3 qrs. 5 lbs.; 14 hrs. 15 min. to the decimal of $3\frac{1}{2}$ days.

(4) 9 lbs. avoirdupois as decimal of 5 lbs. troy; $\frac{3}{8}$ of a rood as decimal of $\frac{1}{2}$ of an acre.

(5) ·26*d*. to decimal of £1.; $2\cdot\dot{9}$ halfcrowns to decimal of 1*s*. 4*d*.; $\frac{2}{3}$ of 1 guinea to decimal of £1. 3*s*. 4*d*.

(6) $\frac{3}{7}$ of $2\frac{4}{5}$ of 5 ac. 3 rds. as the decimal of $\frac{3}{5}$ of 14 ac. 2 rds.; 18*s*. $4\frac{1}{2}$*d*. as decimal of £1000.

(7) Taking 6 ac. 2 rds. 6·69 po. as the unit, what is the value of $\dfrac{1}{\cdot 235}$?

(8) If 5*s*. $1\frac{1}{3}$*d*. is represented by $\cdot 2\dot{5}$, what is the unit?

(9) Half-a-guinea being the unit, find the measure of the difference between $\frac{4}{5}$ of 5*s*. 9*d*. and $\frac{5}{7}$ of 6*s*. 4*d*.

(10) The measures of 3*s*. $11\frac{1}{4}$*d*., £1. 2*s*. $3\frac{1}{4}$*d*., and £1. 4*s*. $0\frac{3}{4}$*d*. are ·1, ·0625, $\cdot 4\dot{7}2\dot{9}$: what are the units?

(11) Reduce $5\frac{1}{4}$ guineas to the decimal of £50.; 11 dwts. to decimal of 1 lb. troy.

(12) 10 drs. to decimal of 1 lb. avoirdupois; 3 qrs. 3 lbs. 1 oz. $12\frac{4}{9}$ drs. to the decimal of 1 cwt.

(13) If 1 cwt. be the unit, express the measure of 5 lbs. 10 ozs. 3 dwts. 15 grs. troy.

(14) What decimal of 1¾ ton is 1 cwt. 3 qrs. 14 lbs.; and 3 wks. 4 dys. 5 hrs. 6 min. 7 sec. of 4 wks.?

(15) ·175 of a ton + ·195 of a cwt. + ·145 qr. + ·15 lb.; take first 1 lb., second 1 ton, as units.

(16) $1\cdot\dot{2}$ of 3 ac. 2 rds. + ·135 of a pole − $21\cdot\dot{9}$ yds. + $310\cdot\dot{1}$ ft.

(17) $\frac{2}{9}$ of 4 ozs. 19 dwts. to the decimal of 2⅔ of 16 dwts. 21 grs.

(18) $\cdot 12\dot{3}6$ of 2 ac. as decimal of 3¼ rds.; $\cdot 14\dot{9}78902953586\dot{4}$ of 39½ yds. of cloth.

(19) £·0375. + ·75*d.* + ·625*s.* + 3*s.* 3·5*d.* as decimal of 7*s.* 6*d.*

(20) $\cdot 35\dot{7}6923\dot{0}$ of £13. + $\cdot\dot{6}1538\dot{4}$ of 13*s.* + $6\dot{9}0476\dot{1}$ of 3*s.* 6*d.* − $\cdot\dot{7}6923\dot{0}$ of £2. 12*s.*

III. DECIMALISATION OF MONEY.

158. Since 1*s.* = £·05.; 6*d.* = £·025.; ∴ 2*s.* = £·1.; 2*s.* 6*d.* = £·05. + £·025. = £·075.; 3*s.* 6*d.* = £·15. + £·025. = £·175.; 19*s.* 6*d.* = £·95. + £·025. = £·975.

Hence the figure in the 1st decimal place expresses the number of florins; for an odd shilling, add 5 in the 2nd place; for 6*d.* over, add 25 in the 2nd and 3rd places; and for 1*s.* 6*d.* over, add 75 in the 2nd and 3rd places.

Exercise 97.

Make a table of every sixpence from 6*d.* to 19*s.* 6*d.*

159. ¼*d.* = $\frac{1}{24}$ of 6*d.* = $\frac{1}{24}$ of £·025. = £·001$\frac{1}{24}$. = £·001. + $\frac{1}{12}$ of ½ of £·001.

∴ ½*d.* = £·002. + $\frac{1}{12}$ of ½ of £·002. = £$\cdot 00208\dot{3}$

∴ ¾*d.* = £·003. + $\frac{1}{12}$ of ½ of £·003. = £·003125

∴ 1¾*d.* = £·007. + $\frac{1}{12}$ of ½ of £·007. = £$\cdot 007291\dot{6}$

∴ 5¾*d.* = £·023. + $\frac{1}{12}$ of ½ of £·023. = £$\cdot 023958\dot{3}$

Exercise 98.

Make a table of every farthing from ¼*d.* to 6*d.*

160. We can now write down any sum of money from ¼*d.* to £1. at sight. Thus, decimalise 17*s.* 10½*d.*

17*s.* 6*d.* = £·875
4½*d.* = £·018 + $\frac{1}{12}$ of £·009. = £·01875
∴ 17*s.* 10½*d.* = £·89375.

Exercise 99.

Reduce to the decimal of £1. :—

(1) 8*s.* 6*d.*; 4*s.* 8¾*d.*; 9*s.* 5¼*d.*; 5*s.* 10¾*d.*; £2. 17*s.* 4½*d.*; £6. 6*s.* 4*d.*

(2) £2. 12*s.* 4¼*d.*; £3. 17*s.* 4½*d.*; £16. 16*s.* 5¼*d.*; £4. 13*s.* 3¾*d.*; £3. 14*s.* 2¾*d.*

(3) £2. 7*s.* 6½*d.*; £7. 15*s.* 8¾*d.*; £9. 2*s.* 10*d.*; £11. 1*s.* 8¾*d.*; £16. 3*s.* 7½*d.*

(4) £3. 16*s.* 8¾*d.*; £1. 17*s.* 2½*d.*; £2. 2*s.* 2½*d.*; £3. 3*s.* 2¼*d.*; £2. 13*s.* 9½*d.*

(5) £10. 7*s.* 9¾*d.*; £11. 16*s.* 7¼*d.*; £12. 11*s.* 8½*d.*; £1. 15*s.* 9¾*d.*; £16. 16*s.* 6¾*d.*

161. To find the value of any decimal of £1. :—

(1) For the shillings double the 1st decimal place, add 1*s.* if the 2nd place be 5 or greater than 5.
(2) Remove these figures, then the remaining figures in the 2nd and 3rd places will be the number of farthings; if they exceed 24 reject 1. The remaining figures being less than a farthing may be omitted. Thus :—

£·784375. = £·75. + £·034. = 15*s.* 8¼*d.*
£·590625. = £·55. + £·040. = 11*s.* 9¾*d.*
£·92708$\dot{3}$. = £·92. + £·027. = 18*s.* 6½*d.*
£·694791$\dot{6}$. = £·65. + £·044. = 13*s.* 10¾*d.*

162. In decimal coinage, the figure in the 1st place of decimals represents florins, in the 2nd cents., in the 3rd mils. So that to convert ordinary money into decimal money is simply the same as Exercise 98; and to convert decimal money into ordinary money employ (**161**).

Ex. 1. Find the cost of 2897 articles at £2. 12*s.* 9*d.* each.

£2. 12*s.* 9*d.* = £2·6375.

2·637500 3 places wanted
7982

5275000
2110000
237375
18462

7640·837 ∴ *Ans.* = £7640. 16*s.* 9*d.*

Ex. 2. Find the cost of 362¾ articles at 6*s.* 7½*d.* each.

362¾ = 362·75 ; 6*s.* 7½*d.* = ·33125.

36275 3 places
52133

108825
10882
363
72
18

120·160 *Ans.* = £120. 3*s.* 2½*d.*

Ex. 3. Find the cost of $27\frac{299}{550}$ articles at £8. 6*s.* $11{\cdot}4\dot{6}$*d.* each.

$27\frac{299}{550} = 27{\cdot}54\dot{3}\dot{6}$; £8. 6*s.* $11{\cdot}4\dot{6}$*d.* = $8{\cdot}34\dot{7}$.

27·54363636 5 places
777777438

220349090
8263091
1101745
192805
19280
1928
193
19
2

229·928:5 *Ans.* £229. 18*s.* 6¾*d.*

Ex. 4. If A owes £248. 14*s*. $5\frac{1}{3}$*d*., and has only £66. 0*s*. $4\frac{8}{9}$*d*., how much will he pay in the £.?

£66. 0*s*. $4\frac{8}{9}$*d*. = £$66{\cdot}02\dot{0}3\dot{7}$. ; £248. 14*s*. $5\frac{1}{3}$*d*. = £$248{\cdot}7\dot{2}$.

```
248722)660203|7 (·26543
        162759|
         13526|
          1090|
            95|
            21|      Ans. 5s. 3⅓d.
```

Exercise 100.

Find the value of—

(1) 969 articles at 19*s*. $11\frac{1}{2}$*d*. each.

(2) 3546 pieces of cloth at £1. 6*s*. $7\frac{3}{4}$*d*. each.

(3) 6723 articles at £1. 8*s*. $8\frac{1}{4}$*d*. each.

(4) 1389 tons of coal at £1. 13*s*. 8*d*. per ton.

(5) 3107 sheep at £1. 14*s*. $7\frac{1}{2}$*d*. each.

(6) $217\frac{1}{4}$ yards at £2. 17*s*. $7\frac{1}{2}$*d*. per yard.

(7) $8172\frac{1}{2}$ yards at £3. 13*s*. 4*d*. per yard.

(8) $4562\frac{1}{4}$ articles at £3. 15*s*. $9\frac{1}{2}$*d*. each.

(9) $7394\frac{3}{4}$ articles at £12. 8*s*. $8\frac{1}{4}$*d*. each.

(10) A bankrupt owes £25962. 10*s*.: what must his assets be worth in order that he may pay 7*s*. $11\frac{1}{2}$*d*. in the pound?

(11) Find the dividend on £3854. 10*s*. at 8*s*. $2\frac{1}{2}$*d*. in the £.

(12) The dividend on £2045. 15*s*. 9*d*. is £609. 9*s*. $5\frac{129}{180}$*d*.: what is that in the pound?

(13) The dividend on £2734. 16*s*. 8*d*. is £1281. 19*s*. $0\frac{3}{4}$*d*.: what is that in the pound?

(14) If a person's estate is worth £1384. 16*s*. per annum, and the rent charges upon it amount to 14*s*. $9\frac{1}{2}$*d*. in the £., what is his net annual income?

(15) A bankrupt owes A £515. 12*s*. 6*d*.; B £407.: C £293. 6*s*. 8*d*.: his estate is worth £911. 19*s*. $4\frac{1}{2}$*d*.: how much can he pay in the £.; and what will A, B, and C receive?

IV. DECIMALISATION OF WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

163. (*a*) If £1. represent 1 ton, 1*s*. will represent 1 cwt. Thus :—

Ex. 1. Find the cost of 87 tons 15 cwts. at £5. 2*s*. 6*d*. per ton.

87 tons 15 cwts. or £87. 15*s*. = 87·75 tons ; £5·125. = £5. 2*s*. 6*d*.

```
 87750
  5215
──────
438750
  8775
  1755
   438
──────
449·718
```

449·718 *Ans.* £449. 14*s*. 4½*d*.

(*b*) 3*d*. will represent 1 qr.

Ex. 2. Find the cost of 57 tons 13 cwts. 3 qrs. at £74. 8*s*. 4½*d*.

57 tons 13 cwts. 3 qrs. (£57. 13*s*. 9*d*.) = 57·6875 tons.

£74. 8*s*. 4½*d*. = £74·41875.

```
 7441875
  578675
────────
 3720937
  520931
   44651
    5954
     521
      37
────────
4293·031
```

4293·031 *Ans.* £4293. 0*s*. 7¼*d*.

(*c*) and we may call every 7 lbs. ¾*d*.

∴ 1 lb. = 1/7 of ·003125 tons
2 lbs. = 1/7 of ·00625 tons
3 „ = 1/7 of ·009375 tons
4 „ = 1/7 of ·0125 tons
5 „ = 1/7 of ·015625 tons
6 „ = 1/7 of ·01975 tons

Ex. 3. Find the cost of 8 tons 3 cwts. 0 qrs. 27 lbs. at £5. 6*s*. 8*d*. per ton.

8 tons 3 cwt. 0 qrs. 21 lbs. (£8. 3*s*. 2¼*d*.) + 6 lbs. = 8·1621 tons.

£5. 6*s*. 8*d*. = 5·3333.

81621
33335

408109
24486
2448
245
24
2

43·5314 *Ans.* £43. 10*s*. 7½*d*.

Ex. 4. Find the cost of 87 tons 13 cwts. 3 qrs. 24 lbs. 13⅝ ozs. at £43. 8*s*. 4*d*.

Perhaps it would be as short here to decimalise the weight in the usual way.

£43. 8*s*. 4*d*. = £$43\cdot41\dot{6}$.

16)13·625
28 { 4)24·85156
7)6·21289
4)3·88755
2,0)13·97188
87·69859

87·69859
6661434

35079436
2630957
350795
87689
5261
526
52
5

3815·4721 *Ans.* £3815. 9*s*. 5¼*d*.

Exercise 101.

Find the cost of—

(1) 56 tons 15 cwts. 2 qrs. 21 lbs. at £24. 6*s*. 8*d*. per ton.

(2) 9 tons 4 cwts. 3 qrs. 21 lbs. at £14. 15*s*. 9*d*. per ton.

(3) 56 tons 4 cwts. 2 qrs. at £58. 7*s*. 6*d*. per ton.

(4) 5 tons 17 cwts. 2 qrs. at £3. 16*s*. 9¾*d*. per ton.

(5) 10 tons 16 cwts. 3 qrs. at £2. 14*s.* 10½*d.* per ton.
(6) 14 tons 15 cwts. 1 qr. at £8. 13*s.* 7¾*d.* per ton.
(7) 21 tons 6 cwts. 3 qrs. at £3. 12*s.* 10¼*d.* per ton.
(8) 35 tons 19 cwts. 3 qrs. at £9. 6*s.* 8*d.* per ton.
(9) 17 tons 12 cwts. 2 qrs. at £9. 6*s.* 8*d.* per ton.
(10) 26 tons 15 cwts. 1 qr. at £9. 6*s.* 8*d.* per ton.
(11) 5 tons 5 cwts. 2 qrs. 14 lbs. at £4. 11*s.* 2*d.* per ton.
(12) 17 cwts. 10½ lbs. at £12. 5*s.* 8*d.* per ton.
(13) 18 cwts. 17½ lbs. at £12. 5*s.* 8*d.* per ton.
(14) 15 cwts. 20 lbs. at £12. 5*s.* 8*d.* per ton.

(*d*) If the price be given per cwt. multiply the price by 20 and proceed as above.

Ex. 5. Find the cost of 3 tons 13 cwts. 2 qrs. 19 lbs. at £1 6*s.* 8*d.* per cwt.

£1. 6*s.* 8*d.* per cwt. = £26. 13*s.* 4*d.* per ton.

3 tons 13 cwts. 2 qrs. 19 lbs. = 3·68346, &c.

```
368346
66662
------
73669|
22100|
 2210|
  221|
   22|
    2|
------
98·226|     Ans. = £98. 4s. 6d.
```

Exercise 102.

Find the cost of—

(1) 2 cwts. 2 qrs. 17 lbs. at £1. 10*s.* 4*d.* per cwt.
(2) 42 cwts. 16 lbs. at £5. 16*s.* 8*d.* per cwt.
(3) 23 cwts. 3 qrs. 12 lbs. at £1. 7*s.* 6*d.* per cwt.
(4) 75 cwts. 3 qrs. 21 lbs. at £5. 15*s.* 6*d.* per cwt.
(5) 179 cwts. 2 qrs. 12 lbs. at £2. 3*s.* 6½*d.* per cwt.
(6) 45 cwts. 3 qrs. 14 lbs. at £2. 9*s.* 6*d.* per cwt.
(7) 353 cwts. 2 qrs. 7 lbs. at 17*s.* 4*d.* per cwt.

HT.

y £1., 1 lb. = £12.; 1 dwt.

wts. 23 grs. by 74.
d by £103·99791$\dot{6}$.

£7695. 16*s.* 11*d.*
3 ozs. 16 dwts. 22 grs.

s. 13 dwts. 16 grs. at £1. 10*s.* 9½*d.*

sented by £8. 13*s.* 8*d.* = 8·68$\dot{3}$.
½*d.* = 1·53958$\dot{3}$.

Ans. = £13. 7*s.* 4½*d.*

mes is 12 lbs. 2 ozs. 16 dwts. 12 grs.
ozs. 15 dwts. 12 grs.?
12 grs. represented by £146. 16*s.* 6*d.* =

dwts. 12 grs. represented by £18646.

5 ÷ 146·825 = 127. *Ans.*

Exercise 104.

dwts. at £7. 11*s.* 4*d.* per pound.

(8) 4 cwts. 3 qrs. 22½ lbs. at £1. 9*s.* 2*d.* per cwt.

(9) 3796 cwts. 3 qrs. 24 lbs. at £1. 13*s.* 3*d.* per cwt.

(10) 75 cwts. 1 qr. 16 lbs. at £2. 4*s.* 11*d.* per cwt.

(11) 19 cwts. 1 qr. 13 lbs. at £2. 11*s.* 4*d.* per cwt.

(12) 38 cwts. 1 qr. 16 lbs. at £2. 12*s.* 6*d.* per cwt.

(13) 6 cwts. 1 qr. 11 lbs. at £2. 17*s.* 9*d.* per cwt.

(14) 3 tons 18 cwts. 3 qrs. 12 lbs. at £2. 17*s.* 9*d.* per cwt.

The following examples in division may be worked the same way :—

Exercise 103.

(1) 84 tons 9 cwts. 3 qrs. 2 lbs. 13 ozs. 2 drs. ÷ 1 ton 2 cwts. 3 qrs. 9 lbs. 7 ozs.

(2) 3934 tons 18 cwts. 10 lbs. 8 ozs. ÷ 70 tons 5 cwts. 1 qr. 8 lbs. 3 ozs.

(3) 5480 tons 15 cwts. 22 lbs. 10 ozs. ÷ 70 tons 5 cwts. 1 qr 8 lbs. 3 ozs.

(4) 10528 tons 15 cwts. 3 qrs. 16 lbs. ÷ 87 tons 14 cwts. 3 qrs. 5 lbs. 8 ozs.

(5) 1320 tons 8 cwts. 3 qrs. 2 lbs. 8 ozs. ÷ 15 tons 3 cwts. 2 qrs. 5 lbs. 8 ozs.

(6) 1400 tons 11 cwts. 1 qr. 14 lbs. ÷ 14 tons 5 cwts. 3 qrs. 9 lbs.

(7) 11468 tons 13 cwts. 1 qr. 3 lbs. 10 ozs. ÷ 154 tons 19 cwts. 2 qrs. 15 lbs. 9 ozs.

(8) 1649 tons 8 cwts. 2 qrs. 3 lbs. ÷ 20 tons 2 cwts. 1 qr. 5 lbs. 8 ozs.

(9) 15284 tons 4 cwts. 2 qrs. 17 lbs. 14 ozs. ÷ 43 tons 15 cwts. 99 lbs. 6 ozs.

(10) 42 tons 4 cwts. 61 lbs. 11 ozs. 8 drs. ÷ 13 cwts. 69 lbs. 10 ozs. 4 drs.

(11) 447 tons 3 cwts. 2 qrs. 15 lbs. 13 ozs. 15 drs. ÷ 49 tons 13 cwts. 2 qrs. 26 lbs. 10 ozs. 7 drs.

(12) 3223 tons 8 cwts. 24 lbs. 3 ozs. 2 drs. ÷ 24 tons 15 cwts. 3 qrs. 17 lbs. 13 ozs. 9 drs.

TROY WEIGHT.

164. If 1 oz. be represented by £1., 1 lb. = £12.; 1 dwt. = 1*s.*; 1 gr. = ½*d.*

Ex. 1. Multiply 8 lbs. 7 ozs. 19 dwts. 23 grs. by 74.

The weight is represented by £103·99791$\dot{6}$.

```
 103·99791̇6
         74
 ──────────
 415·99166̇6
7279·85416̇6
 ──────────
7695·8458̇3        £7695. 16s. 11d.
```

or *Answer* is 641 lbs. 3 ozs. 16 dwts. 22 grs.

Ex. 2. Find the cost of 8 ozs. 13 dwts. 16 grs. at £1. 10*s.* 9½*d.* per ounce.

8 ozs. 13 dwts. 16 grs. represented by £8. 13*s.* 8*d.* = 8·68$\dot{3}$.

£1. 10*s.* 9½*d.* = 1·53958$\dot{3}$.

```
 15396
 33868
 ──────
123168
  9237
  1231
    46
     5
 ──────
13·3687    Ans. = £13. 7s. 4½d.
```

Ex. 3. How many times is 12 lbs. 2 ozs. 16 dwts. 12 grs. contained in 1553 lbs. 10 ozs. 15 dwts. 12 grs.?

12 lbs. 2 ozs. 16 dwts. 12 grs. represented by £146. 16*s.* 6*d.* = 146·825.

1553 lbs. 10 ozs. 15 dwts. 12 grs. represented by £18646. 15*s.* 6*d.* = 18646·775.

18646·775 ÷ 146·825 = 127. *Ans.*

Exercise 104.

Find the cost of—

(1) 13 lbs. 7 ozs. 15 dwts. at £7. 11*s.* 4*d.* per pound.

(2) 15 dwts. 12 grs. at £4. 4s. per ounce.

(3) 2 lbs. 3 ozs. 16 dwts. at £3. 17s. 6d. per ounce.

(4) 10 lbs. 6 ozs. 15 dwts. at 4s. 3¾d. per pound.

(5) 15 lbs. 9 ozs. 17 dwts. at £2. 5s. 8¼d. per pound.

(6) 29 lbs. 3 ozs. 18 dwts. at £3. 15s. 5½d. per pound.

(7) 10 ozs. 15 dwts. 20 grs. at £1. 12s. 4d. per ounce.

(8) 11 ozs. 16 dwts. 4 grs. at £1. 12s. 3½d. per ounce.

(9) 11 ozs. 17 dwts. 23 grs. at 12s. 0½d. per ounce.

(10) 7 ozs. 18 dwts. 21 grs. at £3. 19s. 3¾d. per ounce.

Exercise 105.

The dividends are—

(1) 641 lbs. 3 ozs. 16 dwts. 22 grs.
(2) 701 lbs. 11 ozs. 16 dwts. 15 grs.
(3) 277 lbs. 3 ozs. 18 dwts. 16 grs.
(4) 641 lbs. 3 ozs. 16 dwts. 22 grs.
(5) 849 lbs. 3 ozs. 15 dwts. 22 grs.
(6) 1039 lbs. 11 ozs. 15 dwts.

The divisor is 8 lbs. 7 ozs. 19 dwts. 23 grs. What are the quotients?

(7) 141 lbs. 7 ozs. 19 dwts. ÷ 6 lbs. 8 ozs. 19 dwts.

(8) 148 lbs. 9 ozs. 5 dwts. 21 grs. ÷ 3 lbs. 7 ozs. 10 dwts. 21 grs.

(9) 401 ozs. 7 dwts. 11 grs. ÷ 10 ozs. 16 dwts. 23 grs.

MEASURES OF CAPACITY.

165. (*a*) If 1 gallon is represented by £1., 1 qt. = 5s.; 1 pt. = 2s. 6d.; 1 gill = 7½d.

Ex. 1. Find the cost of 15 gals. 3 qts. 1 pt. 3 gills at £4. 5s. 6d. per gallon.

15 gals. 3 qts. 1 pt. 3 gills is represented by £15. 19s. 4½d. = £15·96875, and £4. 5s. 6d. = 4·275.

15·96875
5724

638750
31937
11178
798

68·2663 ∴ *Ans.* = £68. 5*s.* 3¾*d.*

(*b*) Find the cost of 6 qrs. 3 bus. 1¾ pk. at £3. 18*s.* 8*d.* per quarter.

Here, if 1 qr. = £1., 1 bus. = 2*s.* 6*d.*; 1 pk. = 7½*d.*

∴ 6 qrs. 3 bus. 1¾ pk. = 6·4296; £3. 18*s.* 8*d.* = 3·9333.

64296
33393

192889
57866
1928
192
19

25·2894 *Ans.* = £25. 5*s.* 9½*d.*

Exercise 106.

Find the cost of—

(1) 15 gals. 3 qts. 1 pt. 2 gills at £5. 16*s.* 7¾*d.* per gallon.
(2) 17 gals. 2 qts. 1 pt. 3 gills at £6. 17*s.* 8½*d.* per gallon.
(3) 18 gals. 3 qts. 1 pt. 1 gill at £7. 18*s.* 9¾*d.* per gallon.
(4) 19 gals. 0 qts. 0 pts. 1 gill at £8. 15*s.* 10¼*d.* per gallon.
(5) 20 gals. 1 qt. 1 pt. 1 gill at £9. 16*s.* 11¾*d.* per gallon.
(6) 43 qrs. 5 bus. 3 pks. at £3. 15*s.* 6*d.* per quarter.
(7) 57 qrs. 6 bus. 2 pks. at £6. 13*s.* 11*d.* per quarter.
(8) 63 qrs. 7 bus. 1 pk. at £11. 11*s.* 11¼*d.* per quarter.
(9) 59 qrs. 3 bus. 1 pk. at £12. 12*s.* 11¾*d.* per quarter.
(10) 65 qrs. 5 bus. 2 pks. at £13. 19*s.* 9½*d.* per quarter.

MEASURES OF LENGTH.

166. (*a*) Express the length given as decimal of a yard, or a foot, according as the price is given as per yard, or per foot.

Ex. 1. Find the cost of 9 yds. 2 ft. 10 in. at 5*s.* 7½*d.* per yard.

9 yds. 2 ft. 10 in. = $9{\cdot}9\dot{4}$; 5*s.* 7½*d.* = £·28125.

$$\begin{array}{r} 9{\cdot}9444 \\ 52182 \\ \hline 198888 \\ 79555 \\ 994 \\ 198 \\ 49 \\ \hline 2{\cdot}79684 \end{array} \qquad \textit{Ans.} = £2.\ 15s.\ 11\tfrac{1}{4}d.$$

(*b*) If 1 mile is represented by £1., 1 fur. = 2*s.* 6*d.*, 1 po. = ¾*d.* Decimalise the yards, &c., remembering 1 yd. = $\frac{1}{11}$ of 1½*d.*

Ex. 2. Find the cost of 5 m. 3 fur. 19 po. 1 ft. 6 in. at £15. 14*s.* 11¾*d.* per mile.

5 m. 3 fur. 19 po. 1 ft. 6 in. = 5·43465.
£15. 14*s.* 11¾*d.* = £15·7495.

$$\begin{array}{r} 543465 \\ 594751 \\ \hline 543465 \\ 271732 \\ 38042 \\ 2193 \\ 488 \\ 27 \\ \hline 85{\cdot}5947 \end{array} \qquad \textit{Ans.} = £85.\ 11s.\ 9\tfrac{3}{4}d.$$

Exercise 107.

Find the cost of—

(1) 15 yds. 2 ft. 11 in at £3. 16*s.* 8½*d.* per yard.
(2) 17 yds. 1 ft. 10 in. at £2. 19*s.* 3¾*d.* per foot.
(3) 19 yds. 2 ft. 5 in. at £4. 11*s.* 2½*d.* per yard.
(4) 21 yds. 1 ft. 9 in. at £6. 7*s.* 9¾*d.* per foot.
(5) 23 yds. 2 ft. 7 in. at £7. 8*s.* 10½*d.* per yard.
(6) 9 m. 4 fur. 16 po. at £8. 14*s.* 2½*d.* per mile.
(7) 14 m. 5 fur. 26 po. at £1. 7*s.* 9¾*d.* per mile.
(8) 18 m. 7 fur. 21 po. at £5. 16*s.* 6½*d.* per mile.

(9) 33 m. 3 fur. 25 po. at £6. 10*s.* 1½*d.* per mile.

(10) 84 m. 5 fur. 38 po. at £5. 19*s.* 11¾*d.* per mile.

SURFACE.

167. If £1. is the measure of 1 acre, 5*s.* is the measure of 1 rood; 1½*d.* of 1 pole.

Find the cost of 12 ac. 3 rds. 26 po. at £3 5*s.* per acre

12 ac. 3 rds. 26 po. is measured by £12·9125; £3. 15*s.* = £3·25

```
  12·9125
      325
  -------
   645625
  258250
 387375
 --------
 41·965625    Ans. = £41. 19s. 3¾d.
```

Exercise 108.

Find the cost of—

(1) 25 ac. 3 rds. 26 po. at £3. 6*s.* 7½*d.* per acre.

(2) 42 ac. 2 rds. 36 po. at £2. 11*s.* 6¼*d.* per acre.

(3) 55 ac. 3 rds. 33 po. at £3. 15*s.* 4½*d.* per acre.

(4) 73 ac. 1 rd. 16 po. at £0. 15*s.* 3¾*d.* per acre.

(5) 42 ac. 2 rds. 27 po. at £14. 10*s.* 0¼*d.* per acre.

(6) 9 ac. 1 rd. 38 po. at £10. 0*s.* 1½*d.* per acre.

(7) 25 ac. 3 rds. 25 po. at £11. 11*s.* 10¾*d.* per acre.

(8) 27 ac. 2 rds. 14 po. at £12. 12*s.* 9¼*d.* per acre.

Similar methods may be applied to the other tables.

Miscellaneous Examples F.

(1) After paying £·275 and ·625 of a shilling, I find I have spent ⅔ of the money I had in my purse: what have I left, and how much had I at first?

(2) Multiply $6\frac{1}{8} - 2\frac{1}{4}$ by 2·95, and divide the product by $\frac{1}{23}$ of 108.

(3) Value of £$3\frac{7}{8}$ + 2·0125 of £50. + $7{\cdot}\dot{3}\dot{5}$ of 11 guineas + ${\cdot}03\dot{6}\dot{7}$ of £247. 10*s.*

(4) Divide 21·97 by 1·69, by 169, and by 1690.

(5) What decimal of $1\frac{1}{3}$ of a foot is $\dfrac{2\frac{1}{3}+\dfrac{\frac{3}{8}}{1\frac{1}{2}}}{3\frac{1}{3}\text{ of }\frac{9}{20}}$ of a yard?

(6) Find the difference between £·7095. and $2\frac{1}{3}$ shillings.

(7) Subtract 7·356 from 13·04, and divide the remainder by ·0512.

(8) What is the difference between £(·005. × 7·25) and ($5\frac{3}{8} \div 6\frac{1}{7}$ of 35*s.*)?

(9) What is the square root of 841·48146889?

(10) Multiply 14·068 by 2·007.

(11) Bring ·035 of £5. to the decimal of £1.

(12) I receive $3{\cdot}\dot{3}$ guineas and $£{\cdot}41\dot{6}$, and pay from this amount ·875 of a crown and ·3125 of a guinea: what vulgar fraction of a £ have I still in hand?

(13) Take ·076 from the sum of 1·489 and ·587.

(14) If you paid £·425. for the carriage of $1\frac{3}{5}$ cwts. 50 miles, what would you pay for the carriage of 7·025 times the former weight for three-fourths the distance? (Answer in £ and the decimal of a £.)

(15) A machine throws off in one day 101·56 yards of cotton: how much will it throw off in $84\frac{1}{4}$ days?

(16) Reduce £8. 5*s.* 9*d.* to the decimal of 25 guineas.

(17) Bring (*a*) $\frac{3}{8}$ to a decimal; (*b*) ${\cdot}\dot{8}\dot{1}$ to a vulgar fraction.

(18) Find the difference between ·1 and ${\cdot}\dot{1}$, and multiply the result by 90.

(19) In ${\cdot}\dot{6}$ of a day a man can do ${\cdot}\dot{2}\dot{7}$ of his work: how long will he be in doing the whole of it?

(20) Divide ·06 of £1. by ·0003125.

(21) What number divided by 7·35 will give $6\frac{3}{5}$?

(22) Take square of ·065 from $\frac{1}{2}$ of ·075.

(23) Simplify $\dfrac{(3{\cdot}2-2{\cdot}9)\times 147}{{\cdot}003\times{\cdot}0005}$.

(24) Simplify 12 − 2·105 − 3·05 + ·5; and multiply your answer by ·015.

(25) Divide 4·068 by ·0018.

(26) 214·09 − 78·02104 + ·06935 + 18·2 + 117·5 − 69·92.

(27) Find the value of $\dfrac{{\cdot}\dot{0}09\dot{8}}{{\cdot}\dot{7}65\dot{4}}$ of £12. 13*s.* 4*d.*

(28) 20399600÷·208.

(29) Subtract ·992748 from 1, and divide the result by ·0074.

(30) If £3⅝. is paid for 2·8 yards of velvet, how much should be paid for 14 yards at the same rate?

(31) Divide the difference between 8·3 and ·056 by ·1145.

(32) From the sum of 16·25, ·003, and 5·047 take 13·0625, and multiply your answer by 9·15.

(33) Simplify $\frac{21\cdot05 \times 50}{\cdot00275}$.

(34) Multiply ·9374 by 5·00068.

(35) Divide $6\cdot1\dot{5}$ by $5\cdot\dot{5}\dot{9}$.

(36) If 8 ozs. cost ·5625 shilling, what will ·75 of a ton cost?

(37) 156025 ÷ 87·5.

(38) Reduce ·06875 of 40 guineas to £. *s.* *d.*

(39) Value of $\cdot5\dot{1}$ of 1 qr. 17 lbs. + 6·875 of 1 lb. + 7⅗ of 3½ tons − 17 tons 1 qr. 13¾ lbs.

(40) What decimal of 5*s.* 9½*d.* is £2. 11*s.* 6½*d.*?

(41) Find value of $23\cdot4\dot{5}$ of 3 miles 5 furlongs.

(42) Simplify 6·225 × 8·25÷·0025.

(43) Convert ·0025 into a vulgar fraction, and reduce it to its lowest terms.

(44) Multiply 31·5 by 27·9, and divide the result by 9·765.

(45) Simplify $\frac{5\cdot25 \times \cdot0065}{12\cdot5}$.

(46) Divide the product of $8\cdot\dot{3}$ and ·375 by ⅜.

(47) Value of $\cdot5\dot{7}$ of 1*s.* 10½*d.* + 6·75 of 3*s.* 11*d.* + 15·0025 of £1. 6*s.* 8*d.* − $1\cdot\dot{5}\dot{6}$ of 2*s.* 0¾*d.*

(48) Simplify $\frac{\cdot035 \times \cdot0056}{\cdot007}$.

(49) Find the difference between 5½ half guineas and £4·625, and reduce the result to the decimal of half-a-crown.

(50) From 5·3 take ·0375, and divide the remainder by 12·5.

(51) Multiply the difference between ·1 and ·026795 by 4 times the sum of 81·6 and ·025.

(**52**) Divide the sum of $\cdot025$ and $\cdot3525$ by the difference between 2 and $1\cdot94$.

(**53**) What is the value of $37\cdot02\dot{3}\dot{7}$ of £41. 5*s*. + $9\cdot000\dot{7}$ of £18. 15*s*. + $\cdot\dot{5}\dot{1}$ of £2. 1*s*. 3*d*.?

(**54**) Simplify $\dfrac{5\frac{1}{3} - \cdot042 - 2\cdot4 + 7\frac{5}{6}}{16\frac{2}{15} \div 60\frac{1}{2}}$.

(**55**) If $7\cdot\dot{5}\dot{1}$ ozs. of tea cost £$\cdot176416$., what will be the cost of $3\cdot02\dot{7}$ lbs.?

(**56**) Simplify $\dfrac{2}{3\cdot5}$ of $\dfrac{4}{\cdot\dot{7}1428\dot{5} - \cdot\dot{1}}$ of £2. 2*s*. + $\dfrac{1\frac{7}{8} + \cdot7\dot{5}}{3\cdot8 - \frac{2}{3}}$ of £5.

(**57**) Divide $\cdot04032$, — by $\cdot0048$.

(**58**) Simplify $(3\frac{5}{8} \div \cdot0025) + (\cdot875 \div 1\frac{2}{5})$.

(**59**) Find the value of $(\cdot4\dot{8}\dot{3} + \cdot27\dot{5}\dot{6} - \cdot00\dot{9}\dot{4})$ of £5. 16*s*. 8*d*.

(**60**) Reduce to ounces $5\frac{1}{4}$ of 3 tons 10 cwt. + $2\cdot625$ of 3 qrs. 16 lbs. + $20\cdot\dot{0}\dot{5}$ of 5 lbs. 10 ozs.

(**61**) Simplify £728. $\times\ 2\frac{2}{5} \div 5\frac{7}{35}$.

(**62**) Value of £$16\cdot325$. + $\cdot00875$ of £400. + $\cdot0\dot{0}3\dot{2}$ of £10. 8*s*. $1\frac{1}{2}$*d*. − $3\frac{49}{60}$ of £$\frac{3}{4}$.

(**63**) Find the difference between £3620. 10*s*. $\times\ 3\frac{1}{2} \div 66\frac{2}{3}$ and £3620. $\times\ 4\frac{1}{2} \div 86\frac{2}{3}$.

(**64**) Value of $(3\frac{3}{5} - 2\frac{1}{8} + 5\frac{2}{3} - \frac{3}{4}) \div (9 - 6\frac{20}{23})$ of 5 tons 3 cwts. 2 qrs.

(**65**) If I give away $\frac{3}{8}$ of my money to A, $\frac{1}{5}$ of the remainder to B, what decimal of the original sum do I keep?

(**66**) If 3 qrs. 16 lbs. be the unit of weight, decimalise 3 tons $12\frac{1}{2}$ cwts.

(**67**) Find the difference between £875. $\times\ (\cdot05)^4$, and £$8\cdot75$. $\times\ 4 \times 5$.

(**68**) Find the value of $\sqrt{4762\cdot104064}$ of 15 tons 15 cwts. 3 qrs. 15 lbs.

(**69**) Express as decimals $\frac{3}{5}$, $\frac{2}{3}$, $\frac{7}{9}$, $3\frac{3}{5}$; find their product, and divide $10\frac{1}{5}$ by it.

(**70**) Decimalise £409. 2*s*. $10\frac{7}{10}$*d*. and £1000. 15*s*. $2\frac{3}{4}$*d*., and find their difference.

(**71**) Express as a decimal $\{(11\frac{2}{5} + 2\frac{7}{10}) \div 9\frac{2}{5}\} \div (6\frac{1}{2} + 3\frac{5}{6} - \frac{1}{3})$, the unit being £1.

(**72**) Find the value of £3720. 10*s*. $\times\ \dfrac{100}{95\frac{1}{2}}$.

(73) Find the value of $\sqrt{4904\cdot2009}$ of 5 tons 3 cwts. 14 lbs.

(74) Find the value of $\dfrac{3 \text{ cwts. } 3 \text{ qrs.} \times 51\frac{3}{5} \text{ m.} \times £1}{18s. \times 17\frac{1}{8} \text{ miles}}$.

(75) What decimal of half-a-crown is $\frac{5}{6}$ of a shilling?

(76) Express the difference between $3\frac{1}{2} \div 66\frac{2}{3}$, $4\frac{1}{2} \div 86\frac{2}{3}$ as a decimal, and multiply the result by £3620. 10*s.*

(77) Find the value of $\sqrt[3]{12167}$ of 16 tons 3 cwts. 2 qrs. 13 lbs.

(78) If £107. 14*s.* 6*d.* ÷ £3. 17*s.* 6*d.* be ounces troy, express the result as a decimal of 1 pound.

(79) If the solid content of a cube be 29238037·967943 inches, express its side in yards.

(80) Express in pounds £210. × 86 ÷ $3\frac{1}{2}$.

(81) Find the cube root of 32·706598392 and its value if the unit be £6. 13*s.* 4*d.*

(82) Decimalise £216. 10*s.*, £278. 14*s.* $10\frac{1}{2}$*d.*: find their difference: what factor will make it = $\frac{1}{20}$ of £216. 10*s.*?

(83) If one pint be the unit, what is the value of $\sqrt[3]{67071\cdot878047}$?

(84) Divide 300 into 2 parts, such that $\frac{1}{3}$ of the one added to $\frac{1}{8}$ of the other = 75.

(85) In a school of 500 children $\frac{9}{10}$ passed in reading, ·85 in writing, ·8 in arithmetic: what number passed?

(86) By what factor must $(1\cdot04)^3$ be multiplied to produce £3102. 13*s.* 1·47072*d.*?

(87) A can walk $3\frac{1}{2}$ miles an hour, B 5 miles an hour: if A start at 9 A.M., B at 10.30, when will B catch A?

(88) Express as a decimal $\frac{15}{28}$ of $\frac{7}{195}$ of $8\frac{2}{3} + 7\frac{1}{11} - 3\frac{2}{3}$: what is its value, the unit being 1 pound troy?

(89) Simplify $26 \times 3\frac{1}{2} \times 10 \div 1\frac{1}{2} \times 14$.

(90) What is the value of $\dfrac{\cdot0\dot{9} + 3\frac{1}{5}}{2\frac{1}{5} - \cdot02\dot{9}} \div 3\cdot\dot{1}4285\dot{7}$, the unit being 1 mile?

XIV. PRACTICE.

168. Practice is another way of performing multiplication, and is Simple or Compound according as the quantity whose value is required is simple or compound.

Ex. 1. Find the cost of 5678 articles at 2 shillings.
2*s.* = $\frac{1}{10}$ of £1. ∴ 5678 × 2*s.* = $\frac{1}{10}$ of £5678. = £567. 16*s.*

Ex. 2. Find the cost of 5678 articles at 19 shillings.
5678 × 18 = £5110. 4*s.* or 5678 × £1. = £5678.
5678 × 1 = £283. 18*s.* or 5678 × 1*s.* = £283. 18*s.*
∴ 5678 × 19 = £5394. 2*s.* ∴ 5678 × 19*s.* = £5394. 2*s.*

169. An *aliquot part* of any quantity is any measure of that quantity. The following tables of Aliquot Parts should be learnt:—

Of a Pound Sterling		Of a Shilling
10*s.* is $\frac{1}{2}$	2*s.* is $\frac{1}{10}$	6*d.* is $\frac{1}{2}$
6*s.* 8*d.* is $\frac{1}{3}$	1*s.* 8*d.* is $\frac{1}{12}$	4*d.* is $\frac{1}{3}$
5*s.* is $\frac{1}{4}$	1*s.* is $\frac{1}{20}$	3*d.* is $\frac{1}{4}$
4*s.* is $\frac{1}{5}$	6*d.* is $\frac{1}{40}$	2*d.* is $\frac{1}{6}$
3*s.* 4*d.* is $\frac{1}{6}$	4*d.* is $\frac{1}{60}$	1½*d.* is $\frac{1}{8}$
2*s.* 6*d.* is $\frac{1}{8}$	3*d.* is $\frac{1}{80}$	1*d.* is $\frac{1}{12}$

Ex. 1. Find the value of 4253 articles at £5. 17*s.* 10½*d.* each.

			£	*s.*	*d.*		£	*s.*	*d.*	
A			4253			is value at	1	0	0	each
			5							
B			21265			”	5	0	0	”
C	10*s.*	$\frac{1}{2}$ of £1.	2126	10		”		10	0	”
D	5*s.*	$\frac{1}{2}$ of 10*s.*	1063	5		”		5	0	”
E	2*s.* 6*d.*	$\frac{1}{2}$ of 5*s.*	531	12	6	”		2	6	”
F	3*d.*	$\frac{1}{10}$ of 2*s.* 6*d.*	53	3	3	”			3	”
G	1½*d.*	$\frac{1}{2}$ of 3*d.*	26	11	7½	”			1½	”
H	17*s.* 10½*d.*		25066	2	4½		5	17	10½	”

B is A × 5, C is A ÷ 2, D is C ÷ 2, E is D ÷ 2, F is E ÷ 10, G is F ÷ 2.

Ex. 2. Find the value of 4253⅝ articles at £7. 6*s.* 11¾*d.*

Since ⅝ of £1 = 12*s.* 6*d.*, ∴ cost of 4253⅝ articles at £1. = £4253. 12*s.* 6*d.*

				£	s.	d.		£	s.	d.	
A			∴	4253	12	6	is value at	1	0	0	each
						7					
B				29775	7	6	„	7	0	0	„
C	10s.		$\frac{1}{2}$ of £1.	2126	16	3	„		10	0	„
D	5s.		$\frac{1}{2}$ of 10s.	1063	8	$1\frac{1}{2}$	„		5	0	„
E	1s.		$\frac{1}{5}$ of 5s.	212	13	$7\frac{1}{2}$	„		1	0	„
F		6d.	$\frac{1}{2}$ of 1s.	106	6	$9\frac{3}{4}$	„			6	„
G		4d.	$\frac{1}{3}$ of 1s.	70	17	$10\frac{1}{2}$	„			4	„
H		$1\frac{1}{2}$d.	$\frac{1}{4}$ of 6d.	26	11	$8\frac{7}{16}$	„			$1\frac{1}{2}$	„
K		$\frac{1}{4}$d.	$\frac{1}{6}$ of $1\frac{1}{2}$d.	4	8	$7\frac{13}{32}$	„			$\frac{1}{4}$	„
				33386	10	$6\frac{3}{32}$		7	16	$11\frac{3}{4}$	„

B = A × 7, C = A ÷ 2, D = C ÷ 2, E = D ÷ 5, F = E ÷ 2, G = E ÷ 3, H = G ÷ 4, K = H ÷ 6.

Ex. 3. Find the cost of $389\frac{3}{7}$ articles at £6. 13s. 4d. each.

Since £$\frac{3}{7}$. would give a fraction of a farthing, it will be better to find separately the cost of 389 articles and of $\frac{3}{7}$ of an article, and then add. Thus:

		£	s.	d.		£	s.	d.
		389			is value at	1	0	0
		6						
		2334			„	6	0	0
10s.	$\frac{1}{2}$ of £1.	194	10		„		10	0
3s. 4d.	$\frac{1}{3}$ of 10s.	64	16	8	„		3	4
		2593	6	8	„	6	13	4
		2	17	$1\frac{5}{7}$				
		2596	3	$9\frac{5}{7}$				

	£	s.	d.
	6	13	4
			3
7)	20	0	0
	2	17	$1\frac{5}{7}$

Exercise 109.

Find the cost of—

(1) 8046 at $\frac{1}{4}$d., $\frac{1}{2}$d., $\frac{3}{4}$d. (2) 9063 at $\frac{1}{4}$d., $\frac{1}{2}$d., $\frac{3}{4}$d.
(3) 9989 „ $\frac{1}{4}$d., $\frac{1}{2}$d., $\frac{3}{4}$d. (4) 8754 „ $\frac{1}{4}$d., $\frac{1}{2}$d., $\frac{3}{4}$d.
(5) 3438 „ $1\frac{1}{4}$d., $2\frac{1}{2}$d., $3\frac{3}{4}$d. (6) 7986 „ $4\frac{1}{4}$d., $5\frac{1}{2}$d., $6\frac{3}{4}$d.
(7) 9201 „ $7\frac{1}{4}$d., $8\frac{1}{2}$d., $9\frac{3}{4}$d. (8) 4786 „ $10\frac{1}{4}$d., $11\frac{1}{2}$d., $11\frac{3}{4}$d.
(9) 7500 at 1s., 1s. $0\frac{1}{4}$d., 1s. $0\frac{1}{2}$d.
(10) 6098 „ 1s. $1\frac{3}{4}$d., 1s. 2d., 1s. $2\frac{1}{4}$d.
(11) 1786 „ 1s. $2\frac{1}{2}$d., 1s. $2\frac{3}{4}$d., 1s. 3d.
(12) 1867 „ 1s. $3\frac{1}{4}$d., 1s. $3\frac{1}{2}$d., 1s. $3\frac{3}{4}$d.

(13) 6041 at 1*s.* 4*d.*, 1*s.* 5¼*d.*, 1*s.* 6½*d.*
(14) 6993 „ 1*s.* 7¾*d.*, 1*s.* 8*d.*, 1*s.* 9¼*d.*
(15) 9009 „ 1*s.* 10½*d.*, 1*s.* 11¾*d.*, 2*s.* 1*d.*
(16) 32133 „ 3*s.* 2¼*d.*, 4*s.* 3½*d.*, 5*s.* 4¾*d.*
(17) 62001 „ 6*s.* 5*d.*, 7*s.* 6¼*d.*, 8*s.* 7½*d.*
(18) 48138 „ 9*s.* 8¾*d.*, 10*s.* 9*d.*, 11*s.* 10¼*d.*
(19) 70018 „ 12*s.* 11½*d.*, 13*s.* 0¼*d.*, 13*s.* 1½*d.*
(20) 15801 „ 13*s.* 2¼*d.*, 13*s.* 3½*d.*, 13*s.* 4¾*d.*
(21) 31861 „ 14*s.* 5*d.*, 14*s.* 6¼*d.*, 14*s.* 6½*d.*
(22) 21863 „ 15*s.* 7¾*d.*, 15*s.* 8*d.*, 15*s.* 8¼*d.*
(23) 32016 „ 16*s.* 9½*d.*, 16*s.* 10¾*d.*, 16*s.* 11*d.*
(24) 63201 „ 17*s.* 11¼*d.*, 18*s.* 0½*d.*, 19*s.* 0¾*d.*
(25) 80001 „ 12*s.* 1*d.*, 12*s.* 2¼*d.*, 12*s.* 3½*d.*
(26) 32186 „ 13*s.* 4¼*d.*, 13*s.* 5½*d.*, 13*s.* 6¾*d.*
(27) 82010 „ 14*s.* 7¼*d.*, 14*s.* 8½*d.*, 14*s.* 9¾*d.*
(28) 73219 „ 15*s.* 10¼*d.*, 16*s.* 11½*d.*, 16*s.* 11¾*d.*
(29) 62173 „ 17*s.* 1¼*d.*, 17*s.* 11¾*d.*, 18*s.* 8½*d.*
(30) 48321 „ 19*s.* 9¼*d.*, 19*s.* 6½*d.*, 19*s.* 11¾*d.*

			£	*s.*	*d.*				£	*s.*	*d.*
(31)	7681	at	5	14	9½	(43)	$9876\frac{4}{11}$	at	4	18	6½
(32)	8680	„	6	15	11	(44)	$8673\frac{4}{9}$	„	7	9	11¾
(33)	75078	„	10	16	3¾	(45)	$5549\frac{7}{16}$	„	9	11	8½
(34)	19219	„	11	19	10¾	(46)	$92009\frac{11}{18}$	„	12	14	10¾
(35)	82768	„	15	17	11¼	(47)	$49839\frac{12}{17}$	„	11	8	11¼
(36)	37201	„	17	18	10½	(48)	$62689\frac{3}{19}$	„	15	14	7¾
(37)	19286	„	19	15	6¾	(49)	$24008\frac{11}{12}$	„	91	18	6¾
(38)	82010	„	23	17	11½	(50)	$47326\frac{7}{8}$	„	95	6	9¼
(39)	88919	„	39	16	6½	(51)	$98943\frac{5}{6}$	„	41	7	10½
(40)	51998	„	57	19	9¼	(52)	$55677\frac{4}{5}$	„	51	17	3¾
(41)	82134	„	68	18	4½	(53)	$73260\frac{3}{4}$	„	72	15	4½
(42)	$92016\frac{6}{11}$	„	75	16	7¾	(54)	$19879\frac{5}{8}$	„	92	12	10¼

170. The aliquot parts in Avoirdupois Weight are:—

10 cwt. is $\frac{1}{2}$ of a ton	14 lbs. is $\frac{1}{2}$ of a quarter
5 cwt. is $\frac{1}{4}$,,	7 lbs. is $\frac{1}{4}$,,
4 cwt. is $\frac{1}{5}$,,	4 lbs. is $\frac{1}{7}$,,
2 cwt. is $\frac{1}{10}$,,	8 ozs. is $\frac{1}{2}$ of a pound
2 qrs. is $\frac{1}{2}$ of a cwt.	4 ozs. is $\frac{1}{4}$,,
1 qr. is $\frac{1}{4}$,,	2 ozs. is $\frac{1}{8}$,,

Ex. Find the cost of 2 tons 5 cwts. 3 qrs. 18 lbs. at £15. 10*s*. 8*d*. per ton.

		£	*s.*	*d.*		
		15	10	8	is value of 1 ton	
				2		
		31	1	4	,,	2 tons
5 cwts.	$\frac{1}{4}$ of 1 ton	3	17	8	,,	5 cwts.
2 qrs.	$\frac{1}{10}$ of 5 cwts.		7	$9\frac{1}{5}$	,,	2 qrs.
1 qr.	$\frac{1}{2}$ of 2 qrs.		3	$10\frac{3}{5}$	,,	1 qr.
14 lbs.	$\frac{1}{2}$ of 1 qr.		1	$11\frac{3}{10}$	,,	14 lbs.
4 lbs.	$\frac{1}{7}$ of 1 qr.			$6\frac{23}{35}$	,,	4 lbs.
		35	13	$1\frac{53}{70}$	,,	2 tons 5 cwts. 3 qrs. 18 lbs.

Exercise 110.

Find the value of—

	tons	cwts.	qrs.	lbs.		£	*s.*	*d.*	
(1)	16	7	3	21	at	14	16	8	per ton.
(2)	17	9	2	18	,,	9	15	6	,,
(3)	19	5	1	24	,,	13	13	$11\frac{1}{2}$	,,
(4)	21	11	3	12	,,	7	11	$9\frac{3}{4}$	,,
(5)	31	12	3	14	,,	19	19	$11\frac{3}{4}$	,,

	cwts.	qrs.	lbs.					
(6)	4	1	17	at	5	15	$6\frac{1}{2}$	per cwt.
(7)	5	3	23	,,	6	16	$11\frac{3}{4}$	,,
(8)	46	1	15	,,	9	8	4	,,
(9)	84	3	22	,,	19	11	$10\frac{1}{2}$	,,
(10)	99	3	21	,,	26	13	$9\frac{3}{4}$	,,

	tons	cwts.	qrs.	lbs.					
(11)	5	17	2		at	3	16	$9\frac{3}{4}$	per ton.
(12)	20	16	3	7	,,	5	13	$7\frac{3}{4}$	,,
(13)	24	6	1	16	,,	7	16	$11\frac{1}{4}$	per cwt.

	lbs.	ozs.	drs.		£	s.	d.	
(14)	29	10	3	at	5	10	3¼	per qr.
(15)	33	11	12	,,	8	11	2¾	,,
(16)	43	15	15	,,	3	12	7¼	per lb.
(17)	34	13	11	,,	2	14	5¾	,,
(18)	77	11	11	,,	1	19	6¼	,,
(19)	96	12	8	,,	17	15	11½	,,
(20)	100	9	7	,,	15	12	9¼	,,

171. The aliquot parts in Long Measure are :—

4 furlongs	is ½ of a mile	4 poles	is $\frac{1}{10}$ of a furlong
2 ,,	is ¼ ,,	18 inches	is ½ of a yard
1 ,,	is ⅛ ,,	1 foot	is ⅓ ,,
20 poles	is ½ of a furlong	9 inches	is ¼ ,,
10 ,,	is ¼ ,,	6 ,,	is ⅙ ,,
5 ,,	is ⅛ ,,		

Ex. Find the cost of 11 miles 3 furlongs 55 yards of railway at £32500 per mile.

		£	s.	d.		
		32500				value of 1 mile
		11				
		357500			,,	11 miles
2 furlongs	¼ of a mile	8125			,,	2 furlongs
1 furlong	½ of 2 fur.	4062	10		,,	1 furlong
55 yards	¼ of 1 fur.	1015	12	6	,,	55 yards
		370703	2	6	,,	11 m. 3 fur. 55 yds.

Exercise 111.

Find the value of—

(1) 9 m. 4 fur. 16 po. at £2. 2*s*. 7½*d*. per mile.

(2) 29 m. 3 fur. 6 po. at £1. 11*s*. 3*d*. per mile.

(3) 100 m. 4 fur. 36 po. at £2. 6*s*. 4½*d*. per mile.

(4) 37 m. 7 fur. 1 po. £5. 8*s*. 5*d*. per mile.

(5) 66 m. 7 fur. 5 po. £19. 10*s*. 4½*d*. per mile.

(6) 15 m. 7 fur. 16 po. £3. 9*s*. 5*d*. per mile.

(7) 26 m. 1 fur. 32 po. £8. 2*s*. 6*d*. per mile.

(8) 48 m. 5 fur. 38 po. £6. 18*s*. 4¼*d*. per mile.

(9) 157 m. 3 fur. 24 po. at £11. 10*s*. 0*d*. per mile.

(10) 369 m. 3 fur. 22 yds. at £14. 13*s.* 4*d.* per mile.
(11) 2 m. 6 fur. 9 chains 10 yds. at £3. 18*s.* 4*d.* per mile.
(12) 17 m. 2 fur. 25 po. at £76. 10*s.* 0*d.* per mile.
(13) 28 m. 7 fur. 19 po. at £2. 16*s.* 6¼*d.* per furlong.
(14) 43 m. 5 fur. 16 po. at £2. 13*s.* 8*d.* per furlong.
(15) 90 m. 2 fur. 30 po. at £2. 15*s.* 6¾*d.* per furlong.
(16) 55 m. 3 fur. 33 po. at £14. 15*s.* 10½*d.* per mile.

172. The aliquot parts in Cloth Measure are :—

2 quarters is ½ of 1 yard	2 nails is ½ of 1 quarter
1 quarter is ¼ „	1 nail is ¼ „

1 quarter is ⅛ of an English ell.

Ex. Find the value of 29 yds. 3 qrs. 3 nls. at £19. 16*s.* 4½*d.* per yard.

		£	*s.*	*d.*		
		19	16	$4\frac{1}{2}$	is value of	1 yard
				10		
		198	3	9	„	10 yards
				3		
		594	11	3	„	30 yards
		19	16	$4\frac{1}{2}$		
		574	14	$10\frac{1}{2}$	„	29 yards
2 quarters	$\frac{1}{2}$ of 1 yard	9	18	$2\frac{1}{4}$	„	2 quarters
1 quarter	$\frac{1}{2}$ of 2 qrs.	4	19	$1\frac{1}{8}$	„	1 quarter
2 nails	$\frac{1}{2}$ of 1 qr.	2	9	$6\frac{9}{16}$	„	2 nails
1 nail	$\frac{1}{2}$ of 2 nails	1	4	$9\frac{9}{32}$	„	1 nail
		593	6	$5\frac{23}{32}$	„	29 yds. 3 qrs. 3 nails

Exercise 112.

Find the value of—

	yds.	qrs.	nls.		£	*s.*	*d.*	
(1)	8	3	3	at	5	6	$7\frac{1}{2}$	per yard.
(2)	16	2	1	„	16	4	$11\frac{3}{4}$	„
(3)	13	3	1	„	17	6	$9\frac{1}{4}$	„
(4)	53	3	3	„	19	11	$11\frac{3}{4}$	„
(5)	35	1	3	„	26	13	$9\frac{1}{2}$	„
(6)	17	3	2	„	19	16	$4\frac{1}{4}$	„

	yds.	qrs.	nls.		£	s.	d.	
(7)	16	1	2	at	17	15	3$\frac{1}{2}$	per yard
(8)	15	3	3	„	15	14	10$\frac{1}{4}$	„
(9)	17	0	3	„	13	11	9$\frac{1}{4}$	„
(10)	19	1	3	„	11	19	10$\frac{1}{2}$	„

173. The aliquot parts in Square Measure are :—

2 roods is $\frac{1}{2}$ of an acre
1 rood is $\frac{1}{4}$ „
20 poles is $\frac{1}{8}$ „
16 poles is $\frac{1}{10}$ „

10 poles is $\frac{1}{4}$ of a rood
8 poles is $\frac{1}{5}$ „
5 poles is $\frac{1}{8}$ „
4 poles is $\frac{1}{10}$ „

Exercise 113.

Find the value of—

(1) 5 ac. 3 rds. 4 po. 4$\frac{1}{2}$ yds. at £5. 16*s.* 8*d.* per acre.
(2) 10 ac. 2 rds. 26 po. 9 yds. at £17. 15*s.* 11*d.* per acre.
(3) 50 ac. 2 rds. 20 po. at £53. 13*s.* 9*d.* per acre.
(4) 73 ac. 1 rd. 39 po. 27$\frac{1}{2}$ yds. at £90. 11*s.* 10$\frac{1}{2}$*d.* per acre.
(5) 36 ac. 2 rds. 19 po. 15$\frac{1}{4}$ yds. at £75. 15*s.* 3$\frac{1}{4}$*d.* per acre.
(6) 37 ac. 3 rds. 25 po. at £90. 19*s.* 11$\frac{3}{4}$*d.* per acre.
(7) 1572 ac. 1$\frac{1}{2}$ rd. at £98. 19*s.* 4$\frac{1}{4}$*d.* per acre.
(8) 999 ac. 3 rds. 16 po. at £105. 16*s.* 5$\frac{1}{2}$*d.* per acre.
(9) 25 ac. 3 rds. 25 po. at £19. 11*s.* 6*d.* per rood.
(10) 54 yds. 5 ft. 80 in. at £5. 11*s.* 7$\frac{1}{2}$*d.* per yard.
(11) 72 yds. 3 ft. 130 in. at £7. 12*s.* 9$\frac{1}{4}$*d.* per yard.
(12) 33 yds. 6 ft. 120 in. at £1. 14*s.* 8$\frac{3}{4}$*d.* per yard.

174. The aliquot parts in Cubic Measure are :—

9 feet is $\frac{1}{3}$ of a yard
3 feet is $\frac{1}{9}$ „
144 feet is $\frac{1}{12}$ „
864 inches is $\frac{1}{2}$ of a foot
576 inches is $\frac{1}{3}$ „

432 inches is $\frac{1}{4}$ of a foot
288 inches is $\frac{1}{6}$ „
216 inches is $\frac{1}{8}$ „
192 inches is $\frac{1}{9}$ „

Ex. Find the cost of 34 yds. 19 ft. 864 in. at £7. 11*s.* 6*d.* per yard.

		£	*s.*	*d.*		
		7	11	6	value of 1 yard	
				10		
		75	15	0	,,	10 yards
				3		
		227	5	0	,,	30 yards
		30	6	0	,,	4 yards
		257	11	0	,,	34 yards
9 feet	$\frac{1}{4}$ of 1 yard	2	10	6	,,	9 feet
9 feet	$\frac{1}{4}$ of 1 yard	2	10	6	,,	9 feet
1 foot	$\frac{1}{9}$ of 9 feet		5	$7\frac{1}{3}$	,,	1 foot
576 inches	$\frac{1}{3}$ of 1 foot		1	$10\frac{4}{9}$	,,	576 inches
288 inches	$\frac{1}{2}$ of 576 in.			$11\frac{2}{9}$	,,	288 inches
		263	0	5 .	,,	34 yds. 19 ft. 864 in.

Exercise 114.

Find the value of—

(1) 54 yds. 24 ft. 216 in. at £9. 19*s.* $11\frac{1}{2}$*d.* per yard.
(2) 153 yds. 11 ft. 72 in. at £11. 12*s.* $7\frac{3}{4}$*d.* per yard.
(3) 234 yds. 19 ft. 427 in. at £12. 13*s.* $6\frac{1}{2}$*d.* per yard.
(4) 163 yds. 16 ft. 1512 in. at £13. 14*s.* $7\frac{1}{4}$*d.* per yard.
(5) 299 yds. 7 ft. 504 in. at £15. 16*s.* $5\frac{1}{2}$*d.* per yard.
(6) 77 yds. 17 ft. 1440 in. at £19. 19*s.* $11\frac{3}{4}$*d.* per yard.
(7) 245 yds. 12 ft. 192 in. at £20. 11*s.* $6\frac{1}{4}$*d.* per yard.
(8) 210 yds. 25 ft. 1360 in. at £25. 16*s.* $5\frac{1}{2}$*d.* per yard.
(9) 342 yds. 23 ft. 1643 in. at £13. 12*s.* $7\frac{1}{4}$*d.* per yard.
(10) 118 yds. 10 ft. 216 in. at £25. 13*s.* $9\frac{1}{2}$*d.* per yard.

175. The aliquot parts in Troy Weight are :—

6 ounces is $\frac{1}{2}$ of 1 pound
4 ounces is $\frac{1}{3}$,,
3 ounces is $\frac{1}{4}$,,
2 ounces is $\frac{1}{6}$,,
1 ounce is $\frac{1}{12}$,,
10 pennyweights is $\frac{1}{2}$ of 1 ounce
5 ,, is $\frac{1}{4}$,,
4 ,, is $\frac{1}{5}$,,

2 pennyweights is $\frac{1}{10}$ of 1 ounce
1 pennyweight is $\frac{1}{20}$,,
12 grains is $\frac{1}{2}$ of 1 pennyweight
8 ,, is $\frac{1}{3}$,,
6 ,, is $\frac{1}{4}$,,
4 ,, is $\frac{1}{6}$,,
3 ,, is $\frac{1}{8}$,,
2 ,, is $\frac{1}{12}$,,

Ex. Find the cost of 17 lbs. 7 ozs. 16 dwts. at £1. 17*s.* $4\frac{1}{2}$*d.* per pound.

		£	*s.*	*d.*		
		1	17	$4\frac{1}{2}$	value of 1 pound	
				8		
		14	19	0	„	8 pounds
				2		
		29	18	0	„	16 pounds
		1	17	$4\frac{1}{2}$		
		31	15	$4\frac{1}{2}$	„	17 pounds
4 ounces	$\frac{1}{3}$ of 1 pound		12	$5\frac{1}{2}$	„	4 ounces
3 ounces	$\frac{1}{4}$ of 1 pound		9	$4\frac{1}{8}$	„	3 ounces
10 dwts.	$\frac{1}{6}$ of 3 ounces		1	$6\frac{11}{16}$	„	10 dwts.
5 dwts.	$\frac{1}{2}$ of 10 dwts.			$9\frac{11}{32}$	„	5 dwts.
1 dwt.	$\frac{1}{5}$ of 5 dwts.			$1\frac{139}{160}$	„	1 dwt.
		32	19	$8\frac{1}{40}$	„	17 lbs. 7 ozs. 16 dwts.

Exercise 115.

Find the cost of—

(1) 10 lbs. 6 ozs. 15 dwts. at £1. 14*s.* $6\frac{1}{2}$*d.* per pound.

(2) 15 lbs. 9 ozs. 17 dwts. at £2. 13*s.* $8\frac{1}{4}$*d.* per pound.

(3) 17 lbs. 8 ozs. 19 dwts. at £3. 15*s.* $9\frac{3}{4}$*d.* per pound.

(4) 43 lbs. 11 ozs. 18 dwts. at £5. 17*s.* $6\frac{1}{2}$*d.* per pound.

(5) 35 lbs. 10 ozs. 13 dwts. at £7. 11*s.* $5\frac{3}{4}$*d.* per pound.

(6) 13 ozs. 15 dwts. 23 grs. at £2. 13*s.* $11\frac{1}{4}$*d.* per ounce.

(7) 15 ozs. 14 dwts. 22 grs. at £3. 12*s.* $10\frac{1}{2}$*d.* per ounce.

(8) 11 ozs. 18 dwts. 21 grs. at £4. 11*s.* $9\frac{1}{4}$*d.* per ounce.

(9) 17 ozs. 17 dwts. 19 grs. at £5. 16*s.* $11\frac{1}{4}$*d.* per ounce.

(10) 19 ozs. 13 dwts. 17 grs. at £3. 13*s.* $8\frac{1}{2}$*d.* per ounce.

176. The aliquot parts in Apothecaries' Weight are :—

6 ounces is $\frac{1}{2}$ of 1 pound
4 ounces is $\frac{1}{3}$ „
3 ounces is $\frac{1}{4}$ „
2 ounces is $\frac{1}{6}$ „
1 ounce is $\frac{1}{12}$ „
4 drams is $\frac{1}{2}$ of 1 ounce
2 drams is $\frac{1}{4}$ „

1 dram is $\frac{1}{8}$ of 1 ounce
1 scruple is $\frac{1}{3}$ of 1 dram
10 grains is $\frac{1}{2}$ of 1 scruple
5 grains is $\frac{1}{4}$ „
4 grains is $\frac{1}{5}$ „
2 grains is $\frac{1}{10}$ „
1 grain is $\frac{1}{20}$ „

Exercise 116.

Find the value of—

(1) 25 lbs. 10 ozs. 3 drs. at £2. 15*s.* 2½*d.* per pound.
(2) 35 lbs. 8 ozs. 6 drs. at £3. 7*s.* 9¾*d.* per pound.
(3) 46 lbs. 7 ozs. 7 drs. at £5. 11*s.* 2½*d.* per pound.
(4) 53 lbs. 9 ozs. 7 drs. at £6. 10*s.* 1¼*d.* per pound.
(5) 24 lbs. 3 ozs. 5 drs. at £7. 11*s.* 9½*d.* per pound.
(6) 34 ozs. 5 drs. 1 sc. 15 grs. at £5. 10*s.* 3¼*d.* per ounce.
(7) 72 ozs. 3 drs. 2 sc. 14 grs. at £6. 11*s.* 2½*d.* per ounce.
(8) 47 ozs. 2 drs. 1 sc. 13 grs. at £5. 12*s.* 6½*d.* per ounce.
(9) 41 ozs. 7 drs. 1 sc. 12 grs. at £2. 10*s.* 4¾*d.* per ounce.
(10) 73 ozs. 3 drs. 1 sc. 11 grs. at £1. 13*s.* 9¼*d.* per ounce.

177. The aliquot parts in Wine and Spirit Measure are :—

21 gallons is $\frac{1}{3}$ of 1 hogshead
9 gallons is $\frac{1}{7}$ "
7 gallons is $\frac{1}{9}$ "
3 gallons is $\frac{1}{21}$ "
2 quarts is $\frac{1}{2}$ of 1 gallon

1 quart is $\frac{1}{4}$ of 1 gallon
1 pint is $\frac{1}{2}$ of 1 quart
2 gills is $\frac{1}{2}$ of 1 pint
1 gill is $\frac{1}{4}$ "

Exercise 117.

Find the value of—

(1) 77 hhds. 33 gals. 3 qts. at £5. 6*s.* 8*d.* per hogshead.
(2) 15 hhds. 40 gals. 3 qts. at £6. 7*s.* 1*d.* per hogshead.
(3) 23 hhds. 35 gals. 1 qt. at £17. 13*s.* 8*d.* per hogshead.
(4) 40 hhds. 47 gals. 3 qts. at £12. 15*s.* 2¼*d.* per hogshead.
(5) 54 hhds. 37 gals. 3 qts. at £10. 13*s.* 6½*d.* per hogshead.
(6) 3 hhds. 14 gals. 2 qts. 1 pt. at £1. 3*s.* 6½*d.* per gallon.
(7) 2 hhds. 13 gals. 1 qt. 1 gill at £2. 6*s.* 7¾*d.* per gallon.
(8) 1 hhd. 11 gals. 3 qts. 1 pt. 1 gill at £1. 5*s.* 11½*d.* per gallon.
(9) 4 hhds. 27 gals. 2 qts. 1 pt. at £0. 15*s.* 6¼*d.* per gallon.
(10) 7 hhds. 35 gals. 3 qts. 1 pt. at £3. 11*s.* 7¾*d.* per gallon.

Miscellaneous Examples G.

(1) Find the rent of 39 ac. 2 rds. 20 perches of land at £4. 7*s.* 6*d.* per acre.

(2) Find the price of 16 ingots of gold, each weighing 2 lbs. 6 ozs. 14 dwts. 20 grs. at £3. 15*s.* 9¾*d.* per ounce.

(3) If a man saves £12. 10*s.* a year, how much will he save in 19 years 11 months 2 weeks and 4 days?

(4) What is the cost of 111 yds. 7 ft. 132 in. of carpet at 15*s.* 6¾*d.* a yard?

(5) Find the dividend on £1075. 17*s.* 8*d.* at 9*s.* 6¼*d.* in the £.

(6) If a ton of metal cost £21. 19*s.* 2*d.*, what must be given for 74 tons 13 cwts. 3 qrs. 5 lbs.?

(7) Find the value of ·01625 of £204. 3*s.* 4*d.*

(8) What is the value of a field 14 ac. 3 rds. 26⅔ po. at £52. 7*s.* 6*d.* per acre?

(9) What sum must be added to ·356 of £2. 17*s.* 10*d.* to make up 2⅗ of £8. 19*s.* 7½*d.*?

(10) If the circumference of a wheel be 5 ft. 11¾ in., what distance will be covered in 3678 revolutions?

(11) If my gross income be £2156. 16*s.* 7¾*d.*, and my taxes are 5*s.* 6½*d.* in the £, what is my net income?

(12) Calculate a man's wages for 5 months 3 weeks 6 days at £1. 7*s.* 5*d.* per month.

(13) A person mixed 6 lbs. of tea at 3*s.* 6*d.*; 10 lbs. at 3*s.* 5*d.*; 8 lbs. at 4*s.* 1*d.*; 12 lbs. at 4*s.* 4*d.*; and sold the mixture at 4*s.* 9*d.* a pound: what did he gain by the sale?

(14) Find the value of 5 ac. 2 rds. 11 po. 7 yds. 5 ft. 29 in. at £53. 16*s.* 8*d.* an acre.

(15) If it take 5¾ yds. of cloth for a great coat; 4 yds. for a dress-coat; 2½ yds. for a pair of trousers; ⅞ of a yard for a waistcoat,—what will be the cost of the suit at 15*s.* 6*d.* the yard, and the making £2. 2*s.* 6*d.*?

(16) A gentleman gave 16 tons 3¾ cwts. of coal to a number of poor persons, allowing 1¼ cwt. to each: how many did he relieve, and what did he spend, the price of coal being £1. 6*s.* 8*d.* the ton?

(17) Find the value of 13 m. 5 fur. 9 po. 3 yds. at 7*s.* 6*d.* an inch.

(18) Find the value of $\frac{3}{4}$ of a square mile; $\frac{3}{4}$ of an acre; 99 sq. yds. at 6*s*. 8*d*. a square foot.

(19) How much rent would be paid for 32 ac. 2 rds. 20 po. at £6. 10*s*. an acre?

(20) Add together ·6 of £1.; ·75 of a guinea; ·16 of a crown.

(21) Find the value of $\cdot 5\dot{4}619\dot{6}$ of a guinea and $\cdot 8214215\dot{7}$ of a cwt.

(22) What is the tax on an income of £325. 10*s*. at 7*d*. in the £.?

(23) How much sugar worth £3. 12*s*. 3*d*. a cwt. must be given in exchange for 12 tons 1 qr. 15 lbs. at £27. 10*s*. 4*d*. a ton?

(24) If I buy 19 pieces of silk, each $27\frac{4}{5}$ yards, at $3\frac{2}{3}$ of a shilling per yard, what would the whole cost?

(25) A tradesman failed for £1000.; his effects produced £798. 10*s*.: what did he pay in the £.?

(26) A square foot of iron plate weighs 9·37 pounds: find the weight of a boiler which takes 276 sq. ft., and its cost at 9*d*. a square inch.

(27) What is the value of 5 hogsheads of sugar, each weighing 7 cwts. 3 qrs. 27 lbs. at £3. 10*s*. $10\frac{1}{2}$*d*. a hundredweight?

(28) How many miles will a cart wheel whose circumference is 3 ft. 10 in. go if it turn $34434\frac{18}{23}$ times?

(29) How many yards of cloth at 15*s*. 6*d*. a yard must be given in exchange for 215 yds. at 11*s*. 3*d*.?

(30) A merchant's debts amount to £10864.; he can only pay 6*s*. 9*d*. in the £.: what do his creditors get and lose?

(31) If A spend £1. 1*s*. 2*d*. a day, B £1. 4*s*. 8*d*., C £2. 5*s*. 9*d*., how much will they spend between them in 2054 days?

(32) If a man can pay 7*s*. 4*d*. in the £., and his debts amount to £6375., what is the value of his effects?

(33) I mix 120 gals. of wine at 12*s*. 3*d*. a gal. with 98 gals. at 17*s*. 6*d*., and sell the mixture at 15*s*. 3*d*. the gal., what profit do I make?

XV. BILLS OF PARCELS.

178. *An Invoice* is a statement in detail, sent by a Seller to a Buyer, at the time the goods are delivered to the buyer, of the quantity, description, and price of the goods.

An Account is a statement sent by the Seller to the Buyer at the end of a term of credit, showing the totals and dates of each Invoice and the sum total of the whole.

Each separate article of an Invoice or amount in an Account is called an *Item.*

A Detailed Account is a full statement, sent by the Seller to the Buyer at the end of a term of credit, showing the dates of delivery, the quantities, description, prices, and the sum total of the goods delivered by the Seller to the Buyer during that term of credit.

When an Account has been made out and sent in to the Buyer it is said to be 'rendered.'

SPECIMEN OF AN INVOICE.

Oxford, *Aug.* 28, 1883.

Thomas Williams, Esq.,

Bought of H. Brown & Co.,

34 Cornmarket.

	£	s.	d.
3¼ lbs. of tea at 2*s.* 8*d.* per lb.		8	8
1½ lbs. of coffee at 1*s.* 8*d.* per lb.		2	6
6¾ lbs. of loaf sugar at 5*d.* per lb.		2	9¾
¾ lb. of butter at 1*s.* 5*d.* per lb.		1	0¾
£		15	0½

SPECIMEN OF AN ACCOUNT.

Oxford, *Sept.* 30, 1883.

Thomas Williams, Esq.,

To H. Brown & Co.,

34 Cornmarket.

1883.		£	s.	d.
Aug. 28.	To goods, as per Invoice . .		15	0½
Sept. 1.	To goods, ditto . .		17	6
„ 13.	To goods, ditto . .		15	0
	£	2	7	6½

SPECIMEN OF A DETAILED ACCOUNT.

Oxford, *Sept.* 30, 1883.

Thomas Williams, Esq.,

To H. Brown & Co.,
34 Cornmarket.

1883.		£	*s.*	*d.*
Aug. 28.	3¼ lbs. of tea at 2*s.* 8*d.* per lb. .		8	8
"	1½ lbs. of coffee at 1*s.* 8*d.* per lb. .		2	6
"	6¾ lbs. of loaf sugar at 5*d.* per lb. .		2	9¾
"	¾ lb. of butter at 1*s.* 5*d.* per lb. .		1	0¾
Sept. 1.	5½ lbs. of soap at 4½*d.* per lb. .		2	2¼
"	3½ lbs. of candles at 6½*d.* per lb. .		1	10¾
"	2 lbs. of butter at 1*s.* 5½*d.* per lb.		2	11
Sept. 13.	3 pots of marmalade at 10*d.* .		2	6
"	11 lbs. of biscuit at 1*s.* 0*d.* per lb.		11	0
"	1 jar of pickles at 1*s.* 6*d.* . .		1	6
	£	2	7	6½

Exercise 118.

Make out Invoices of the following Sales, supplying names and dates of your own selection :—

(1) 27 yds. of flannel at 3*s.* 4*d.* per yard ; 32 yds. of calico at 5½*d.* per yard ; 3½ dozen of stockings at 8*s.* 4*d.* per dozen ; 6 pairs of gloves at 3*s.* 6*d.* per pair ; 4 collars at 5½*d.* each.

(2) 5 yds. of muslin at 1*s.* 2½*d.* per yard ; 39 yds. of merino at 4*s.* 3¼*d.* per yard ; 3 dozen buttons at 4¾*d.* per dozen ; 198 yds. of braid at 1*s.* 3½*d.* per dozen yards.

(3) 2 dozen pairs of gloves at 2*s.* 11½*d.* a pair ; 37 yds. of chintz at 9¾*d.* a yard ; 1 gross of buttons at 3¼*d.* a dozen ; 164 yds. of fringe at 1*s.* 3¾*d.* a dozen yards.

(4) 12 bars of soap, each weighing 3½ lbs., at 5½*d.* per pound ; 164 lbs. of tea at 30*s.* per dozen pounds ; 12 packets of candles, each weighing 6½ lbs., at 11½*d.* per pound.

(5) 10 lbs. of sugar at 5*d.* per pound ; 6 lbs. of tea at 3*s.* 8*d.* per pound ; 8 lbs. of coffee at 1*s.* 4*d.* per pound ; 12 lbs. of currants at 5¾*d.* per pound ; 10 lbs. of rice at 4½*d.* per pound ; 9 lbs. of cheese at 11*d.* per pound.

(6) 24 yds. of cloth at 3*s.* 5*d.* per yard ; 17½ yds. of floor cloth at 2*s.* 7½*d.* a yard ; 96 yds. of braid at 3*d.* a piece of 12 yds.;

177 yds. of flannel at 1*s.* 1½*d.* a yard; 4 gross of balls of cotton at 7½*d.* a dozen.

(7) 73 ft. of stout pipe at 6½*d.* per foot; 18 joints at 1*s.* 3*d.* each; 37 pipe hooks at 1*s.* 3*d.* each; 157½ yds. of wire fencing at 2*s.* 1*d.* per yard; time, 4 men, each 2 days, at 3*s.* 6*d.*

(8) 48 pairs of blankets at 12*s.* 6¾*d.* a pair; 3000 yds. of calico at 4*s.* 5*d.* per dozen yards; 936 reels of cotton at 15*s.* per gross; 15 yds. of velvet at 9*s.* 11*d.* per yard.

(9) 17 yds. of calico at 7¾*d.* per yard; 9 yds. of trimming at 2*s.* 7½*d.* per yard; 1 silk umbrella at half-a-guinea; 19 yds. of flannel at 1*s.* 10½*d.* per yard.

(10) 17 lbs. of candles at 4¾*d.* per pound; 7¾ lbs. of tea at 2*s.* 3*d.* per pound; 27 lbs. of rice at 3¾*d.* per pound; 42 lbs. of sugar at 21*s.* per hundredweight; 11½ lbs. currants at 2*d.* per pound; ¼ cwt. of soap at 3¼*d.* per pound; 220 oranges at 7½*d.* per dozen.

Make out Accounts of the following Sales, supplying names and dates:—

(11) Sirloin of beef, 18½ lbs., at 11*d.* per pound; leg of mutton, 10¼ lbs. at 10½*d.* per pound; shoulder of mutton, 7½ lbs. at 8½*d.* per pound; fillet of veal, 16 lbs. at 7½*d.* per pound; ham, 14¾ lbs. at 10*d.* per pound; 3 tongues at 2*s.* 6*d.* each.

(12) 97 lbs. of loaf sugar at 6¼*d.* per pound; 83 lbs. of moist sugar at 5½*d.* per pound; 72 lbs. of tea at 3*s.* 1¾*d.* per pound; 64 lbs. of raisins at 4½*d.* per pound; 57 lbs. of muscatels at 1*s.* 3½*d.* per pound.

(13) 7 dozen and 4 eggs at 9*d.* per dozen; 19 lbs. of soap at 5½*d.* per pound; 18 lbs. of butter at 1*s.* 2*d.* per pound; 13½ lbs. of cheese at 7½*d.* per pound; 5 lbs. of bacon at 10*d.* per pound; 8 four-pound loaves at 4½*d.* each; ½ lb. pepper at 1¼*d.* per ounce.

(14) 19 lbs. of beef at 11*d.* per pound; 13¼ lbs. of mutton at 9*d.* per pound; 9½ lbs. of mutton at 9½*d.* per pound; 6¾ lbs. of lamb at 11*d.* per pound; 17¼ lbs. of beef at 10*d.* per pound; 3½ lbs. of suet at 8½*d.* per pound.

(15) 2 quarter chests of tea (weight 101 lbs. each) at 2*s.* 3*d.* per pound; 2 do. of gunpowder tea (weight 101 lbs. each) at 3*s.* 11*d.*; 101 cwts. 19 lbs. rice at 27*s.* 6*d.* per hundredweight.

(16) 13½ yds. ribbon at 1*s.* 4½*d.* per yard; 47 yds. of calico at 8½*d.* per yard; 2½ dozen pair gloves at 2*s.* 8*d.* per pair; 5 table-cloths at a guinea and a half each; 25 reels of cotton at 1½*d.* per reel.

(17) 5½ stone of flour at 2*s.* 8*d.* per stone; 8½ lbs. cheese at 8*d.* per pound; 5½ lbs. of coffee at 1*s.* 5*d.* per pound; 19¾ lbs. pork at 9*d.* per pound; 11¼ lbs. of butter at 1*s.* 7*d.* per pound; 2¼ lbs. of tea at 3*s.* 4*d.* per pound.

(18) 12½ tons of hay at £3. 15*s.* per ton; 8¼ quarters of beans at 2 guineas a quarter; 12¾ quarters of oats at £1. 6*s.* a quarter; 5 sacks of flour at 27*s.* 6*d.* a sack.

(19) 67½ ft. of window-sashes at 9½*d.* per foot; 154 cub. ft. of oak at 4*s.* 7¾*d.* per cubic foot; 245 ft. of ash at 8¼*d.* per foot; 127½ ft. of deal at 3½*d.* per foot.

(20) 31 ells of Holland at 5*s.* 8*d.* per ell; 39 yds. of Irish cloth at 2*s.* 4*d.* per yard; 17½ yds. of muslin at 7*s.* 2½*d.* per yard; 13¾ yds. cambric at 10*s.* 6*d.* per yard; 27 yds. of linen at 2*s.* 5*d.* per yard.

(21) 6 chests of Congou tea, each 2 qrs. 17 lbs., at 3*s.* 9*d.* per pound; 13 hhds. brown sugar, each 13 cwts. 1 qr. 18 lbs., at 36*s.* 4*d.* per hundredweight; 3 casks molasses, each 7 cwts. 2 qrs. 14 lbs., at 3*s.* 9*d.* per hundredweight.

XVI. RATIO AND PROPORTION.

179. *Ratio* is the relation which two quantities of *the same kind* bear to one another with respect to magnitude, the comparison being made by considering what multiple, part, or parts the first quantity is of the second. Thus, in comparing 8 with 4, we observe that 8 has a certain magnitude with respect to 4, which it contains twice; again, in comparing 8 with 2, we see that 8 has now a different *relative* magnitude, for it contains 2 four times; or 8 is greater when compared with 2 than it is when compared with 4.

180. Of these two quantities (or 'terms') the first is called the *antecedent*, and the second is called the *consequent*.

Thus in the ratio of 5 to 7, usually expressed by two points placed between them, thus $5:7$, or as a fraction $\frac{5}{7}$, 5 is the antecedent, 7 is the consequent. We have, therefore, the following sets of interchangeable terms:—

(1) Numerator,	Denominator,	Fraction.
(2) Dividend,	Divisor,	Quotient.
(3) Antecedent,	Consequent,	Ratio.

Hence, whatever holds true of fractions must also hold true of ratios, *e.g.* **37** (1), (2), and since $\frac{4}{7}=\frac{8}{14}$, $4:7=8:14$, and $\frac{5}{6}>\frac{2}{3}$, $5:6>2:3$.

181. A ratio of *greater inequality* is that in which the antecedent is *greater* than the consequent, and is therefore an *improper* fraction, as 7 : 5 or $\frac{7}{5}$.

A ratio of *less inequality* is that in which the antecedent is *less* than the consequent and is a proper fraction, as 5 : 7 or $\frac{5}{7}$.

A ratio of *equality* is that in which the antecedent *equals* the consequent, as 7 : 7 or $\frac{7}{7}$ or 1.

182. Ratios are *compounded* by multiplying together the fractions by which they are represented, and expressing the result as a ratio.

Thus the ratio compounded of 14 : 25, 35 : 51, 17 : 49 is

$$\frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{25}}}\times\frac{\overset{\cancel{7}}{\cancel{35}}}{\underset{3}{\cancel{51}}}\times\frac{\cancel{17}}{\underset{\cancel{7}}{\cancel{49}}}=\frac{2}{15}\text{ or }2 : 15.$$

183. When two ratios are equal, the four terms are said to be in *Proportion.*

Thus, since $\frac{4}{8}=\frac{12}{24}$, the four numbers 4, 8, 12, 24 are in proportion.

We usually write this 4 : 8 :: 12 : 24 ; and read it 4 is to 8 as 12 is to 24.

In every proportion there must be four terms, two antecedents (4 and 12), and two consequents (8 and 24). The first and fourth are called the *Extremes* ; the second and third are called the *Means.*

184. When four numbers are in proportion, the product of the extremes = the product of the means. Thus if 5 : 7 :: 10 : 14, $5\times14=7\times10$—

$$5 : 7=\frac{5}{7}=\frac{5\times14}{7\times14}\ (\mathbf{37}, \text{i.})\text{ and }10 : 14=\frac{10}{14}=\frac{10\times7}{14\times7};$$

but

$$5 : 7 :: 10 : 14\ \therefore\ \frac{5\times14}{7\times14}=\frac{10\times7}{14\times7},$$

and these fractions have the same denominators, ∴ their numerators must be equal, or $5\times14=10\times7$.

Hence, if we are given three terms of a proportion we can always find the fourth. For, putting x for the missing term, we get the four following cases:—

(1) x : 7 :: 10 : 14. (2) 5 : x :: 10 : 14. (3) 5 : 7 :: x : 14.
(4) 5 : 7 :: 10 : x.

In (1) $14x=7\times10$ or $x=\frac{\cancel{7}\times\overset{5}{\cancel{10}}}{\underset{\cancel{2}}{\cancel{14}}}=5.$

In (2) $10x = 5 \times 14$ or $x = \dfrac{\overset{\cancel{1}}{\cancel{5}} \times \overset{7}{\cancel{14}}}{\underset{\cancel{2}}{\cancel{10}}} = 7.$

In (3) $7x = 5 \times 14$ or $x = \dfrac{5 \times \overset{2}{\cancel{14}}}{\cancel{7}} = 10.$

In (4) $5x = 7 \times 10$ or $x = \dfrac{7 \times \overset{2}{\cancel{10}}}{\cancel{5}} = 14.$

N.B.—In all these cases the right-hand side of the equality is the *Dividend*, x is the *Quotient*, and the other factor the *Divisor.*

185. Three numbers are said to be in *Continued Proportion* when the ratio of the first to the second is equal to the ratio of the second to the third.

Thus 6, 12, 24 are in continued proportion, $\therefore \frac{6}{12} = \frac{12}{24}$.

The second number is called a *Mean Proportional* between the first and the third.

Ex. 1. Find a mean proportional between 9 and 16.

$$9 : x :: x : 16, \quad \therefore x^2 = 144, \quad \therefore x = \sqrt{144} = 12;$$

$\therefore$ 12 is the required number.

Ex. 2. The ratio of x to y is 4 : 6; of y to z is 10 : 12; find the ratio of x to z. Here

$$\frac{x}{y} = \frac{4}{6}, \text{ and } \frac{y}{z} = \frac{10}{12};$$

$$\therefore \frac{x}{z} = \frac{x}{y} \times \frac{y}{z} = \frac{\cancel{4}}{\underset{3}{\cancel{6}}} \times \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{12}}} = \frac{5}{9} \text{ or } x : z :: 5 : 9.$$

Ex. 3. The ratio of x to y is 4 : 6; of y to z is 10 : 12; and of z to w is 14 : 16; find the continued ratio of x, y, z, w.

$$\frac{x}{y} = \frac{4}{6}; \ \frac{x}{z} = \frac{5}{9}. \ \therefore \ \frac{x}{w} = \frac{70}{144} \text{ by Ex. 2.}$$

$$x : y : z : w :: 70 : 105 : 126 : 144.$$

Ex. 4. A mixture is composed of 9 parts brandy and 1 water: 4 gallons of water are added, and the mixture contains 6 times as much brandy as water: how many gallons of brandy does it contain?

Since the quantity of brandy is the same in both cases, ∴ the quantity of water in the 1st case : the quantity of water in the 2nd case : : 6 : 9 : : 2 : 3, *i.e.* if the first contain 2 measures of water, the second will contain 3 measures, and by the problem the difference between them is 4 gallons, which is 1 measure, ∴ there are 8 gallons of water in the 1st case and 72 gallons of brandy.

Ex. 5. A greyhound pursues a hare and takes 3 leaps for every 4 leaps of the hare, but 2 leaps of the hound are equal to 3 of the hare : compare the rates of hound and hare.

Suppose the hare's leap = 2 yds., the hound's leap = 3 yds., ∴ the hound will go 9 yds. while the hare goes 8 yds., ∴ the rates are 9 : 8.

Exercise 119.

(1) The last three terms of a proportion being 4, 6, 8, what is the first term?

(2) Find a third proportional to 25 and 400.

(3) Three numbers are in continued proportion, the first is 3, the third is 1083 : what is the second?

(4) Of two equal ratios the two consequents are 2 tons 9 cwt., and £114. 16*s.*; one of the antecedents is 26 lbs., what is the other?

(5) The first three terms of a proportion are 7, 80, 3 cwt. 2 qrs. : what is the fourth?

(6) What is x when 60 : x : : 25 : 4?

(7) 16 gals. : 24 gals. : : x : £45.

(8) x : £255. : : 123 yds. : £205.

(9) x : 19 bar. 24 gals. 3 qts. of beer : : 10½*d.* : 3 qts.

(10) 2 cwt. 2 qrs. 10 lbs. : x : : 3 cwt. 1 qr. : £61. 9*s.* 1*d.*

(11) 86 cwt. 1 qr. 9 lbs. : £43. 13*s.* 1½*d.* : : 2 cwt. 2 qrs. 13 lbs. : x.

(12) 3 yds. 1 qr. 2 nls. : 5*s.* 3*d.* : : 36 yds. 3 qrs. : x.

(13) 66 cwt. 1 qr. 9 lbs. : £48. 3*s.* 8*d.* : : 51 cwt. 2 qrs. 14 lbs. : x.

(14) £162. 10*s.* 9*d.* : x : : £64. 10*s.* 6*d.* : £1290. 10*s.*

(15) 3 bush. 2 pks. : £1. 11*s.* 6*d.* : : x : £143. 11*s.*

(16) 5 gallons : 3 tuns 2 hhds. 25 gals. of wine : : £9. 6*s.* 8*d.* : x.

(17) 1 cwt. 14 lbs. : £2. 7*s.* 3*d.* : : x × 2 cwt. 1 qr. : £1767. 3*s.*

(**18**) 10*s.* 8*d.* : x :: £13067. 11*s.* 4*d.* : £24501. 13*s.* 9*d.*

(**19**) 89 tons 15 cwt. : £4. 9*s.* 9*d.* :: x : £1. 9*s.* 9*d.*

(**20**) 1000 : 128 :: x : 2 lbs. 9 ozs.

(**21**) 89 tons 15 cwt. : 29 tons 15 cwt. :: £4. 9*s.* 9*d.* : x.

(**22**) A : B :: 3 : 4; B : C :: 4 : 5; C : D :: 5 : 6; find the ratio of A, B, C, D.

(**23**) A : B :: 3 : 4; B : C :: 4 : 5; C : D :: 5 : 6; find the value of A : D.

(**24**) A gains ¼*d.* in sixpence per day, B gains 2*s.* 3*d.* in ten shillings per week, and C 150 out of 200 in 4 weeks: compare their respective gains.

(**25**) A's money : B's money :: 9 : 13; A has £10. 3*s.* 3*d.*: how much have they together?

(**26**) A : B : C : D :: 3 : 7 : 9 : 11; A = £35049. 12*s.*: find value of A + B + C + D.

(**27**) A : B : C :: 15 : 2 : 3; C = 1 cwt 3 qrs. 5¾ lbs.: find value of A + B + C.

(**28**) In England gunpowder is made of 75 parts of nitre, 10 of sulphur, 15 of charcoal; in France, of 77 of nitre, 9 of sulphur, and 14 of charcoal: if a ton of each be mixed together, what weight of nitre will there be in the compound?

(**29**) A : B : C :: one guinea : 3 half-crowns : 4*d.*, and C = 4*d.*: find the value of 16A + 13B + 19C.

(**30**) £114. was divided between A, B, C, D; A had twice as much as D, and C twice as much as A; B had thrice as much as C: what was D's share?

(**31**) If in one year one in every 75 of the inhabitants die, what is the population of a town where the number of deaths is 896?

(**32**) A grocer has 1 cwt. of mixed coffee and chicory, mixed in proportion of 5 : 2; 16 lbs. of chicory are added: what is now the proportion?

(**33**) A : B :: 3 : 4; B : C :: 4 : 5; A : D :: 9 : 18; find C : D.

(**34**) A : B :: 3 : 4; C : D :: 5 : 6; A : D :: 1 : 2; find B : C.

(**35**) A : B : C : D :: 3·125 qrs. : 1 cwt. : £11·625. : x; find x when B = 1 cwt.

(**36**) Find B when A : B :: B : C. (i.) When A = 9, C = 16. (ii.) When A = 4½, C = 12½.

(37) If the two antecedents of a proportion be $\frac{11}{14}$ of a cwt. and $\frac{2}{5}$ of a ton, and the first consequent be £5. 6*s*., what is the second?

(38) Find x when x : one minute :: $\frac{2}{3}$ of $\frac{19}{20}$ of 118·75 miles : $8\frac{2}{3}$ hours.

(39) A, B, C met. A and B were provided with meat, C with money; C divided his money between A and B in proportion to the quantity of meat he had received from each. If all received 7 lbs. of meat each, B 20*d*., and A 8*d*., how much meat had A at first?

(40) Find x when 2 qrs. $6\frac{1}{3}$ lbs. : $\frac{2}{3}$ of $\frac{1}{9}$ of £63. 19*s*. :: $\frac{1}{3}$ of $4\frac{1}{7}$ of 12 cwts. 1 qr. 11 lbs. : x.

(41) Find the ratio compounded of $2\frac{1}{2}$ of $4\frac{1}{7} : 3\frac{1}{3} \times \frac{17}{19}$ and $1\frac{5}{29} : \frac{1}{9}$.

(42) Prove that the ratio compounded of $3\frac{1}{4} \times 2\frac{1}{2} : 1\frac{1}{2} - \frac{18}{19}$, $\frac{21}{38} : \frac{3}{4} \div \frac{2}{5}$ is the same as that of ·01 : ·0006 : 1 : 15.

(43) $a : b$ :: 3 ac. 2 rds. 12 po. : 7 ac. 3 rds. 36 po. :: $c : d$; c is 15 qrs. 3 bush. 2 pks.: what is d?

(44) $a = {}$·23 of $6·\dot{3}$; $b = \dfrac{76}{112\frac{1}{2}}$; $d = 11\frac{3}{4}$; if the difference between $\dfrac{a}{b}$ and $\dfrac{c}{d}$ be $15\frac{17}{96}$, what is c?

(45) I mix milk and water in the proportion of 4 : 5, and by adding 9 gallons of each the mixture becomes 7 : 8; how many gallons of milk did the original mixture consist of?

(46) If the antecedent of a ratio be $3\frac{1}{3}$ inches, and the ratio be $\frac{7}{126720}$, what is the consequent?

(47) ·003261 : x :: ·78264 : £1.; find x.

(48) 6 tons 3 cwts. 1 qr. : £149. 3*s*. $8\frac{1}{8}$*d*. :: x : 1 cwt.; find x.

(49) The second, third, and fourth terms of a proportion are 6*s*. $3\frac{1}{4}$*d*., $219\frac{129}{307}$, 1 : what is the first?

(50) What quantity bears the same ratio to 3 qrs. 2 nls. that 7 does to 48?

(51) $(2\frac{1}{3} + 1\frac{1}{9}) \div 23·\dot{6} : x :: 179417 : (1\frac{1}{3} - \frac{18}{19}) \div (6\frac{1}{3} \div 2\frac{1}{7})$; find x.

(52) Express as a decimal 3 mi. 6 fur. : 4 fur. 36 po.

(53) 2 qrs. 18 lbs. 14 ozs. : x :: £14. 17*s*. $11\frac{9}{32}$*d*. : 3*s*. $11\frac{3}{4}$*d*.; find x.

(54) 64 cub. yds. 23 ft. 1215 in. : 1 cub. yd. :: £572. 0*s*. $5\frac{8}{27}$*d*. : x; find x.

(55) If $1\cdot\dot{8}28571\dot{4} : x :: 1 :$ 2 mi. 1 fur. 20 ft., find x.

(56) A cask of 84 gallons consists of 7 parts brandy and 5 parts water: how much brandy must be added that it may consist of 10 parts brandy and 7 water?

(57) A hare pursued by a greyhound was 130 yards before him at starting, and is caught after running 325 yards: compare their rates.

(58) Two numbers, whose sum is 45, are in the ratio of 2 : 3; what number must be added to each that their ratio may become 3 : 4?

(59) How is the ratio of 9 : 16 affected by adding 3 to both terms?

(60) Two vessels, A and B, each contain a mixture of wine and water—A in the ratio of 3 : 2, and B in the ratio of 7 : 3. If 2 gallons be taken from A and 14 from B, how many gallons of wine will there be in the mixture?

(61) A post is $\frac{1}{5}$ in the earth, $\frac{3}{7}$ in the water, and 13 feet out of the water: find its length.

(62) After paying away $\frac{1}{5}$ and $\frac{1}{6}$ of my money, I had £190. left: what had I at first?

(63) The tail of a fish is 9 lbs.; the head weighs as much as the tail and half the body, and the body weighs as much as the head and tail together: what is the weight of the fish?

(64) A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3; but 2 of the greyhound's = 3 of the hare's: how many leaps must the greyhound take to catch the hare?

XVII. CAUSE AND EFFECT.

186. The Rule by which we shall work questions usually given under the head of Rule of Three, or Double Rule of Three, is this:—

1st effect : 1st cause :: 2nd effect : 2nd cause,

the word 'effect' usually meaning work done, food or something consumed, money spent. The method will be gradually explained in the following examples.

Ex. 1. If 6 bushels of wheat cost £12., what will be the cost of 1 bushel?

Here the 1st effect is £12., 1st cause is 6 bushels; the 2nd effect is x, the 2nd cause is 1 bushel;

$\therefore$ by Rule, £12. : 6 bushels :: x : 1 bushel.

Then by (**184**) $6 \times x = £12$;

$\therefore x = \frac{12}{6} = £2$. *Ans.*

Ex. 2. If 57 oxen are bought for £513., how many can be bought for £387. at the same rate?

1st effect is £513., 1st cause 57 oxen; 2nd effect £387., 2nd cause x;

$\therefore$ £513. : 57 oxen :: £387. : x;

$\therefore 513 \times x = 57 \times 387$;

$$\therefore \quad x = \frac{\cancel{57} \times \overset{43}{\cancel{387}}}{\underset{9}{\cancel{513}}} = 43 \text{ oxen. } \textit{Ans.}$$

Exercise 120.

(1) If 5 chairs cost £2. what would be the cost of 25 such chairs?

(2) If 90 horses cost £30. a week, how many can I keep for £6. a week?

(3) If 30 ounces of gold cost £105., what is the value of 18 ounces?

(4) If the fare for 24 miles be 6 shillings, how far can I travel for 17 shillings?

(5) 45 articles cost me £50., what will 54 cost me?

(6) If 375 articles are sold for 12 shillings, how many may be sold for 32 shillings?

(7) 8 lbs. of coffee cost 10 shillings: how much must I give for 64 lbs.?

(8) If 15 days' wages come to 35 shillings, what will 9 days' wages be?

(9) 42 acres cost £66.: what number can I rent for £143.?

(10) 35 articles cost £100.: what will 63 cost?

(11) £88. buy 40 dozen of wine: what sum will buy 15 dozen?

(12) What will 45 sheep fetch at the rate of £51. for 27?

187. In the above exercise the 'effect' was money spent; we will now give an example where the 'effect' is food or something consumed.

Ex. If 6 bushels feed 12 horses, how much will 36 horses require?

Here the 1st effect is 6 bushels, the 1st cause 12 horses, the 2nd effect x, the 2nd cause 36 horses;

$$\therefore 6 \text{ bushels} : 12 \text{ horses} :: x : 36 \text{ horses};$$
$$\therefore 12 \times x = 36 \times 6,$$

or
$$x = \frac{\overset{3}{\cancel{36}} \times 6}{\cancel{12}} = 18 \text{ bushels.}$$

Exercise 121.

(1) If 35 bushels feed 7 horses, how many will be required for 18 horses?

(2) If 35 bushels last 20 days, how long will 91 bushels last?

(3) If 16 horses consume 19 quarters of corn, how much will 48 horses consume?

(4) If 20 loaves are required for a family of 9 persons, how many will you want for 81?

(5) If 90 acres of grass feed 30 horses, how many will be required for 6 horses?

(6) If 28 gallons of beer last 14 days, how long will 34 gallons last?

(7) If 30 cannon consume 250 barrels of powder, how much will 150 cannon require?

(8) 12 pecks of malt are required to brew 36 gallons of beer: what is this for 24 gallons?

(9) If 57 oxen require 513 cwt. of provisions, how many can be kept on 387 cwt.?

(10) If 939 soldiers consume 351 quarters of wheat, how many will consume 1404 quarters?

(11) How many bushels will serve 72 people if 4 bushels serve 6 people?

(12) If 858 men consume 234 quarters of wheat, how many quarters will be required for the consumption of 979 men?

188. The next kind of question we have to consider is that in which the 'effect' is 'work done.' We will give 2 examples, from which it will be seen that the method of working is the same as in the two previous exercises.

EX. I. If 12 horses plough 96 acres, how many horses will plough 64 acres?

The 1st effect is 96 acres, the 1st cause 12 horses, the 2nd effect 64 acres, the 2nd cause x;

$$\therefore 96 \text{ acres} : 12 \text{ horses} :: 64 \text{ acres} : x;$$
$$\therefore 96 \times x = 64 \times 12,$$

or $$x = \frac{\overset{8}{64} \times \cancel{12}}{\underset{8}{\cancel{96}}} = 8 \text{ horses.} \quad \textit{Ans.}$$

EX. 2. If 96 acres can be ploughed in 6 days, how many can be ploughed in 8 days?

The 1st effect is 96 acres, the 1st cause 6 days, the 2nd effect is x, the 2nd cause 8 days;

$$\therefore 96 \text{ acres} : 6 \text{ days} :: x : 8 \text{ days};$$
$$\therefore 96 \times 8 = 6 \times x;$$
$$\therefore x = \frac{\overset{16}{\cancel{96}} \times 8}{\cancel{6}} = 128 \text{ acres.} \quad \textit{Ans.}$$

Exercise 122.

(1) If a man walk 62 miles in 4 days, how far will he walk in 6 days?

(2) If 12 men reap 4 acres, how much will 33 men reap?

(3) If 40 acres can be mowed by 8 men, how many acres will be mowed by 24 men?

(4) If I require 5 days to travel 150 miles, how long shall I require to travel 600 miles?

(5) If 6 bricklayers build a wall 18 yards long, how many will build one 33 yards long?

(6) 8 men plant a field of 10 acres: how many will be required for 15 acres?

(7) If 72 men dig a trench, how many will be required for one three times as great?

(8) If 60 men turn out 20 chairs, how many will turn out 30 chairs?

(9) If 150 weavers manufacture 6000 yards of calico, how many yards will 200 produce?

(10) In 8 days I travel 656 miles: how far can I go in 12 days?

(11) If 8 men build 24 square yards of walling, how many will build 18 square yards?

(12) If 12 men build a wall 60 feet long, how many will it take to build a wall 100 feet long?

189. Hitherto the 'effect' has been produced by one cause only; if there be more than one cause, the final cause is measured by the product of the several causes. Thus:—

Ex. 1. If 2 loads of hay will serve 3 horses for 4 weeks, how long will 5 loads serve 6 horses?

1st effect is 2 loads, the 1st cause 3 horses × 4 weeks;
2nd effect is 5 loads, the 2nd cause x weeks × 6 horses;

$\therefore$ 2 loads : 3 horses × 4 weeks :: 5 loads : x weeks × 6 horses;
$\therefore x \times 2 \times 6 = 5 \times 3 \times 4;$

$$\therefore x = \frac{5 \times \cancel{3} \times \overset{\cancel{2}}{\cancel{4}}}{\cancel{2} \times \underset{\cancel{3}}{\cancel{6}}} = 5 \text{ weeks. } \textit{Ans.}$$

Ex. 2. If 12 horses can plough 96 acres in 6 days, how many horses will plough 64 acres in 8 days?

1st effect is 96 acres, the 1st cause 12 horses × 6 days;
2nd effect is 64 acres, the 2nd cause x horses × 8 days;

$\therefore$ 96 acres : 12 horses × 6 days :: 64 acres : x horses × 8 days;
$\therefore x \times 8 \times 96 = 12 \times 6 \times 64;$

$$\therefore x = \frac{\cancel{12} \times 6 \times \overset{\cancel{8}}{\cancel{64}}}{\cancel{8} \times \underset{\cancel{8}}{\cancel{96}}} = 6 \text{ horses. } \textit{Ans.}$$

Exercise 123.

(1) If 8 men set up a work of 6 sheets in 10 days of 9 hours long, how many will set up a work of 8 sheets in 16 days of 12 hours long?

(2) If 12 men build 18 square yards of walling in 3 days, how many will build 24 square yards in 6 days?

(3) If a man can travel 450 miles in 20 days of 6 hours each, how many hours a day must he walk to complete 360 miles in 12 days?

(4) If 7 men mow 22 acres in 8 days, working 11 hours a day, how many acres will 12 men, working 10 hours a day, mow in 84 days?

(5) 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days: how many barrels will be consumed in 2 days by 66 cannon, firing 40 rounds an hour for 5 hours a day?

(6) If I pay £120. for the hire of 4 horses for 5 months, how long can I hire 3 horses with £18.?

(7) If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount if they work 25 days of 8 hours each?

(8) If a family of 7 persons can live on £140. for 28 weeks, how long can a family of 9 persons live on £135.?

(9) If the wages of 4 men for 12 days be £6., what would be the wages of 12 men for 10 days?

(10) If a man earn 30 shillings in 8 days, working 9 hours a day, in how many days will he earn 50 shillings, working 10 hours a day?

(11) How many horses would be required to plough 117 acres of land in 35 days if 10 horses can plough 13 acres in 7 days?

(12) If I can dig a trench 200 yards long, 3 yards wide, and 2 yards deep, for £1080., how wide may a trench 100 yards long, 3 yards deep, be made for £2160.?

190. In some cases the 'effect' itself is a product.

Ex. If 288 men in 5 days of 11 hours each can dig a trench 264 yds. long, 5 ft. wide, and 2 ft. deep, in how many days of 9 hours each can 112 men dig a trench 420 yds. long, 8 ft. wide, and 3 ft. deep?

Here 1st effect is 264 yds. × 5 ft. × 2 ft., 2nd effect is 420 yds. × 8 ft. × 3 ft.;

1st cause is 288 men × 5 days × 11 hours, 2nd cause is 112 men × x days × 9 hours;

$$\therefore 264 \times 5 \times 2 : 288 \times 5 \times 11 :: 420 \times 8 \times 3 : 112 \times x \times 9;$$

$$\therefore 264 \times 5 \times 2 \times 112 \times x \times 9 = 288 \times 5 \times 11 \times 420 \times 8 \times 3;$$

$$\therefore x = \frac{\overset{\overset{6}{\cancel{12}}}{\cancel{288}} \times \cancel{5} \times \cancel{11} \times \overset{\overset{10}{\cancel{30}}}{\cancel{420}} \times \cancel{8} \times \cancel{3}}{\underset{\cancel{24}}{\cancel{264}} \times \cancel{5} \times \cancel{2} \times \underset{\cancel{14}}{\cancel{112}} \times \underset{\cancel{3}}{\cancel{9}}} = 60 \text{ days.}$$

Exercise 124.

(1) If a field 792 yds. long, 55 yds. broad, can be mown by 22 men in 15 hours, how long will it take 33 men to mow a field 121 yds. broad, 180 yds. long?

(2) If 5 men can reap a field 800 ft. long and 700 ft. broad in 7 days of 7 hours each, in how many days of 12 hours each can 7 men reap a field 1800 ft. long and 960 ft. broad?

(3) If 6 men in 5 days, working 8 hours a day, can reap a field 1000 ft. broad and 1280 ft. long, what is the length of a field whose breadth is 800 ft. which 5 men can reap in 6 days, working 6 hours a day?

(4) If 18 men can dig a trench 200 yds. long, 3 yds. wide, and 2 yds. deep in 6 days of 10 hours each, in how many days of 8 hours each will 5 men dig a trench 100 yds. long, 4 yds. wide, 3 yds. deep?

(5) If 20 men build a wall 800 ft. long, 10 ft. high, and 18 in. thick in 14 days, working 8 hours a day, how thick a wall could 15 men build 450 ft. long and 15 ft. high in 21 days, working 9 hours a day?

(6) If 252 men can dig a trench 210 yds. long, 3 yds. wide, and 2 yds. deep in 5 days of 11 hours each, in how many days of 9 hours each will 22 men dig a trench 420 yds. long, 5 yds. wide, and 3 yds. deep?

(7) If 20 masons build a wall 50 ft. long, 2 ft. thick, and 14 ft. high in 12 days of 7 hours each, how high will 60 masons in 64 days of 10 hours each build a wall 500 ft. long and 4 ft. thick?

(8) 125 men make an embankment 100 yds. long, 20 ft. wide, and 4 ft. high in 4 days, working 12 hours a day: how many hours a day must 2400 men work to build one 1000 yds. long, 16 ft. wide, and 6 ft. high in 3 days?

(9) 7 men, working 16 days, can mow a field 1320 yds. long and 880 wide: what will be the length of a side of a field 1320 yds. broad which 4 men can mow in 42 days?

(10) 12 men, working 12 hours a day for 24 days, build a wall 60 ft. long, 4 ft. thick, and 20 ft. high: how long a wall, 3 ft. thick and 12 ft. high, can 18 men build in 18 days, working 8 hours a day?

(11) If 3 men, working 11 hours a day, can reap a field 220 yds. long and 440 yds. broad in 11 days, how many men, working 12 hours a day, will reap a field 360 yds. long and 320 yds. broad in 4 days?

(**12**) If 20 men can dig a trench 100 yds. long, 4 yds. wide, and 3 yds. deep in 40 days of 10 hours each, in how many days of 9 hours each would 10 men dig a trench 45 yds. long, 6 yds. wide, and 4 yds. deep?

191. In all cases where the 'effects' are the same the 'causes' are equal.

Ex. 1. What time would 24 men, working 8 hours a day, require to build a wall which 16 men, working 6 hours a day, can build in 9 days?

Here the 'effect' in either case is the wall built.

The 1st cause is 24 men × 8 hours × x days; the 2nd cause is 16 men × 6 hours × 9 days, and these must be equal.

$$\therefore 24 \times 8 \times x = 16 \times 6 \times 9;$$

or

$$x = \frac{\overset{\cancel{2}}{\cancel{16}} \times \cancel{6} \times 9}{\underset{\underset{2}{\cancel{4}}}{\cancel{24}} \times \cancel{8}} = \frac{9}{2} = 4\tfrac{1}{2} \text{ days.}$$

Ex. 2. How long must I lend a man £800. for the loan of £600. for 4 months?

Here the effect is the debt, which is the same in both cases, *i.e.*, 1st cause = 2nd cause, or

$$£600. \times 4 \text{ months} = £800. \times x \text{ months};$$

or

$$x = \frac{\overset{3}{\cancel{600}} \times \cancel{4}}{\underset{\cancel{200}}{\cancel{800}}} = 3 \text{ months.}$$

Exercise 125.

(**1**) In how many days might a house be built by 42 men, working 10 hours a day, if 12 men, working 8 hours a day, can build it in 70 days?

(**2**) 49 men can do a piece of work in 130 days of 8 hours each: how many men will be required to do the same in 26 days of 10 hours each?

(**3**) 4 men, working 8 hours a day, can copy a manuscript in 30 days: how long will 8 men, working 10 hours a day, be in copying it?

(4) If 48 men can do a piece of work in 16 days of 9 hours each, in how many days of 12 hours each can 64 men accomplish the same?

(5) 60 men can do in 18 days what 45 men take 30 days of 12 hours each to do: how many hours a day did the first set of men work?

(6) How many days of 10 hours each would 60 men take, when 45 men require 40 days of 12 hours each?

(7) What is the depth of a well 15 ft. broad, 49 ft. long, which contains as many cubic feet of water as one 7 ft. deep, 21 ft. broad, and 30 ft. long?

(8) How many yards of carpet 4 ft. wide shall I require for a floor 18 ft. long, 16 ft. broad?

(9) A lends B £700. for 9 months: how much should B lend A 7 months in return?

(10) What must be the length of a beam 2 ft. broad and 4 in. thick, which weighs as much as a similar beam 3 ft. broad, 8 in. thick, and 20 ft. long?

(11) What time should 24 men take to perform a piece of work which 8 men can perform in 12 days?

(12) How much tea at 5*s.* per lb. must be given in exchange for 25 lbs. of coffee at 2*s.* a lb.?

(13) A field was mown by 50 men in 10 days of 12 hours each: in how many days of the same length would it be mown by 20 men?

(14) If a field contain sufficient pasture for 7 oxen 36 days, how many oxen would it serve 28 days?

(15) A farmer borrowed 16 qrs. of wheat of a neighbour when wheat was 72*s.* a quarter, and paid him back 18 qrs.: to what price had the wheat fallen?

(16) A garrison has provisions to last 3000 soldiers 5 months: how many must be sent away in order that it may last 8 months?

192. Hitherto the terms have consisted of *simple quantities*; we will now work examples in which *compound quantities* occur. Care must be taken before the proportion is stated that all corresponding expressions are reduced to the same denomination.

EX. 1. If the carriage of 1 cwt. 3 qrs. 21 lbs. for 52¼ miles come to 17*s.* 5*d.*, what will be charged for 2½ tons for 46½ miles?

1 cwt. 3 qrs. 21 lbs. = 217 lbs.; 2½ tons = $\frac{5}{2} \times 20 \times 112$ lbs.; 52¼ miles = $\frac{209}{4}$ miles; 46½ miles = $\frac{93}{2}$ miles; 17*s.* 5*d.* = 209*d.*

$$\therefore\ 209d. : 217 \times \tfrac{209}{4} :: x : \tfrac{5}{2} \times 20 \times 112 \times \tfrac{93}{2};$$

$$\therefore\ x \times 217 \times \tfrac{209}{4} = 209 \times \tfrac{5}{2} \times 20 \times 112 \times \tfrac{93}{4};$$

$$\therefore\ x = \frac{\cancel{209} \times 5 \times \overset{10}{\cancel{20}} \times \overset{16}{\cancel{112}} \times \overset{3}{93} \times \overset{2}{\cancel{4}}}{\underset{\cancel{31}}{\cancel{217}} \times \cancel{2} \times \cancel{209} \times \cancel{2}} = 4800d. = £20. \quad \textit{Ans.}$$

Ex. 2. If 5 horses require as much corn as 8 ponies, and 15 qrs. last 12 ponies for 64 days, how long may 25 horses be kept for £41. 5*s.*, when corn is 22*s.* a quarter?

Here the 1st effect is 15 × 22 shillings, the 1st cause is 12 ponies × 64 days; the 2nd effect is 825 shillings, the 2nd cause is 25 horses × x, or 40 ponies × x.

$$\therefore\ 15 \times 22 : 12 \times 64 :: 825 : 40 \times x;$$

$$\therefore\ 15 \times 22 \times 40 \times x = 12 \times 64 \times 825;$$

$$\therefore\ x = \frac{\overset{6}{\cancel{12}} \times \overset{8}{\cancel{64}} \times \overset{\cancel{165}\ \cancel{33}\ \cancel{3}}{\cancel{825}}}{\underset{\cancel{3}}{\cancel{15}} \times \underset{\cancel{2}}{\cancel{22}} \times \underset{\cancel{5}}{\cancel{40}}} = 48 \text{ days.} \quad \textit{Ans.}$$

Ex. 3. If the twopenny loaf weigh 15 oz. when wheat is at 8*s.*, how is wheat selling when the 3½*d.* loaf weighs 2 lbs.?

The 1st effect is 2*d.*, the 1st cause is 15 oz. × 8*s.*; the 2nd effect is 3½*d.*, the second cause is 2 lbs. × x.

$$\therefore\ 2 : 15 \times 8 :: 3\tfrac{1}{2} : 2 \times 16 \times x;$$

$$\therefore\ 2 \times 2 \times 16 \times x = 15 \times 8 \times 3\tfrac{1}{2};$$

$$\therefore\ x = \frac{15 \times \cancel{8} \times 7}{\cancel{2} \times \cancel{2} \times 16 \times \cancel{2}} = \frac{105}{16} = 6s.\ 6\tfrac{3}{4}d. \quad \textit{Ans.}$$

The following exercise is a miscellaneous set of examples on the previous rules.

Exercise 126.

(1) If the rent of a house for 3 months 2 weeks 4 days be £24. 17*s.* 3*d.*, what is that per month of 4 weeks?

(2) A can mow 1⅗ acres in 8⅔ hours, and B ⅞ acres in 2¼ hours: how many acres will they mow together in 21 hours?

(3) A does ⅗ of a task in 12 days, B can do the whole in 9 days: how many days must A ask B to help him to finish the task?

(**4**) A shilling loaf weighs 3 lbs. when wheat is at 60*s*. per qr.: what should be the price of a 12 oz. loaf when wheat is 40*s*. per qr.?

(**5**) 3 cows eat as much as 7 sheep. 40 cows can be pastured for 12 weeks for £50. 10*s*.: what would be the cost for 6 weeks' pasturage of 76$\frac{24}{101}$ sheep?

(**6**) If the board and lodging of a scholar cost eleven guineas per quarter of a year, and his education £3. 10*s*., how many scholars can be maintained one year for £2408?

(**7**) 15$\frac{3}{4}$ lbs. cost £2. 9*s*.: how many hundredweights can be bought for £65. 6*s*. 8*d*.?

(**8**) How many acres of land will 4 men plough in 3 days, working 8 hours a day, if they will plough 18 acres in 8 days working 4$\frac{1}{2}$ hours a day?

(**9**) If 6$\frac{7}{8}$ acres cost £30$\frac{3}{5}$., 13$\frac{3}{4}$ acres of another kind of land cost £48. 19*s*. 2$\frac{2}{5}$*d*., how many acres of the latter = 2 of the former?

(**10**) If 13$\frac{1}{2}$ lbs. cost £$\frac{15}{8}$., how many parcels each containing 5$\frac{5}{8}$ lbs. can I buy with £2. 0*s*. 7$\frac{1}{2}$*d*.?

(**11**) $\frac{2}{3}$ of a ship cost £3333$\frac{1}{3}$.: what decimal fraction of it will cost £4000.?

(**12**) 5 cows eat as much as 11 sheep. 25 cows and 50 sheep consume 2663 cwt. 3 qrs. 9$\frac{19}{169}$ lbs. of hay in 35 weeks: how many sheep with 16 cows can I keep 10 days on 140 cwts. of hay?

(**13**) If the fourpenny loaf weigh 3·35 lbs. when wheat is sold for 4·75*s*. per bushel, how many pounds of bread can I buy with 13*s*. 4*d*., when wheat is at 13·4*s*. a bushel?

(**14**) An embankment 300 yds. long, 3 yds. wide, and 4 ft. high, is made by 250 men in 81 days of 8 hours each; half as many men again, working 12 hours a day, make another embankment 6 ft. wide and 2$\frac{1}{2}$ ft. high, which presents double the difficulty of the former, in 20 days: how long is it?

(**15**) If 2 ozs. 5 dwts. of silver cost 7*s*. 10$\frac{1}{2}$*d*., how many silver plates, each weighing 3 lbs. 2$\frac{2}{3}$ ozs., cost £27. 1*s*. 4*d*.?

(**16**) 10 men can make 72 pairs of boots in 18 days, working 9 hours a day: how many hours a day must 12 men work to make 130 pairs in 20$\frac{5}{16}$ days?

(**17**) If $\frac{1}{3}$ of $\frac{9}{10}$ of an estate be worth £1000., what is the value of it?

(**18**) If 63 acres produce 100 quarters of wheat, how many quarters will 840 acres produce, taking the produce per acre of the former to be to the latter as two to three?

(19) If 6 bricklayers build a wall 18 yds. long in 3 days of 9 hours each, how long a wall of the same height and thickness could 3 men build in 9 days of 11 hours each?

(20) If $\frac{3}{7}$ of a ship be worth £125. 5*s.*, what portion of it is worth £36. 10*s.* 7½*d.*?

(21) If 12 cwts. 2 qrs. be carried 17⅕ miles for £1., what weight should be carried 51⅗ miles for 18*s.*?

(22) 28$\frac{2}{7}$ women can be kept 14 months for £2750.: how many men can be kept eight-sevenths of that time for £4000., the cost of two men being equal to that of three women?

(23) What is the tax on £335. 7*s.* 6*d.* at 3*s.* 6*d.* in the £.?

(24) If 188 tons 2 cwt. cost £238. 0*s.* 6*d.*, what weight can I buy with £297. 10*s.* 7½*d.*?

(25) The pasture of 9 cows, 15 sheep, and 3 horses for 16 weeks costs £150.: how many sheep can I keep 4 weeks 4⅔ days for £200., if 1 horse eat as much as 2 cows, or 5 sheep?

(26) What sum will gain £24. 10*s.* in 15 months, if £1575. gain £42. in 8 months?

(27) What should be the price of a loaf weighing 5 lbs. 6⅘ ozs. when wheat is at 40*s.* a quarter, if the 3*d.* loaf weigh 2¼ lbs. when wheat is at 37*s.* per quarter?

(28) 125 men can make an embankment 100 yds. long, 20 ft. wide, and 4 ft. high, in 4 days, working 12 hours a day: how many hours a day must 2400 men work to make an embankment 1000 yds. long, 16 ft. wide, and 6 ft. high in 3 days?

(29) If 22 men earn £1. 0*s.* 8$\frac{5}{12}$*d.* in 4 days, how much will 32 men earn in 7½ days?

(30) Travelling 10$\frac{2}{7}$ hours a day, I perform a journey in 27⅜ days: how many hours a day must I travel to perform the same in 20$\frac{256}{385}$ days?

(31) 24 men, working 8 hours a day, do a piece of work in 10 days: how many men will be required to do the same in 15 days of 8 hours each?

(32) If either 4 oxen or 6 cows eat up the grass of a meadow in 54 days, how many cows with 10 oxen will it last 18 days?

(33) What should be the cost for carriage of 3·5 tons, for 96 miles, if 2 tons can be taken 27 miles for £4·5?

(34) How many hours a day must 45 men work to dig a trench 90 yds. long, 2 ft. 6 in. broad, and 3 ft. deep, in 27 days, if 135 men dig a trench 48 yds. long, 1 ft. 3 in. broad, and 3 ft. 9 in. deep, in 5 days of 6 hours each?

(**35**) How many compositors would set up a work of 70 pages of 36 lines in a page and 40 letters in a line, in 28 days, working 10 hours a day, if 11 would set up a work of 120 pages of 44 lines in a page and 45 letters in a line, in 20 days of 9 hours each?

(**36**) If 100 men drink £20. worth of wine at 4*s*. 6*d*. per bottle, what is the price of a bottle when 324 men drink £72. worth under the same circumstances?

(**37**) If 36 horses eat 19 bus. of oats in 13 days, how long would 38 bus. last 18 horses?

(**38**) How much tea can I buy with £14., when 3 qrs. 7 lbs. cost £11. 7*s*. 6*d*.?

(**39**) What sum will gain £37. 10*s*. in 12 months if £560. gain £45. in 9 months?

(**40**) If ·125 of a ton be carried 12·5 miles for 1·25 of one shilling, how many miles can 12½ tons be carried for 37·5 shillings?

(**41**) 4 needlewomen can make 17 shirts in 20 days, working 8½ hours a day: how many needlewomen can make 6 shirts in 4 days of 12 hours each?

(**42**) If I borrow £300. for 8 months when money is worth 5 per cent., how long ought I to lend in return £266. 13*s*. 4*d*. when money is worth 3 per cent.?

(**43**) If $\frac{3}{16}$ of $\frac{5}{36}$ of a ship be worth £75., how much is $\frac{5}{12}$ of it worth?

(**44**) What weight can I have carried 120 miles for 1*s*. 8*d*., when the cost of 16 lbs. 5 ozs. for 90 miles is 3*s*. 9*d*.?

(**45**) If 6 men can pick 84 bushels of apples in 3 days, in how many days will 5 men pick 210 bushels?

(**46**) How many tons can be carried 100 miles for £132. 3*s*. 9*d*., when £5. 5*s*. 9*d*. is charged for carrying 94 tons 20 miles?

(**47**) If a man walk 600 miles in 24 days, walking 10 hours a day, how far would he walk in 25 days of 8 hours each?

(**48**) If a man walk $\frac{3}{8}$ of a mile in $\frac{1}{6}$ of $1\frac{3}{4}$ hours, how far would he walk in 2 hrs. 8 min. 20 sec.?

(**49**) If £100. make £5. in one year, how much would £25. 2*s*. 6*d*. make in 5 years?

(**50**) If 30 cannon consume 250 barrels of powder in 4 days, how long will 3750 barrels last 150 cannon?

(**51**) If the quartern loaf be sold for 10¼*d*. when the price of wheat is 65*s*., what should be its price when wheat is 47*s*. per quarter?

(52) 5 horses eat as much as 8 ponies; 15 quarters of corn last 12 ponies for 64 days: how many horses can be kept 48 days for £41. 5*s.* when corn is 22*s.* a quarter?

(53) What is the cost per pound of beef when 3 cwt. 3 qrs. 3 lbs. cost £8. 14*s.* $0\frac{9}{16}$*d.*?

(54) If the corn for 4 horses, when oats are at 22*s.* 6*d.*, is £12. 18*s.*, what is the price of oats when the cost of corn for 5 horses is £15. 1*s.*?

(55) If I pay £18. 8*s.* $6\frac{69}{73}$*d.* for the loan of £312. 10*s.*, for 1 y. 35 w. 5 d., what must I pay for the loan of £100. for 1 year?

(56) If 8 ozs. of bread are sold for 6*d.* when wheat is £15. a load, what should be the price of wheat when 12 ozs. are sold for 4*d.*?

(57) If 3 horses eat as much as 50 sheep, and the cost of keeping 21 horses and 217 sheep for 10 days be £56. 8*s.* 4*d.*, for how many days may I keep 9 horses and 60 sheep for £56. 8*s.* 4*d.*?

(58) If $\frac{2}{57}$ of $3\frac{1}{6}$ of $7\frac{3}{5}$ of $\frac{4}{7}$ of 36 lbs. cost 12*s.* $6\frac{1}{4}$*d.*, what weight can I buy with £1305. 18*s.* $0\frac{1}{4}$*d.*?

(59) If 18 men can dig a trench 150 ft. long, 6 ft. broad, and 4 ft. 6 in. deep, in 12 days, how many men will be required to dig a trench 210 ft. long, 5 ft. broad, and 4 ft. deep in 14 days?

(60) If 7 women do a piece of work in 10·625 days of 9·6 hours each, how many will be required to do twice the same work in 47·6 days of 10 hours each?

(61) How much shall I be able to pay on a debt of £1537. 3*s.* 4*d.*, when I can only pay £3. 2*s.* $4\frac{1}{2}$*d.* on £276. 11*s.* 6*d.*?

(62) How many guineas per ton is hay worth when £40. 9*s.* $9\frac{3}{4}\,\frac{13}{25}$*d.* is paid for the supply of hay for 17 oxen and 40 sheep for 31 days, supposing 6 oxen or 13 sheep eat 26 cwts. 3 qrs. 20 lbs. of hay in 29 days?

(63) How many vibrations will a pendulum make in 2·75 seconds, that vibrates 172800 times in one day?

(64) When corn is at 44*s.* per quarter the value of the produce of $13\frac{1}{2}$ acres is £108. 18*s.*: what is the price of corn when 16 acres produce £134. 4*s.*?

(65) If 7 men can do as much work as 11 women, how many men would do in $10\frac{20}{187}$ days what 30 women require 9 days for?

(66) 12 men build a wall 60 ft. long, 4 thick, and 20 high, in 24 days, working 12 hours a day: how many hours a day must

18 men work to build a wall 100 ft. long, 3 thick, and 12 high, in 18 days?

(67) If 5 boys or 3 men can dig 28 yds. of earth in 10 hours, how many boys must be employed with eight men to dig 120 yds. in $13\frac{347}{406}$ hours, if the work in the latter case be $\frac{1}{4}$ more difficult than in the former?

(68) If $\frac{13}{15}$ of a hundredweight be worth £7. 3*s.*, what fraction of a ton can be bought for £90.?

(69) How many men, working 9 hours a day for 56 days, will do as much as 270 men in 35 days of 8 hours each?

(70) If $4\frac{61}{66}$*s.* be paid for 23·75 lbs. of bread, when wheat is 5·416*s.* per bushel, what must be paid for 6·875 lbs. of bread when wheat is at 4·75*s.* per bushel?

(71) If the sun move through 360 degrees in 365 d. 6 h. 9 min. 9·6 sec., through how many seconds will it move in 20·2166025 minutes?

(72) When the day is 10·164 hours long I require $4\frac{1}{2}$ days to travel 142·2 miles: what distance can I travel in 19·36 days when the length of a day is 8·4 hours?

(73) If the weight of a cubic fathom of water be 6 tons 0 cwt. 0 qrs. $23{\cdot}58\dot{8}57142\dot{2}$ lbs., how many grains does a cubic inch weigh?

(74) How many men at 3*s.* 6*d.* a day will earn in 10 days the same amount that 9 men earn in 14 days at 3*s.* 4*d.* per day?

(75) How many men will perform a piece of work in 12 days of 10 hours each, if 1000 men will perform twice that amount in a tenth part of the time, supposing that 2 of the second set can do as much work in an hour as 3 of the first set?

(76) A tenant pays corn rent of 30 quarters of wheat and 12 of barley, Winchester measure: the value of his rent is £124. 10*s.* $2\frac{1}{4}$*d.* + $\frac{8}{11}$: what is the price per quarter Imperial measure of the wheat, the barley being 64*s.*, supposing a Winchester bushel to be to an Imperial bushel as 32 : 33?

(77) If the 4*d.* loaf weigh 1 lb. $9\frac{1}{3}$ ozs. when wheat is at 9*s.* 3*d.* a bushel, what is the price of 26 lbs. $0\frac{1}{4}$ oz. of bread when wheat is at 76*s.* a quarter?

(78) How many men can dig a trench 100 yds. long, 3 deep, and 2 wide, in 7 days, working 10 hours a day, if $746\frac{2}{3}$ women require 11 days of 12 hours each to dig a trench 320 yds. long, 4 deep, and 3 wide; the work of 5 women being equal to that of 3 men?

(**79**) What is the length of A's step if he go 155 yds. while B, who takes 2·8 ft. to a step, runs 220 yards; A taking 7 steps to B's 11?

(**80**) 15 men, 12 women, and 9 boys can do a piece of work in 50 days: how many boys must help 9 men and 15 women to do twice this work in 104 days, the parts done by each in the same time being as the numbers 3, 2, 1?

(**81**) What is the value of 11 cwts. 3 qrs. $11\frac{106}{133}$ lbs. if $\frac{3}{7}$ of 24 lbs. cost $\frac{10}{3}$ of 5*s.*?

(**82**) How many acres will 3 men reap in 11 days of 11 hours each, when 9 men, working 12 hours a day, reap a field 384 yds. long and 300 yds. broad in 4 days?

(**83**) What is the price of 4·35 lbs. of bread when wheat is at 5·75*s.* a bushel, 9*s.* $0\frac{31}{45}$*d.* being paid for 49·3 lbs. of bread when wheat is at 9·2*s.* a bushel?

(**84**) How many hours are required for travelling a given distance at a given rate, 3 hours being required when the distance is diminished by one-fourth, and the rate is increased by one-half?

(**85**) If a loaf weighing 48 ozs. cost $8\frac{1}{2}$*d.*, when wheat is at 60*s.* the quarter, what should be the weight of a 6*d.* loaf when the price of wheat is $62\frac{38}{51}$*s.* a quarter?

(**86**) A pole, one-third of which is in the ground, projects a shadow of 3·5 ft.: what is its whole length if a tower 100 ft. high has at the same moment a shadow of $23\frac{1}{3}$ yds.?

(**87**) If 30 guns, firing 7 rounds in 5 minutes, kill 210 men in 3 hours, how many guns would kill, at the same rate, 420 men, firing for 6 hours, 2 rounds in a minute?

(**88**) If 48 men in 5 days of $12\frac{1}{2}$ hours long can dig a trench 139·75 yds. long, 4·5 yds. wide, and 2·5 yds. deep, how many men must dig 42 days of 12·8 hours each to finish a trench 1636·6875 yds. long, 4·875 yds. wide, and 3·2 yds. deep?

(**89**) How many men in 8 days of $10\frac{1}{2}$ hours each will mow as much as 7 men in 6 days working 10 hours a day?

(**90**) How many acres of turnips will 24 oxen eat in 10 weeks, if $13\frac{1}{2}$ acres would supply 6 score sheep for 15 weeks, on the supposition that 3 oxen eat as much as 10 sheep?

(**91**) What length of thread can be spun from 40 ozs. of silk, when a thread 90000000 miles long weighs 424607 tons 2 cwts. 3 qrs. 3 lbs.?

(**92**) If the 4*d.* loaf weighs 24 ozs. when wheat is at 50*s.* per quarter, what is the weight of the 6*d.* loaf when wheat is 60*s.* per quarter?

(93) 60 men engage to dig 3150 yds. in 50 days; but after 35 days, only 1800 yds. have been dug: how many extra men must be put on that the job may be completed in time?

(94) A bar 10 ft. $6\frac{1}{2}$ in. long, $3\frac{5}{8}$ in. broad, and $3\frac{1}{2}$ in. thick, weighs 3 cwts. 3 qrs. $24\frac{2}{3}$ lbs.: what is the weight of a similar bar 35 ft. long, $4\frac{2}{3}$ in. broad, and $4\frac{1}{8}$ in. thick?

(95) 20 horses and 196 sheep can be kept for 18 days for £15. 10*s.*: how many days can 15 horses and 72 sheep be kept for £4. 2*s.* 8*d.*, if 5 horses eat as much as 76 sheep?

(96) What fraction of a hundredweight of sugar cost £2. 2*s.* $5\frac{1}{2}$*d.*, the duty being £$\frac{7}{12}$. per hundredweight, if when the duty is £$\frac{5}{13}$. per hundredweight $\frac{5}{12}$ of 1 cwt. cost $\frac{3}{4}$ of $\frac{7}{8}$ of £3.?

(97) 780 men on full rations consume 91 qrs. of wheat in 49 days: how long will 69 qrs. last 6440 men on half rations?

(98) If 7 women earn as much as 4 men, and 48 men, assisted by 14 women, earn 121 guineas in 17 days, what number of men assisting 14 women will earn £21. 3*s.* 6*d.* in one-third of the time?

(99) The price of barley and oats being as 4 : 3, and the quantity of each produced on an acre being equal, what number of acres would produce oats to the value of £152. 5*s.* when the barley produced on 21 acres is worth £147.?

(100) 12 men, working 8 hours a day, take 15 days to do what a certain number of boys working 10 hours a day take 18 days to do: if the work of 1 man be equivalent to that of 2 boys, how many boys were there?

XVIII. INTEREST.

I. SIMPLE INTEEST.

193. When money is lent in business, the borrower is charged a certain sum for the use of it: this is called the *Interest.* The sum lent is called the *Principal*, and the sum returnable is called the *Amount*, which equals the principal + the interest. The sum charged for the use of £100. for 1 year is called the rate per cent. per annum, and its sign is %; thus 5% means that £5. is charged for the use of £100. for 1 year. and is read 5 per cent.

The method for calculating interest will be evident from the following example:—

Ex.—Find the Interest on £450. for 5 years at 4%.

This may be expressed thus: £100. in 1 year produces £4., how much will £450. produce in 5 years?

The first effect is £4., the first cause is £100. × 1 **year**; **the** second effect is required; the second cause is £450. × 5.

$$\therefore \text{ by } (\mathbf{186})\ 100 \times 1 : 450 \times 5 :: 4 : x$$

$$\text{or } x = \frac{\cancel{4} \times \overset{90}{\cancel{450}} \times \cancel{5}}{\underset{\underset{\cancel{5}}{\cancel{25}}}{\cancel{100}} \times 1} = £90.$$

Hence the ordinary rule for calculating Interest: 'Multiply the Principal by the number of years and rate per cent. and divide by 100.'

The above example worked according to this *rule* would appear thus:—

$$\begin{array}{r} £ \\ 450 \\ \underline{\quad 4} \\ 1800 \\ \underline{\quad 5} \\ 90{,}00 \end{array} \qquad \textit{Ans.} = £90.$$

Let each of the following examples be worked in *both* these ways.

Exercise 127.

Find the Interest and Amount of—

(1) £1750. for 4 years at 5%.
(2) £350. „ 5 „ 4%.
(3) £875. „ 8 „ 6%.
(4) £925. „ 2 „ 4%.
(5) £715. „ 4 „ 5%.
(6) £125. „ 3 „ 4%.
(7) £795. „ 10 „ 2%.
(8) £325. „ 2 „ 6%.

194. Find the Interest on £533. 6*s*. 8*d*. for 3 years at 5%.

This may be done in three ways:—

(i.) Cause and effect—

$$\frac{5}{100 \times 1} = \frac{x}{533 \cdot 6 \cdot 8 \times 3}$$

$$\text{or } x = \frac{5 \times 3 \times 533 \cdot 6 \cdot 8}{100} = \frac{5 \times 1600}{100} = £80.$$

(ii.) By Rule (**193**)—

£	s.	d.
533	6	8
		3
1600	0	0
		5
80,00	0	0

Ans. £80.

(iii.) Or by reduction of Principal to pounds—

$533\cdot\dot{3}$
3
1600
5
80,00 *Ans.* £80.

Each of the following examples may be worked in these three ways.

Exercise 128.

Find Interest and Amount of—

(1) £533. 6*s*. 8*d*. for 2 years at 3%.
(2) £466. 13*s*. 4*d*. „ 3 „ 4%.
(3) £202. 1*s*. 8*d*. „ 4 „ 12%.
(4) £506. 5*s*. 0*d*. „ 4 „ 4%.
(5) £212. 10*s*. 0*d*. „ 8 „ 1%.
(6) £556. 13*s*. 4*d*. „ 6 „ 5%.
(7) £912. 10*s*. 0*d*. „ 4 „ 2%.
(8) £119. 3*s*. 4*d*. „ 12 „ 10%.

195. To find the number of years, or time, in which £733. 6*s*. 8*d*. will amount to £821. 6*s*. 8*d*. at 4%.

Here the Interest is £88.: therefore we may state the question thus:—

If £100. in 1 year produce £4., in what time will £733. 6*s*. 8*d*. produce £88.?

1st method.—

$$\frac{4}{100\times 1}=\frac{88}{733.6.8\times x}$$

$$\text{or}\ \frac{\overset{1}{\cancel{4}}}{\cancel{100}\times 1}=\frac{\overset{\cancel{12}\ 3}{\cancel{264}}}{\underset{22}{\cancel{2200}}\times x}$$

$$\therefore\ x=3\ \text{years.}$$

2nd method.—Reduce the Principal to pounds: calculate the Interest for 1 year and then divide the given Interest by it. Thus:—

$$\begin{array}{r}733\cdot\dot{3}\\ 4\\ \hline 2933\cdot 3\end{array}\quad \therefore\ \text{Int. for 1 year}=£29\cdot\dot{3}.$$

$$\therefore\ \textit{Answer}=\frac{£88}{£29\cdot\dot{3}}=3\ \text{years.}$$

Each of the following examples may be worked in either of these ways.

Exercise 129.

In how many years will—

(1)	£933. 6*s*. 8*d*.	amount to	£1045. 6*s*. 8*d*.	at	6%?
(2)	£866. 13*s*. 4*d*.	„	£970. 13*s*. 4*d*.	„	3%?
(3)	£202. 1*s*. 8*d*.	„	£299. 1*s*. 8*d*.	„	8 ?
(4)	£1506. 5*s*. 0*d*.	„	£1747. 5*s*. 0*d*.	„	4%?
(5)	£1312. 10*s*. 0*d*.	„	£1417. 10*s*. 0*d*.	„	2%?
(6)	£1556. 13*s*. 4*d*.	„	£2023. 13*s*. 4*d*.	„	5%?
(7)	£219. 3*s*. 4*d*.	„	£482. 3*s*. 4*d*.	„	10%?
(8)	£1912. 10*s*. 0*d*.	„	£2065. 10*s*. 0*d*.	„	2%?

196. To find the rate per cent. at which £466. 13*s*. 4*d*. will amount to £494. 13*s*. 4*d*. in 3 years.

Here the Interest is £28.; therefore the question may be stated thus:—

If £100. in 3 years at 1% produce £3., at what rate will £466. 13*s*. 4*d*. produce £28 in the same time?

1st method.—

$$\frac{3}{100\times 1}=\frac{28}{466\cdot 13\cdot 4\times x}$$

$$\frac{\cancel{3}}{\cancel{100}\times 1}=\frac{\overset{2}{\cancel{28}}\times 3}{\cancel{1400}\times x}$$

$$\therefore x=2.\quad \textit{Ans.}$$

2nd method.—Calculate the Interest on the Principal, reduced to pounds, at 1%, and divide the given Interest by it. Thus:—

$$\begin{array}{r} 466\cdot\dot{6} \\ 3 \\ \hline 1399\cdot\dot{9} \end{array}\quad \therefore \text{Int.}=£14.$$

$$\therefore \textit{Ans.}=\frac{28}{14}=2.$$

Exercise 130.

At what rate per cent. will—

(1)	£133. 6*s.* 8*d.*	amount to	£149. 6*s.* 8*d.*	in	3 years?
(2)	£666. 13*s.* 4*d.*	„	£746. 13*s.* 4*d.*	„	2 „
(3)	£902. 1*s.* 8*d.*	„	£1335. 1*s.* 8*d.*	„	6 „
(4)	£2506. 5*s.* 0*d.*	„	£2907. 5*s.* 0*d.*	„	4 „
(5)	£312. 10*s.* 0*d.*	„	£337. 10*s.* 0*d.*	„	4 „
(6)	£956. 13*s.* 4*d.*	„	£1243. 13*s.* 4*d.*	„	10 „
(7)	£319. 3*s.* 4*d.*	„	£702. 3*s.* 4*d.*	„	15 „
(8)	£112. 10*s.* 0*d.*	„	£121. 10*s.* 0*d.*	„	4 „

197. To find what Principal, or what sum of money, will in 3 years at 4% amount to £1045. 6*s.* 8*d.*

£100. in 3 years at 4% amounts to £112.: what sum will amount to £1045. 6*s.* 8*d.*?

$$\frac{100}{112}=\frac{x}{1045\cdot 6\cdot 8}$$

decimalise the Principal—

$$\begin{array}{l} 112)104533\cdot\dot{3}(933\cdot\dot{3} \\ \quad\ 1008 \\ \hline \quad\quad 373 \\ \quad\quad 336 \\ \hline \quad\quad\ 373 \\ \quad\quad\ 336 \\ \hline \quad\quad\quad 37 \end{array}\qquad \textit{Ans.}=£933.\ 6s.\ 8d.$$

Rule, therefore, is 'multiply the Principal by 100 and divide the product by the amount of £100.'

Exercise 131.

Find the Principal which will amount to

(1)	£970. 13*s*. 4*d*.	in	2	years at	6%
(2)	£482. 3*s*. 4*d*.	„	20	„	6%
(3)	£2065. 10*s*. 0*d*.	„	4	„	2%
(4)	£299. 1*s*. 8*d*.	„	12	„	4%
(5)	£149. 6*s*. 8*d*.	„	4	„	3%
(6)	£2907. 5*s*. 0*d*.	„	4	„	4%
(7)	£1335. 1*s*. 8*d*.	„	4	„	12%
(8)	£1243. 13*s*. 4*d*.	„	10	„	3%

198. It should be noticed generally that if $P = $ Principal, $n = $ time, $r = $ rate per cent., $I = $ Interest, and $A = $ Amount, we deduce the following relations :—

(i.) $P \times n \times r = 100 \times I$. (ii.) $P(100 + nr) = 100 \times A$.

Three of these quantities are always given, and the problem is to find the fourth. We will work out a few examples where special methods are adopted, but previously draw attention to these remarks.

(1) In working by decimals the results need be correct to the *third* place.

(2) When the time is given in *months* and *days*, reckon 12 months to the year, and 30 days to the month.

(3) When the time is given in *weeks*, multiply by the number of weeks and divide by 5200.

(4) When the time is given in *days*, multiply by double the rate and number of days, and divide by 73000, which is done by this *rule*.

Divide the dividend (neglecting the decimal part) by 3, the quotient by 10, and this new quotient by 10; add these quotients and the dividend together, and point off 5 places of decimals, and deduct $\frac{1}{10000}$.

Thus :— 3418893·1791÷73000.

(1) 3418893—dividend.
(2) 1139631—dividend divided by 3.
(3) 113963—(2) divided by 10.
(4) 11396—(3) divided by 20.
46·83883
468—(4) divided by 10000.
46·83415

(5) When the time is given in *years* and *days*, the interest may be found separately for the years and days, and the results added together; or the time may be reduced to days as in remark (4).

Ex. 1. Find the Simple Interest on £5260. 10*s.* 2*d.* at $2\frac{7}{8}$% for 6 yrs. 5 m. 21 days.

4	$\frac{1}{3}$	5260·50833
		6
		31563049
1	$\frac{1}{4}$	1753502
15	$\frac{1}{2}$	438375
6	$\frac{2}{5}$	219187
		87675

$\frac{1}{8}$) 34061·791
3
102185373
4257724
979·27649 ∴ *Ans.* = £979. 5*s.* 6·3576*d.*

Ex. 2. In what time will £458. 2*s.* 5*d.* amount to £465. 13*s.* 10*d.* at $4\frac{3}{4}$%?

950 is 100 × double the rate %.

By Rk. (4)
950
316
36
3
·013 is the interest on £100. in one day.

The Interest on the Principal £458·120 is £7·5716.

$$\therefore \frac{\cdot013}{100 \times 1} = \frac{7\cdot5716}{458\cdot120 \times x}$$

$$\therefore x = \frac{75716000}{595556} = 127 \text{ days.}$$

EX. 3. What sum of money in 8¾ years at £6. 7*s*. 6·084*d*.% will amount to £538. 12*s*. 6*d*.?

£6. 7*s*. 6·084*d*. = £6·37535; £538. 12*s*. 6*d*. = £538·625; 8¾ = 8·75.

6·37535
578
51003
4462
318
55·783

∴ £100. amounts to £155·783: what sum will amount to £538·625?

(198. ii.) $1\cancel{5}\cancel{5}\cancel{7}\cancel{8}\cancel{3}$)538625(345754
467349
71276
62313
8963
7789
1174
1090
84
77
7

£
345·754
20
15·080
12
·960

Ans. = £345. 15*s*. 1*d*. nearly.

EX. 4. £420. 13*s*. 8*d*. was put out at Interest on March 19, 1868 at 4½% : by what date will it amount to £467. 10*s*. 4¼*d*.?

The Interest = £46. 16*s*. 8¼*d*. = £46·8341.

£46·8341 × 73000 = £3418893·1791.

The Principal = £420. 13*s*. 8*d*. = £420·6833; this multiplied by double the rate = 3786·1497.

∴ No. of days = 3418893·1791 ÷ 3786·1497 = 903

37861497)3418893I791(903 days
340753473
113584491
113584491

Now 903 days = 730 + 173 = 2 years 173 days; and as the day on which the money is lent and the day on which it is paid make only *one* day, we reckon only 12 days in March, 30 in April, 31 in May, 30 in June, 31 in July, 31 in August, and 8 in September,

∴ the date required is Sept. 8, 1880.

Ex. 5. At what rate % will the interest on £498. 16*s*. 8*d*. amount to £10. 9*s*. $3\frac{1}{4}$*d*. in 125 days?

£498. 16*s*. 8*d*. = £$498\cdot8\dot{3}$: £10. 9*s*. $3\frac{1}{4}$*d*. = £10·46354;

$\frac{498\cdot833 \times 2 \times 125}{73000}$ will give the interest at 1%; and $2 \times 125 = \frac{1000}{4}$;

$$\begin{array}{rl} 4\underline{)498833} & \\ \overline{124708} & \\ 41569 & \\ 4156 & \\ \underline{415} & \\ 1\cdot70848 & \\ \underline{17} & \text{—deduct } 17 \\ 1\cdot70831 & \end{array}$$

∴ rate required = $\frac{10\cdot46354}{1\cdot70831} = 6\cdot125$. ∴ *Ans.* = £6. 2*s*. 6*d*.

Exercise 132.

(i.) Find the Interest and Amount on—

(1) £8540. 13*s*. 9*d*. for 1 yr. 50 wks. at $4\frac{7}{8}$%.

(2) £460. 3*s*. 6*d*. for 3 yrs. $8\frac{3}{4}$ m. at $4\frac{3}{4}$%.

(3) £727. 16*s*. $11\frac{3}{4}$*d*. for 210 days at 3%.

(4) £284. 10*s*. for 2 yrs. 4 m. 25 dys. at $3\frac{1}{2}$%.

(5) £533. 13*s*. 4*d*. for 584 days at $4\frac{1}{2}$%.

(6) £194. from March 3 to Dec. 18 at 5%.

(7) £113. 6*s*. 8*d*. for 213 days at $3\frac{1}{2}$%.

(8) 200 guineas for 4 yrs. 7 m. 25 dys. at $4\frac{1}{2}$%.

(9) £463. 14*s*. 9*d*. for 1 yr. 5 m. at $2\frac{3}{4}$%.

(10) £433. 13*s*. 4*d*. for 3 yrs. 146 dys. at $4\frac{1}{2}$%.

(ii.) In what time will—

(1) £5764. 12*s*. 4*d*. at $3\frac{3}{5}$% amount to £6495. 4*s*. $5\frac{387}{912}$*d*.?

(2) $500\frac{1}{2}$ guineas at $2\frac{1}{4}$% amount to £544. 16*s*. $9\frac{0}{4}\frac{261}{800}$*d*.?

(3) £716. 17*s*. 9*d*. at $3\frac{3}{4}$% amount to £749. 11*s*. $9\frac{1}{2}$*d*.?

(4) £257. 10s. 6d. at 4½% amount to £272. 3s. $4\frac{20083}{29200}$d.?

(5) £213. 18s. 9¼d. at 5¾% amount to £224. 6s. 4½ $\frac{7251}{36500}$d.

(6) £225. 10s. at 5% amount to £228. 17s. $11\frac{37}{73}$d.?

(7) £550. at 4% amount to £550. 12s. $0\frac{48}{73}$d.?

(8) £17. 5s. at 4¾% amount to £17. 10s. 3d.?

(9) £540. at 6% amount to £552. 3s.?

(10) £327. 15s. 8½d. at 4¼% amount to £339. 0s. 10½ $\frac{12211}{14600}$d.?

(iii.) What sum of money will amount to—

(1) £460. 10s. in 292 days at 3¾%?

(2) £244. 4s. 7¼ $\frac{829}{832}$d. in 1 yr. 23 wks. at 4¾%?

(3) £8187. 7s. 9¾d. in 31 days at 3½%?

(4) £457. 12s. 6d. in 6 yrs. 10 m. at 4½%?

(5) £393. 8s. 1¾ $\frac{17}{23}$d. in 146 days at 6%?

(6) £629. 17s. $3\frac{3}{16}$d. in 1 yr. 10½ m. at 5%?

(7) £275. 1s. 9·76875d. in 3¾ years at 4¼%?

(8) £108. 12s. 7d. in 117 days at 4¾%?

(9) £450. 17s. 3d. in 1 yr. 10 m. at 4⅜%?

(10) £1035. 1s. $7\frac{0}{4}$ $\frac{7}{365}$d. in 191 days at 5%?

(iv.) On what date will be due—

(1) £1653. 18s. $6\frac{39968}{116800}$d. if £1633. 15s. 1½d. be placed out on Jan. 4, 1862, at 4⅜%?

(2) £1441. 14s. 2¾d. if £1327. 3s. 8d. be placed out on Oct. 18, 1869, at 5⅜%?

(3) £270. 3s. $5\frac{7}{10}$d. if £250. 12s. 6d. be placed out on March 26, 1870, at 3%?

(4) £1000. if £960. 15s. $3\frac{24359}{25327}$d. be placed out on Jan. 1 at 5½%?

(5) £1496. 9s. $9\frac{0}{4}$ $\frac{3}{5}$d. if £1450. 17s. 6d. be placed out on March 1 at 4½%?

(6) £942. 4s. $0\frac{41}{125}$d. if £924. 12s. 8d. be placed out on March 18 at 4¾%?

(7) £7611. 14s. $7\frac{5}{73}$d. if £7500. be placed out on May 5 at 5⅛%?

(8) £3262. 14s. $7\frac{0}{4}$ $\frac{1}{5}$d. if £3234. 16s. 3d. be placed out on Jan. 12 at 5%?

(9) £1778. 18s. 8d. if £1703. 6s. 8d. be placed out on July 1, 1874, at 4½%?

(10) £1254. 5*s.* $0\frac{1}{4}\frac{1}{5}$*d.* if £1231. 17*s.* 6*d.* be placed out on March 1 at $6\frac{1}{2}$%?

(v.) At what rate % will the Interest on—

(1) £659. 2*s.* 6*d.* in 5 yrs. 11 m. amount to £136. 9*s.* $10\frac{41}{80}$*d.*?

(2) £743. 10*s.* in 87 days amount to £7. 19*s.* 6*d.*?

(3) £57. 19*s.* from Sept. 4 to Dec. 19 amount to 16*s.* $9\frac{3}{4}$*d.*?

(4) £280. 10*s.* in 2 years amount to £19. 10*s.*?

(5) £684. 7*s.* 6*d.* from July 5 to Nov. 20 amount to £7. 2*s.* $3\frac{3}{4}$*d.*?

(6) £1158. 17*s.* 6*d.* for 1 yr. 115 dys. amount to £38. 2*s.*?

(7) £17310. 10*s.* for 12 days amount to £19. 18*s.* 4·54356*d.*?

(8) £712. 6*s.* for 8 months amount to £35. 12*s.* $3\frac{1}{2}$*d.*?

(9) £713. 10*s.* 9*d.* for 79 days amount to £7. 2*s.* $10\frac{1}{4}$*d.*?

(10) £743. 10*s.* for 154 days amount to £15. 19*s.*?

II. PRESENT WORTH AND DISCOUNT.

199. The *Present Worth* of a sum of money, due some time hence, is that Principal which, placed out at Interest for the given time at the given rate %, would amount to the given sum. Thus, since £100. in 3 years at 4% amounts to £112., £100. is the *Present Worth* of £112.

The *Discount* on a sum of money is the difference between the sum and its Present Worth. £12. would be the Discount on £112. due 3 years hence at 4%. It is plain, therefore, that instead of stating a question, as in (**197**), What Principal would in 3 years at 4% amount to £1045. 6*s.* 8*d.*? we might state it, 'Find the Present Value or Worth of £1045. 6*s.* 8*d.* due 3 years at 4%;' and Ex. 131, Ex. 132 (iii.) become examples in 'Present Worth.'

200. If P = the sum of money; I = the Interest on it; D = the Discount on it; n = time; r = rate %; we may deduce generally some useful results.

£100. in n years at r% amounts to £$(100+nr)$, ∴ £nr is the discount on £$(100+nr)$,

$$\therefore 100+nr : nr :: \mathrm{P} : \mathrm{D}, \text{ or } \mathrm{D}=\frac{\mathrm{P}nr}{100+nr}, \text{ but } \mathrm{I}=\frac{\mathrm{P}nr}{100},$$

$$\therefore (100+nr)\,\mathrm{D}=100\,\mathrm{I}, \text{ both being} = \mathrm{P}nr,$$

$$\therefore \frac{\mathrm{D}}{\mathrm{I}}=\frac{100}{100+nr}, \text{ a \textit{proper} fraction.}$$

$$\therefore \text{ D is always } < \text{I, and is the Present Worth of I} \quad . \quad . \quad (1)$$

Again we have—

$$D(100+nr)=Pnr, \text{ or } nr(P-D)=100D, \quad \therefore nr=\frac{100D}{P-D}=\frac{100I}{P},$$

$$\therefore P.D=I(P-D), \quad \therefore P(I-D)=I.D,$$

$$\therefore P=\frac{I.D}{I-D};\ I=\frac{P.D}{P-D};\ D=\frac{P.I}{I+P} \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\frac{Pn^2r^2}{100(100+nr)}=I-D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

201. I of (**200**) is called in business transactions 'Banker's discount,' and is always greater than D, the 'true Discount;' hence the banker has an advantage: the distinction between them is, that *Interest* is calculated on the Amount, while the Discount is calculated on the Present Worth of the Amount. From (2), (3) (**200**) we obtain these rules:—

(1) Given the Interest and Discount, to find the Principal divide their product by their difference.

(2) To find the difference between the Interest and Discount on a sum of money, multiply it by the square of the Interest of £100., and divide by the Product of £100. and the Amount of £100.

202. Discount questions occur most frequently in connection with Bills of Exchange or Promissory Notes.

A bill of exchange is a written instrument in which one person orders another to pay him a sum of money on a certain date. Thus:—

£750. Plymouth: Dec. 15, 1884.

Six months after date pay A. B., on order, Seven hundred and fifty pounds, value received.

Accepted C. D.

To C. D. E. F.;
London.

i.e., C. D. promises to pay A. B., or his order, £750. at the end of 6 months from Dec. 15, 1884.

A promissory note is a written instrument in which A. B. promises to pay C. D. a sum of money at a specified time. Thus:—

£200. Plymouth: Dec. 15, 1884.

Two months after date, I promise to pay C. D., or order, Two hundred pounds, value received. A. B.;

i.e., A. B. engages to pay C. D. £200. at the end of 2 months from Dec. 15, 1884.

A bill of exchange or promissory note always runs 3 days beyond the time specified, called days of grace.

Ex. 1. Find the Present Worth of £635. 7*s*. 6*d*. due 9 years hence, at 5½%.

$$£635.\ 7s.\ 6d. = £635{\cdot}375\ ;\ £5\tfrac{1}{2}. = 5{\cdot}5.$$

Since ∴ £100. in 9 years at 5½% amounts to £149·5, what sum will amount to £635·375?

$$\therefore 100 : x :: 149{\cdot}5 : 635{\cdot}375 ;$$

$$\therefore \text{Present Worth} = \frac{635{\cdot}375 \times 100}{149{\cdot}5} = \frac{\overset{\overset{9005}{\cancel{127075}}}{\cancel{635375}}}{\underset{\underset{23}{\cancel{299}}}{\cancel{1495}}} = £425.$$

Ex. 2. What would be the value on Jan. 18, 1872, of £509. 5*s*. 11¼*d*. due May 17, 1872, at 6⅛%?

There are 18 days in Jan., 29 in Feb., 31 in March, 30 in April, 17 in May; *i.e.*, we have to find the P. W. of £509. 5*s*. 11¼*d*. due 125 days hence at 6⅛%. £509. 5*s*. 11¼*d*. = £509·296.; from (**198**, 4) we find amount of £100.

$$\begin{array}{l r l}
4) & 100 & \\
 & \underline{12} & \\
 & 1200 & \\
 & \underline{25} & \\
 & 1225 & \text{Add 2 cyphers and divide by 8.} \\
 & \underline{125} & \\
 & 153125 & \\
 & 51041 & \\
 & 5104 & \\
 & \underline{510} & \\
 & 2{\cdot}09780 & \text{Deduct 20.} \\
 & \underline{20} & \\
 & 209760 &
\end{array}$$

$$\begin{array}{r}
10\cancel{2}{\cdot}\cancel{0}\cancel{9}\cancel{7}\cancel{6})5092960(4988327 \\
1009056 \\
90178 \\
8500 \\
333 \\
27 \\
7
\end{array}
\qquad\qquad
\begin{array}{r}
498{\cdot}8327 \\
\underline{20} \\
16{\cdot}654 \\
\underline{12} \\
7{\cdot}848
\end{array}$$

Ans. £498. 16*s*. 8*d*.

Ex. 3. Find the Discount on £538. 12*s.* 6*d.* due in 8$\frac{3}{4}$ years at 6$\frac{3}{8}$%.

£6$\frac{3}{8}$. = 6·375; £538. 12*s.* 6*d.* = £538·625.

$$\begin{array}{rl}
4) & 6{\cdot}375 \times 8\tfrac{3}{4}\ (=9-\tfrac{1}{4}) \\
 & \underline{9} \\
 & 57{\cdot}375 \\
 & \underline{1{\cdot}593} \\
 & 55{\cdot}782
\end{array}$$

∴ Discount on £155·784 is £55·784

$$\begin{array}{r}
538{\cdot}625 \\
\underline{48755} \\
2693125 \\
269312 \\
37703 \\
4309 \\
\underline{215} \\
1\cancel{5}\cancel{5}\cancel{7}\cancel{8}\cancel{4})3004664(1928750 \\
144682 \\
4477 \\
1362 \\
116 \\
8 \\
0
\end{array}$$

∴ Discount required is £192·875 or £192. 17*s.* 6*d.*

Ex. 4. What should be deducted from £387. 14*s.* 10*d.* due August 8, paid June 4, the rate of interest being 4%?

From June 4 to August 8 is 65 days. £387. 14*s.* 10*d.* = £387·742.

$$\begin{array}{rl}
65 & \\
\underline{800} & \\
52000 & \\
17333 & \\
1733 & \\
\underline{173} & \\
{\cdot}71239 & \text{Deduct 7.} \\
\underline{7} & \\
{\cdot}71232 &
\end{array}$$

∴ Discount on £100·71232 is £·71232.

387742
23217
271419
3877
775
116
8

$$100\cancel{7}\cancel{1}\cancel{2}\cancel{3})\overline{276195}(2742$$

74771
4273
245
44

∴ Discount = £2·742 = £2. 14*s*. 10¼*d*.

Ex. 5. I am entitled to £457. 12*s*. 6*d*. when I 'come of age,' but its present value at 4½ % is £350.: how old am I now?

This question may be stated thus: £100. in 1 year realises £4·5, in what time will £350. realise £107·625?

$$\frac{4\cdot5}{100\times1}=\frac{107\cdot625}{350\times n},$$

$$\therefore \text{ time required}=\frac{107625}{45\times350}=\frac{41}{6}=6\tfrac{5}{6}\text{ yrs.}=6\text{ yrs. }10\text{ m.}$$

∴ I am 14 yrs. 2 m. old.

Ex. 6. On March 14, 1873, I drew out from the bank £566. 8*s*. 6·66*d*.: when did I put in £536. 11*s*., reckoning the interest at 7½%?

100
15
1500
500
50
5
·02054

∴ £100. in one day at 7½ % makes £·02054: in how many days will £536. 11*s*. or £536·55 make £29·87775?

536·55
4502
107310
2683
213

$$11\cdot0206)\overline{2987775}(271$$

78365
1221
19

i.e., 271 days.

Now the money was in the bank 14 days in March, 28 in February, 31 in January, 31 in December, 30 in November, 31 in October, 30 in September, 31 in August, 31 in July, and 14 in June;

∴ I must have put it in on June 17, 1872.

Ex. 7. What is the annual rate of Interest when the true Present Worth of £107. for 83 days is £105. 15*s.* $11\frac{7}{23}$*d.*?

£105. 15*s.* $11\frac{7}{23}$*d.* = £105·7972.; the discount off this is £1·2028.

The interest on it @ 1% for 83 days—

$$\begin{array}{rl}
105797 & \\
\underline{661} & \\
105797 & \\
63478 & \\
\underline{6348} & \\
3)175623 & \\
58541 & \\
5854 & \\
\underline{585} & \\
\cdot 239603 & \text{Interest at } 1\%.
\end{array}$$

$$\therefore \frac{1\cdot2028}{\cdot24} = 5 \text{ is the rate \% required.}$$

Ex. 8. The sum of £2795. 4*s.* 4*d.* is payable on December 3, what is the rate per cent. per annum if its true cash value on August 17 preceding is £2767. 18*s.* 4*d.*?

From Aug. 17 to Dec. 3 is 108 days. £2767. 18*s.* 4*d.* = £2767·916.

£2795. 4*s.* 4*d.* = £2795·216. ∴ the discount = £27·3.

£2767·916 at 1% for 108 days is thus found:—

$$\begin{array}{rl}
2767916 & \\
\underline{612} & \\
5535833 & \\
276791 & \\
\underline{166075} & \\
597869\cdot9 & \\
1992899 & \\
199289 & \\
\underline{19928} & \\
8\cdot19081 & \text{Deduct 81.} \\
\underline{81} & \\
8\cdot19 &
\end{array}$$

∴ The discount at £1.% would have been £8. 19*s.*,

$$\therefore \text{ rate } \% = \frac{27\cdot3}{8\cdot19} = \frac{10}{3} = 3\tfrac{1}{3}. \quad \textit{Ans.}$$

Ex. 9. The interest on £522. 10*s.* for a certain time is £34. 7*s.* 6*d.*: find the discount on the same sum for the same time.

From (**200**, 2)—
$$D = \frac{522\frac{1}{2} \times 34\frac{3}{8}}{522\frac{1}{2} + 34\frac{3}{8}}$$

$$= \frac{1045 \times 275}{2 \times 8 \times 556\frac{7}{8}} = \frac{1045 \times \overset{\overset{5}{\cancel{5}}}{\cancel{275}}}{2 \times \underset{\underset{\underset{4}{\cancel{81}}}{\cancel{891}}}{\cancel{4455}}}$$

$$\begin{array}{r|l} 2 & 5225 \\ \hline 9 & 2612 - 10 \\ \hline 9 & 290 - 5 - 6\frac{2}{3} \\ \hline & £32 - 5 - 0\frac{20}{27} \end{array}$$

Ans. £32. 5*s.* $0\frac{20}{27}$*d.*

This might be worked by decimals, thus:—

£522. 10*s.* = £522·5 ; £34. 7*s.* 6*d.* = £34·375 ;

```
∴         34375
           5225
         ------
         171875
           6875
            687
            172
         ------
556875)179609(32·253
          12547
           1410
            296
             18
              2
```

Ans. = £32. 5*s.* $0\frac{3}{4}$*d.*

Ex. 10. The interest on a sum of money is £33. 17*s*. 7·92*d*, the discount on the same sum is £27. 6*s*. 6*d*.: what is the sum?

The interest = £33·883; the discount = £27·325; ∴ by (**200**, 2)—

$$P = \frac{27·325 \times 33·883}{6·558}$$

$$= \frac{925·852975}{6·558} = 141·1791\dot{6} = £141.\ 3s.\ 7d.$$

Ex. 11. The discount on £574. 13*s*. is £63. 17*s*.: find the interest on the same sum.

Principal = £574·65; Discount = £63·85;

(**200**, 2) ∴ Interest $= \frac{574·65 \times 63·85}{510·8} = \frac{36691·4025}{510·8} = 71·83125$

$= £71.\ 16s.\ 7\frac{1}{2}d.$ *Ans.*

Ex. 12. Find the difference between the Interest and Discount of £1114. 11*s*. 8*d*. in 1¾ years at 4%.

From (**200**)—

$$I : D :: 107 : 100,\ \therefore\ I - D : D :: 7 : 100,\ \therefore\ I - D = \frac{7D}{100},$$

$$\text{and } D = \frac{1114·58\dot{3} \times 7}{107},$$

$$\therefore\ I - D = \frac{1114·58\dot{3} \times 7 \times 7 \times 100}{107} = \frac{546·1458\dot{3}}{107} = 5·1041\dot{6}$$

$= £5.\ 2s.\ 1d.$ *Ans.*

Ex. 13. A bill of £649. is dated on June 23, 1853, at 6 months, and is discounted on July 8, at 3%: what does the banker gain thereby?

This is the same question as finding the difference between the Interest and Discount on £649. for 171 days, *i.e.* from June 23 to December 23 + 3 days' grace.

$\mathrm{I} - \mathrm{D} = \dfrac{\mathrm{P}n^2r^2}{100(100 + nr)}$ ∴ (**198**, 4) 650
171
650
4550
650
111150
37050
3705
370
152275
15
∴ $nr = 1{\cdot}5226$ Square this.
62251
15226
7613
314
30
9
2·3192
946
139152
9277
2087
$10\cancel{1}{\cdot}\cancel{5}\cancel{2}\cancel{2}6)15{\cdot}0516({\cdot}14826$
3 48993
8384
264
61
0 *Ans.* 2*s.* 11*d.* nearly.

Exercise 133.

(i.) Find the Present Worth of—

(1) £930. due in $2\frac{3}{4}$ years at $3\frac{79}{187}$%.
(2) £11421. 2*s.* $7\frac{463}{1460}$*d.* due in 5 yrs. 297 dys. at $3\frac{1}{4}$%.
(3) £200. 1*s.* $2\frac{1}{2}$*d.* due in $189\frac{701}{18816}$ days at 4%.
(4) £250. due in $13\frac{1}{3}$ years at $7\frac{1}{2}$%.
(5) £805. 6*s.* $2\frac{5}{8}$*d.* due in $4\frac{1}{2}$ years at $2\frac{1}{2}$%.
(6) £226. 17*s.* $7\frac{33}{128}$*d.* due in 3 yrs. $341\frac{12637}{260148}$ dys. at $4\frac{3}{8}$%.
(7) £191. 7*s.* $3\frac{23}{80}$*d.* due in 5 months at $4\frac{1}{2}$%.
(8) £1165. 10*s.* 5*d.* due in 2 yrs. 1 m. at $5\frac{3}{8}$%.

(ii.) What would be the value on—

(1) June 1 of £126. 16*s*. $9\frac{83}{365}$*d*. due Dec. 3 at $5\frac{1}{3}$%?

(2) March 3 of £201. 14*s*. $1\frac{47}{73}$*d*. due Dec. 18 at 5%?

(3) March 11 of £1870. 5*s*. $9\frac{1}{2}$*d*. due Aug. 4 at $4\frac{3}{4}$%?

(4) April 6 of £1402. 14*s*. 4·125*d*. due Aug. 30 at $4\frac{3}{4}$%?

(5) March 18 of £942. 4*s*. 0·328*d*. due Aug. 11 at $4\frac{3}{4}$%?

(6) May 5 of £7611. 14*s*. $7\frac{5}{73}$*d*. due Oct. 26 at $3\frac{1}{8}$%?

(7) June 16, 1872, of £566. 8*s*. 6·66*d*. due March 14, 1873, at $7\frac{1}{2}$%?

(8) Jan. 7, 1860, of £267. 1*s*. $10\frac{3}{4}$*d*. due Sept. 12, 1860, at 4%?

(iii.) Find the Discount on—

(1) £800. due in $1\frac{1}{2}$ year at $3\frac{8028}{9057}$%.

(2) £598. 9*s*. 9*d*. due in 1 yr. 115 dys. at $2\frac{1}{2}$%.

(3) £1346. 10*s*. 2*d*. due in 250 days at £3. 13*s*.%.

(4) £927. 3*s*. 8*d*. due in 313 days at 4%.

(5) £1117. 16*s*. due in 106 days at $5\frac{3}{4}$%.

(6) £450. 17*s*. $3\frac{227}{320}$*d*. due in 1 yr. 10 m. at $4\frac{3}{8}$%.

(7) £257. 8*s*. $8\frac{1}{4}$*d*. due in 210 days at $4\frac{1}{2}$%.

(8) £325. 10*s*. due in 146 days at $3\frac{3}{4}$%.

(iv.) What should be deducted from—

(1) £4761. 12*s*. due Jan. 3, 1883, paid Aug. 6, 1882, at $4\frac{2}{3}$%?

(2) £1311. 2*s*. $10\frac{7744}{9125}$*d*. due Aug. 18, paid March 23, at $4\frac{1}{2}$%?

(3) £1965. 11*s*. $0\frac{7944}{9125}$*d*. due Oct. 19, paid April 18, at $3\frac{1}{2}$%?

(4) £14478. 14*s*. $0\frac{403}{1825}$*d*. due Nov. 6, paid June 1, at $3\frac{1}{4}$%?

(5) £10. 0*s*. $4\frac{8}{73}$*d*. due March 28, paid March 3, at $2\frac{1}{2}$%?

(6) £103. 17*s*. $3\frac{1}{73}$*d*. due March 9, 1860, paid June 1, 1859, at 5%?

(7) £1035. 3*s*. $10\frac{49}{365}$*d*. due May 28, paid April 13, at 5%?

(8) £565. 16*s*. $2\frac{8}{365}$*d*. due Oct. 20, 1861, paid Jan. 1, 1861, at 5%?

(v.) In what time will—

(1) £54. 15*s*. be the present value of £60. 15*s*. at $12\frac{4}{13}$%?

(2) £934. 18*s.* 2*d.* be the present value of £980. 14*s.* $4\frac{261}{500}$ at $2\frac{3}{5}$% ?

(3) £87. 19*s.* 10*d.* be the present value of £90. 12*s.* $8\frac{231}{416}$*d.* at $3\frac{1}{8}$% ?

(4) £280. 10*s.* be the present value of £316. 8*s.* $6\frac{36}{73}$*d.* at $5\frac{1}{2}$% ?

(5) £700. be the present value of £743. 10*s.* $5\frac{67}{73}$*d.* at $4\frac{1}{4}$% ?

(6) £1091. 4*s.* be the present value of £1257. 12*s.* $0\frac{1152}{9124}$*d.* at $2\frac{1}{5}$% ?

(7) £730. be the present value of £734. 3*s.* $2\frac{2}{5}$*d.* at $3\frac{1}{4}$% ?

(8) £498. 19*s.* 4*d.* be the present value of £538. 5*s.* $9\frac{5101}{45625}$*d.* at $3\frac{3}{5}$% ?

(vi.) At what dates must the following sums of money be put out to Interest—

(1) £4961. at $6\frac{1}{2}$% to amount on Dec. 8, 1875, to £5252. 10*s.* $10\frac{178}{365}$*d.* ?

(2) £300. at 3% to amount on Nov. 3 to £307. 6*s.* $5\frac{1}{2}+\frac{26}{73}$*d.* ?

(3) £243. 6*s.* 8*d.* at $7\frac{1}{2}$% to amount on Nov. 7 to £252. 2*s.* 8*d.*?

(4) £684. 7*s.* 6*d.* at $2\frac{3}{4}$% to amount on Nov. 20 to £691. 9*s.* $9\frac{3}{4}$*d.*?

(5) £321. at $3\frac{1}{4}$% to amount on Sept. 12 to £326. 8*s.* $7\frac{197}{365}$*d.*?

(6) £3764. at $3\frac{1}{8}$% to amount on Feb. 19 to £3818. 15*s.* $8\frac{16}{73}$*d.*?

(7) £186. 17*s.* 6*d.* at 5% to amount on Feb. 6, 1866, to £206. 9*s.* $8\frac{13}{73}$*d.* ?

(8) £226. 17*s.* $11\frac{5}{404}$*d.* at 4% to amount on Dec. 3 to £230. 15*s.* ?

(vii.) What is the rate of Interest when the true Present Worth of—

(1) £20. due 9 months hence is £17. 10*s.* ?

(2) £825. 13*s.* $0\frac{4}{5}$*d.* due $3\frac{1}{4}$ years hence is £730. 13*s.* 4*d.*?

(3) £3190. 12*s.* $10\frac{27}{40}$*d.* due 2 yrs. 2 m. hence is £2980. 15*s.* ?

(4) £858. 17*s.* $8\frac{567}{600}$*d.* due 3 yrs. 7 m. hence is £735. 16*s.* 9*d.* ?

(5) £899. 15*s.* $1\frac{91}{200}$*d.* due 11 months hence is £875. 13*s.* 6*d.* ?

(6) £826. 19*s.* $7\frac{1}{5}$*d.* due 3 yrs. 5 m. hence is £672. ?

(7) £2015. 4*s.* due 4 yrs. 10 m. hence is £1760. ?

(8) £1101. 8*s.* $3\frac{77}{100}$*d.* due 7 yrs. 4 m. hence is £845. 12*s.* 4*d.* ?

(viii.) Find the rate of Interest when the true cash value on—

(1) July 1 of £230. 15*s.* payable Dec. 3 is £226. 17*s.* 11$\frac{8}{404}$*d.*

(2) Feb. 7 of £123. 5*s.* 2$\frac{266}{365}$*d.* payable Oct. 13 is £120.

(3) March 14 of £358. 15*s.* 10$\frac{26}{73}$*d.* payable Dec. 1 is £350.

(4) July 17 of £188. 2*s.* 0$\frac{0}{4}$·8*d.* payable Dec. 5 is £184. 3*s.* 9*d.*

(5) June 18 of £860. 11*s.* 2$\frac{1}{2}$·8*d.* payable Sept. 25 is £853. 0*s.* 10*d.*

(6) April 6 of £3138. 18*s.* 1$\frac{1}{4}$*d.* payable Oct. 27 is £3057. 14*s.* 7*d.*

(7) April 14 of £476. 18*s.* 9$\frac{3}{4}$*d.* payable July 6 is £473. 3*s.* 6*d.*

(8) Nov. 9, 1867, of £167. 15*s.* 7$\frac{1}{2}$*d.* payable March 3, 1868, is £164. 15*s.* 11*d.*

(ix.) Find the Discount when the Interest on—

(1) £908. 5*s.* 9*d.* is £129. 8*s.* 7$\frac{173}{400}$*d.*

(2) £500. is £8. 8*s.* 5$\frac{3}{4}$ $\frac{49}{73}$*d.*

(3) £843. 12*s.* 6*d.* is £5. 12*s.* 8$\frac{65}{584}$*d.*

(4) £1074. 10*s.* 4*d.* is £20. 19*s.* 6·037*d.*

(5) £419. 12*s.* 1*d.* is £12. 9*s.* 5$\frac{1}{2}$ $\frac{87}{365}$*d.*

(6) £1510. 11*s.* 6*d.* is £23. 11*s.* 9·552*d.*

(7) £457. 18*s.* is £7. 16*s.* 6$\frac{3}{4}$ $\frac{729}{9125}$*d.*

(8) £503. 7*s.* 9*d.* is £14. 6*s.* 10$\frac{2}{5}$*d.*

(x.) Find the Principal when the Interest and Discount are respectively—

(1) £3. 9*s.* 4$\frac{0}{4}$ $\frac{20}{73}$*d.*; £3. 9*s.* 0$\frac{1}{2}$ $\frac{478}{733}$*d.*

(2) £1. 7*s.* 7$\frac{0}{4}$ $\frac{124}{365}$*d.*; £1. 7*s.* 6*d.*

(3) £3. 1*s.* 2$\frac{1}{2}$ $\frac{5374}{9375}$*d.*; £1. 13*s.*

(4) £24. 10*s.* 3·8*d.*; £23. 14*s.* 6*d.*

(5) £114. 4*s.* 8$\frac{1}{4}$*d.*; £101. 10*s.* 10*d.*

(6) £41. 13*s.* 10$\frac{1}{5}$*d.*; £38. 5*s.*

(7) £31. 12*s.* 2·49*d.*; £24. 6*s.*

(8) £364. 17*s.* 9·3*d.*; £351. 14*s.*

(xi.) Find the Interest when the Discount on—

(1) £126. 13*s.* 4*d.* is £1. 13*s.* 4*d.*

(2) £250. is £2. 3*s.* 4$\frac{1}{4}$ $\frac{211}{269}$*d.*

(3) £4120. 8*s.* 7*d.* is £120. 0*s.* 3*d.*

(4) £520. 17*s.* 6*d.* is £70. 17*s.* 6*d.*

(5) £151. 17*s.* 6*d.* is £26. 17*s.* 6*d.*

(6) £214. 10*s.* is £5. 4*s.* $7\frac{1}{2}\frac{18}{41}$*d.*

(7) £170. is £2. 15*s.* $8\frac{3}{4}\frac{25}{61}$*d.*

(8) £27723. 10*s.* is £2803. 10*s.*

(xii.) Find the difference between the Interest and Discount on—

(1) £100. for 5 years at 5%.

(2) £690. 3*s.* 9*d.* for 9 months at 3%.

(3) £243. 10*s.* for 5 months at $3\frac{1}{2}$%.

(4) £6192. 2*s.* for 4 months at $3\frac{3}{4}$%.

(5) £923. 11*s.* 9*d.* for 3 months at $7\frac{1}{2}$%.

(6) £598. 9*s.* 9*d.* for 1 yr. 115 dys. at $2\frac{1}{2}$%.

(7) £376. 16*s.* 3*d.* for 5 months at $3\frac{3}{4}$%.

(8) £210. 12*s.* 1*d.* for $3\frac{1}{2}$ years at $4\frac{1}{4}$%.

(xiii.) What does a bank gain by discounting a bill for—

(1) £500. drawn Jan. 1 at 4 months at 5%.

(2) £843. 12*s.* 6*d.* drawn July 17 at 2 months at $3\frac{3}{4}$%.

(3) £843. 12*s.* 6*d.* drawn July 17 at 2 months, discounted Aug. 11 at $3\frac{3}{4}$%.

(4) £500. drawn Feb. 1 at 4 months at 5%.

(5) £419. 12*s.* 1*d.* drawn March 6 at 7 months at 5%.

(6) £419. 12*s.* 1*d.* drawn March 6 at 7 months, discounted Sept. 15 at 5%.

(7) £500. drawn March 1 at 4 months at 5%.

(8) £457. 18*s.* drawn Sept. 12 at 5 months at 4%.

III. COMPOUND INTEREST.

203. In *Compound Interest* the interest is not drawn, but is added from time to time as it becomes due to the principal, and the amount forms a new principal for the next period; and the difference between the final amount and the original principal is called the *Compound Interest.*

Ex. 1. Find the amount of £45. 18*s.* 9*d.* for 4 years at 7%.

£1. in 1 year would amount to £1·07.

To find what £1·07. in 1 year would amount to at 7%, multiply by 7, divide by 100, and add the quotient to £1·07. Thus—

1·0700
701
10700
749
1·1449 amount in 2 years
multiply by 7, and divide by 100 80143
1·225043 amount in 3 years
8575301
1·31079601 amount in 4 years

∴ amount of £45. 18*s*. 9*d*. will be the product of this and 45·9375.

1·31079601
573954
524318404
65539800
11797164
393238
91755
6553
60·2146914 = £60. 4*s*. 3½*d*. + —.

Ex. 2. Find the amount of £2554. 12*s*. 9*d*. in 3 years at 6¾%.

£1. in one year would amount to £1·0675.

Product by	·71	1·0675	
”	·06	64050	
”	·007	74725	
”	·0005	53375	
		1·13955625	amount in 2 years
”	·06	6837337	
”	·007	797689	
”	·0005	56977	
∴ ”	1·0675	1·21647628	amount of £1. in 3 years
£2554. 12*s*. 9*d*. =	2554·6375	1·57364552	
		243295256	
		60823814	
		6082381	
		486599	
		72988	
		3649	
		851	
		61	
		3107·65590	= £3107. 13*s*. 1·416*d*.

x. 3. Find the Interest of £917. 5*s.* 9*d.* for 2¾ years at 4¾%.

unt of £1. in 1 year is £1·0475.

Product by ·04 419

" ·007 7325

" ·0005 52375

1·09725625 amount in 2 years

" ·04 4389025

" ·007 768079

" ·0005 54862

∴ ·05211966 = interest for 3rd year

3

4)·15635898

·03908974 interest for ¾ of 3rd year

∴ 1·13634599 amt. of £1. in 2¾ years

5782719

1022711391

11363460

7954432

227269

90907

7954

568

∴ Amount of Principal is £1042·355981.

Principal is £917·2875.

∴ Interest required is £125·068481.

or £125. 1*s.* 4¼*d.* + —.

Ex. 4. What will £1600. amount to in 5 years at 4%, the rest being paid quarterly?

Here there are 20 periods of a quarter each, and the rate £4. = 1 %.

1st Principal £1.

Amount at end of 1st period	1·01
" " 2nd "	1·0201
" " 3rd "	1·030301
" " 4th "	1·040604
" " 5th "	1·05101

Amount at the end of 10th period $= (1·05101)^2$

= 1·0510100

525505

10510

105

1·104622

Amount at the end of 20th period $= (1·104622)^2$

$= 1·104622$
110462
4418
662
22
2

1·220188
1600

1952·3008 $=$ £1952. 6*s*. 0¼*d*.

Ex. 5. What is the Compound Interest on £1380. 17*s*. 4*d*. for 4 yrs. 6 m. 3 wks. at 2⅕%?

1·022
2044
2044

1·044484 amount in 2 yrs. of £1.
4844401

1044484
41779
4178
418
83
4

1·090946 amt. in 4 yrs. of £1. (1)

·021819
2181

6 months $= \frac{1}{2}$ | ·024000 int. in the 5th yr. of £1.
3 weeks $= \frac{1}{8}$ | ·012000 (2)
·001500 (3)

1·104446 amt. of £1. 4 yrs. 6 m. 3 w. (1) + (2) + (3)

£1380. 17*s*. 4*d*. $=$ £1380·86̇ 6680831

1104446
331333
88355
883
66
7

1525·090 amount of £1380. 17*s*. 4*d*.
1380·866

144·223 int. $=$ £144. 4*s*. 5½*d*.

Ex. 6. What sum will amount to £405. 3*s.* 4½*d.* in 4 years at 5%, or what is its present worth?

Amount of £1. at the end of 1st year = 1·05
" " 2nd " = 1·1025
" " 3rd " = 1·157625
" " 4th " = 1·21550625

∴ P. W. of £1·215506 is £1.; what is the P. W. of £405. 3*s.* 4½*d.* = £405·16875.?

1215506)40516875(333333
405169
40518
4053
407
43
7 *Ans.* = £333. 6*s.* 8*d.*

Ex. 7. In what time will £75. 14*s.* 9*d.* amount to £84. 11*s.* 1½*d.* at 4¼%?

£75. 14*s.* 9*d.* = £75·7375.; £84. 11*s.* 1½*d.* = £84·5563.

757375)845563(111644
88188
12451
4878
334
32

∴ $(1\cdot0425)^n = 1\cdot11644$ (1)
1·0425
417
21
5

∴ $(1\cdot0425)^2 = 1\cdot0868$ (2)
434
21
5

∴ $(1\cdot0425)^3 = 1\cdot1328$ (3)

∴ (1) lies between (2) and (3).

∴ time required $= \frac{(1)-(2)}{(3)-(2)} = \frac{\cdot02964}{\cdot0460} = \cdot644 = 7$ m. 21 dys.,

or, 2 yrs. 7 m. 21 days is the Answer.

Ex. 8. Find the *Discount* on £93. 10*s*. $8\frac{1}{4}$*d*. for $1\frac{1}{2}$ years at $4\frac{3}{4}$% payable half-yearly.

Here we have 3 periods at $2\frac{3}{8}$% per period.

$$\begin{array}{lr}
\text{Amount of £1. 1st period} = & 1{\cdot}02375 \\
 & 2047 \\
 & 307 \\
 & 71 \\
 & \underline{5} \\
\text{,, \quad ,, \quad 2nd period} = & 1{\cdot}04806 \\
 & 2096 \\
 & 314 \\
 & 73 \\
 & \underline{5} \\
\text{,, \quad ,, \quad 3rd period} = & 1{\cdot}07295
\end{array}$$

$$\therefore \text{Present Worth} = \frac{\text{£}93.\ 10s.\ 8\frac{1}{4}d. \times 1}{1{\cdot}07295} = \frac{93{\cdot}5347}{1{\cdot}07295}$$

$$\begin{array}{l}
107295)935347(871750 = \text{£}87.\ 3s.\ 6d. \\
\quad\quad 76987 \\
\quad\quad 1881 \\
\quad\quad 808 \\
\quad\quad 57 \\
\quad\quad 5 \\
\quad\quad 1
\end{array}$$

∴ Discount £6. 7*s*. $2\frac{1}{4}$*d*.

Ex. 9. At what rate per cent. will £250. amount to £270. 12*s*. 1·9296*d*. in 2 years, the Interest being payable every half year?

Here we have four periods.

£270·60804.÷£250. = 1·08243216, the 4th root of this is 1·02,

∴ the rate per cent. is £4.

This Problem is always possible when the period is a factor of 2 or 3, because the square or cube root of the amount of £1. for the whole time can be extracted.

204. These examples are illustrations of the equation $P \times (1{\cdot}0\,r)^n = A$, when 3 quantities are given to find the 4th, also if P in n years amounts to A, and in m years to B: given n, m, A, B, find P, $P\,(1{\cdot}0\,r)^n = A$, $\quad P\,(1{\cdot}0\,r)^m = B$, $\quad \therefore (1{\cdot}0\,r)^{m-n} = B \div A$,

$$\therefore 1{\cdot}0\,r = \left(\frac{B}{A}\right)^{\frac{1}{m-n}}, \qquad \therefore P = A \times \left(\frac{A}{B}\right)^{\frac{n}{m-n}} = \left(\frac{A^m}{B^n}\right)^{\frac{1}{m-n}}$$

Exercise 134.

Find the amount of—

(i.) (1) £250. for 3 years at 5%.
(2) £1953. 2*s.* 6*d.* for 4 years at 4%.
(3) £1000. for 4 years at 5%.
(4) £830. 6*s.* 8*d.* for 2 years at 3%.
(5) £1167. 3*s.* 9*d.* for 2 years at 4%.
(6) £6250. for 3 years at 5%.

(ii.) (1) £150. for 3 years at $2\frac{2}{3}$%.
(2) £1001. 5*s.* for 3 years at $3\frac{1}{3}$%.
(3) £2800. for 4 years at $4\frac{1}{2}$%.
(4) £1500. for 6 years at $5\frac{1}{4}$%.
(5) £3650. 10*s.* for 3 years at $5\frac{1}{2}$%.
(6) £5000. for 3 years at $4\frac{1}{2}$%.

(iii.) (1) £1785. 13*s.* 8*d.* for $3\frac{3}{4}$ years at $4\frac{1}{4}$%.
(2) £850. 12*s.* 6*d.* for $4\frac{1}{2}$ years at 5%.
(3) £10. for $2\frac{1}{2}$ years at 5%.
(4) £300. for $2\frac{1}{2}$ years at $3\frac{3}{4}$%.
(5) £917. 5*s.* 9*d.* for $2\frac{3}{4}$ years at $4\frac{3}{4}$%.
(6) £439. 18*s.* 4*d.* for $4\frac{5}{12}$ years at $5\frac{3}{4}$%.

(iv.) (1) £875. in 2 years at 5%, interest payable quarterly.
(2) £550. in $2\frac{1}{2}$ years at 4%, interest payable half-yearly.
(3) £1670. in $2\frac{1}{2}$ years at 5%, interest payable half-yearly.
(4) £1441. 9*s.* 4·924*d.* in 1 year at 6%, interest payable quarterly.
(5) £1073. 8*s.* 8·29*d.* in $1\frac{1}{2}$ years at 6%, interest payable half-yearly.
(6) £525. in $1\frac{1}{2}$ years at 5%, interest payable quarterly.

(v.) (1) £875. 15*s.* 6*d.* in 3 years 7 months at $3\frac{1}{3}$%.
(2) £1198. 18*s.* 9·0384*d.* in 3 years at $4\frac{1}{2}$%.
(3) £20000. in 4 years 146 days at 5%.
(4) £854. 3*s.* 6*d.* for 4 years 5 months at £$4\frac{1}{4}$%.
(5) £230. 15*s.* for 3 years 7 months at £$4\frac{1}{2}$%.
(6) £45. 12*s.* 6*d.* for $3\frac{1}{2}$ years at $3\frac{1}{2}$%.

Find the Present Worth of—

(vi.) (1) £178. 8*s.* $0\frac{1}{2}$*d.* in 5 years at 4%.

(2) £452. 11*s.* $3\frac{803}{3200}$*d.* in $2\frac{1}{2}$ years at 5%, payable half-yearly.

(3) £397. 18*s.* 0·86*d.* in 4 years at $3\frac{1}{2}$%.

(4) £695. 3*s.* 9·66*d.* in 4 years at $3\frac{3}{4}$%.

(5) A sum which in 1 year amounts to £653. 2*s.* 6*d.*, and in 3 years to £713. 4*s.* 6·91875*d.*

(6) £893. 8*s.* $4\frac{1}{8}$*d.* in 3 years at $3\frac{3}{4}$%.

In what time will—

(vii.) (1) The interest on £4500. at 4% become £663. 2*s.* 6·1824*d.*?

(2) £325. at 5% amount to £395. 0*s.* $9\frac{39}{80}$*d.*?

(3) £399. 11*s.* 11*d.* at 5% amount to £509. 19*s.* $11\frac{1}{4}$*d.*?

(4) £184. 12*s.* 6·93*d.* produce £48. 9*s.* 2·6*d.* interest at 6%?

(5) £180. amount to £205. 8*s.* 2·376*d.* at $4\frac{1}{2}$%?

(6) £2645. amount to £2807. 14*s.* $4\frac{3}{4}$*d.* at 4%, payable quarterly?

Find the discount on—

(viii.) (1) £4525. 12*s.* 7·884375*d.* for $2\frac{1}{2}$ years at 5%, interest payable half-yearly.

(2) £1721. 5*s.* $8\frac{1}{4}$*d.* due 4 years hence at $3\frac{1}{2}$%.

(3) £893. 0*s.* 1·56768*d.* due 2 years hence at 4%, payable half-yearly.

(4) £5264. 4*s.* 2·15625*d.* due 3 years hence at $3\frac{1}{8}$%.

(5) £270. 12*s.* 1·9296*d.* due 2 years hence at 4%, payable half-yearly.

(6) £146. 8*s.* $0\frac{1}{4}$*d.* due 3 years hence at 4%, payable half-yearly.

At what rate % will—

(ix.) (1) £333. 6*s.* 8*d.* amount to £405. 3*s.* $4\frac{1}{2}$*d.* in 4 years?

(2) £1500. amount to £1823. 5*s.* $2\frac{1}{4}$*d.* in 4 years?

(3) £267. 13*s.* $5\frac{1}{2}$*d.* amount to £304. in 3 years?

(4) £1260. amount to £1532. 19*s.* 7·84*d.* in 5 years?

(5) £399. 11*s.* 11*d.* amount to £462. 11*s.* $7\frac{3}{4}$*d.* in 3 years?

(6) £246. 13*s.* 4*d.* amount to £303. 4*s.* 3·9*d.* in 6 years?

Find the difference between the Simple and Compound Interest on—

(x.) (1) £945. 16*s.* 8*d.* in 4 years at 6%.
(2) £6400. in 3 years at 3¾%.
(3) £415. 10*s.* in 4 years at 2¼%.
(4) £1640. in 3 years at 3½%, payable every 6 months.
(5) £100. in 2 years at 5%, reckoned quarterly.
(6) £2000. in 3 years at 2½%.

205. The following examples worked out will be found useful:—

Ex. 1. The difference between the Simple and Compound Interest of a certain sum for 3 years at 4½% is £8. 13*s.* 7¾*d.*; find the sum.

£8. 13*s.* 7¾*d.* = £8·682291$\dot{6}$.

Amt. of £1. Com. Int. = $(1·045)^3$ = 1·045
4180
5225
1·092025
43681
5460125
1·141166125
Amount of £1. Simple Interest = 1·135
∴ Difference = ·006166125

6166125)8682292(1408·0628
2516167
49717
388
18
5
0

∴ sum required = £1408. 1*s.* 3*d.*

Ex. 2. The difference between the Simple and Compound Interest of £527. 17*s.* 6*d.* at 4¾% is £3. 3*s.* 9¼*d.*: for what time is the Interest calculated?

£527. 17*s.* 6*d.* = £527·875.; £3. 3*s.* 9¼*d.* = £3·188542.

∴ the difference on the Interests of £1 = 3·188542 ÷ 527·875.

[illegible]

∴ the difference [illegible]

and the difference on given sum = [illegible]

∴ sum required = [illegible]

[illegible]

[illegible] find the sum and rate [illegible]

[illegible] = 1·1025 = (1·05)²

[illegible]

[illegible] and the rate 5%

EX. 5. £650. is put out at 4% for 1 year: what must be the Interest of the 2nd year that the amount for the 2 years may be £706. 8*s.* 4·8*d.*

The amount of £1. for the 2 years = £706·42 ÷ 650 = 1·0868;

$$\therefore\ 1{\cdot}04 + 1{\cdot}04 \times ({\cdot}0\,r) = 1{\cdot}0868$$

$$1{\cdot}04 \times ({\cdot}0\,r) = {\cdot}0468\,;$$

$$\therefore\ {\cdot}0\,r = {\cdot}0468 \div 1{\cdot}04 = {\cdot}045\,;$$

$$\therefore\ r = 4\tfrac{1}{2}\%.$$

Exercise 135.

(1) The difference between the simple and compound interest for 2½ years at 5% on a certain sum is 1*s.* $0\frac{3}{20}$*d.*: on what sum is the interest calculated?

(2) The difference between the simple and compound interest at 4% of £1250. is £12. 6*s.* $5\frac{71}{125}$*d.*: for how many years is the interest calculated?

(3) The difference between the simple and compound interest on £195. 6*s.* 3*d.* for 3 years is 19*s.*: at what rate was the interest charged?

(4) The difference between the simple and compound interest on £230. 15*s.* at 4½% is £2. 5*s.* 7·695*d.*: for how long is the interest calculated?

(5) The compound interest on a sum of money in 3 years at 4% is increased by £2. 10*s.* 8·634*d.* when the interest is made payable half-yearly instead of yearly: what is the sum?

(6) What rate of simple interest will produce the same amount in 4 years as 6% compound interest in the same time?

(7) The amount of a sum of money in 3 years is £926. 2*s.*, and in 5 years is £1021. 0*s.* $6\frac{3}{50}$*d.*: what is the sum and rate %?

(8) Find the present worth at simple interest of the amount of £515. 12*s.* 4·293*d.* in 3 years at 4% compound interest.

(9) £300. at 3¾% compound interest amounts to £328. 19*s.* $6\frac{51}{128}$*d.*: what will it amount to at the same time and rate at simple interest?

(10) The difference between the simple and compound interest on £2750. at 5% is £72. 5*s.* 5¾, $\frac{13}{40}$*d.*: what is the number of years?

(11) Find the amount at simple interest of the present worth of £5705. 16*s.* $7\frac{12}{45}$*d.* for 3 years at 4½% compound interest.

(12) £1500. at compound interest amounts in 3 years to

£1591. 13*s.* 2·16*d.*, the rates of the 2nd and 3rd years are 2% and 1% respectively: what is the rate % the 1st year?

(13) If £650. amounts, at compound interest, to £699. 13*s.* 2·4*d.* in 2 years, and the rate for the 2nd year is 3½%: what is the rate for the 1st year?

(14) A man saves every year a certain sum, and invests it at the end of the year at 5% compound interest. The amount of his savings at the end of the 4th year is £646. 10*s.* 4½*d.*: what sum does he save every year?

(15) If the compound interest on a certain sum in 3 years at 4% is £1. 5*s.* $11\frac{10321}{15625}$*d.*: what is the simple interest on the same sum?

(16) If the simple interest on a certain sum in 4 years at 5% is £4. 6*s.*: what is the compound interest on the same sum?

(17) The amount of £7123. 4*s.* 7*d.* at 4% compound interest is £7781. 10*s.* $7\frac{52}{625}$*d.*: what is the amount at simple interest?

(18) The amount of 10 guineas at simple interest in 3 years is £11. 11*s.* 6*d.*: what is the amount at compound interest?

(19) The difference in compound interest on a sum in 3 years at 4%, and in 4 years at 3%, is 12*s.* 10·7544*d.*: what is the sum?

(20) What sum due in 2½ years will have the same present value at 4% simple interest as £2758. 1*s.* 7·2*d.* due at the same time and at the same rate compound interest?

(21) A sum of money was put out to compound interest; the 1st year's interest was £195. 6*s.* 3*d.*, and the fourth year's interest was £219. 14*s.*: what was the sum and the rate %?

XIX. CHAIN RULE.

206. In *Chain Rule* we have a series of quantities of different kinds, and we have given the relations between the 1st and 2nd, the 2nd and 3rd, &c. to find what quantity of the last kind equals a given quantity of the 1st kind.

Ex. 1. Thus: if 12 of A = 13 of B, 6 of B = 18 of C, and 13 of C = 2 of D: how many of A = 100 D?

$$100\,D = 650\,C = \frac{6 \times 650}{18}\,.\,B = \frac{\cancel{6} \times \overset{50}{\cancel{650}} \times \overset{4}{\cancel{12}}}{\underset{3}{\cancel{18}} \times \cancel{13}}\,.\,A = 200\,A.$$

This can be most easily done thus—

$$\begin{aligned} 12\,\text{A} &= 13\,\text{B} \\ 6\,\text{B} &= 18\,\text{C} \\ 13\,\text{C} &= 2\,\text{D} \\ 100\,\text{D} &= x\,.\,\text{A}. \end{aligned}$$

Now multiply all these equations together, and it will be seen

$$12 \times 6 \times 13 \times 100 = 13 \times 18 \times 2 \times x.$$

$$\therefore\ x = 200.$$

Ex. 2. If a journey of 506 kilomètres cost 56 fr. 80 c.: find the rate per mile in pence, if 5 francs = 4 shillings, and 8 kilomètres = 5 inches.

$$\begin{aligned} 506\ \text{k.} &= 56{\cdot}8\ \text{fr.} \\ 5\ \text{in.} &= 8\ \text{k.} \\ 5\ \text{fr.} &= 4s. \\ 1s. &= 12d. \\ \therefore\ 12650\ \text{in.} &= 21831{\cdot}2d. \end{aligned}$$

$$\begin{array}{r} 12650)21831(1{\cdot}7242d. \\ 2101 \\ 306 \\ 53 \\ 3 \\ 1 \end{array}$$

Ans. = 1·7242*d.*

Ex. 3. 1 mètre = 39·37 inches, express inches per hour in mètres per second. Let x be the number of inches per hour, y the number of mètres per second.

$$\begin{aligned} x\ \text{inches} &= y\ \text{mètres} \\ 1760\ \text{yards} &= 1\ \text{mile} \\ 3\ \text{feet} &= 1\ \text{yard} \\ 12\ \text{inches} &= 1\ \text{foot} \\ 1\ \text{hour} &= 60 \times 60\ \text{seconds.} \\ 1\ \text{mètre} &= 39{\cdot}37\ \text{inches.} \end{aligned}$$

$$\therefore\ y = x\,.\,\frac{1760 \times 3 \times 12}{60 \times 60 \times 39{\cdot}37} = \frac{1760}{393{\cdot}7} = {\cdot}447\ \text{the multiplier.}$$

Ex. 4. Find the weight in tons per mile of wire which weighs ·03 of an ounce troy to the inch run.

$$\begin{aligned} x\ \text{tons} &= 1\ \text{mile} \\ 2240\ \text{lbs.} &= 1\ \text{ton} \\ 1\ \text{mile} &= 1760 \times 3 \times 12\ \text{inches} \\ 7000\ \text{grs. troy} &= 1\ \text{lb. avoirdupois} \\ 1\ \text{oz. troy} &= 24 \times 20\ \text{grs. troy} \\ 1\ \text{inch} &= {\cdot}03\ \text{oz. troy} \end{aligned}$$

$$\therefore\ x = \frac{{\cdot}03 \times 1760 \times 3 \times 12 \times 24 \times 20}{2240 \times 7000} = {\cdot}05817.$$

Ex. 5. The 'acceleration' of a falling body due to gravity is proportional to the square of the time; it is 32 when the units of space and time are feet and seconds: what would it be when the units are miles and hours?

1 second = 32 feet
1760×3 feet = 1 mile
x miles = $(60 \times 60)^2$ seconds;

$$\therefore x = \frac{60 \times 60 \times 60 \times 60 \times 32}{1760 \times 3} = 78545{\cdot}45.$$

Ex. 6. An acceleration which is represented by 32·2 when a second and a foot are units, is represented by 9660 when a yard is the unit of length: find the unit of time.

1 second = 32·2 feet
3 feet = 1 yard
9660 yards = (t^2) seconds;

$$\therefore t = \sqrt{\frac{9660 \times 3}{32{\cdot}2}} = 30 \text{ seconds.}$$

207. The dimensions of the principal mechanical units are tabulated below—

Length = L, Mass = M, Time = T (1)
Area = L^2 (2)
Volume = L^3 (3)
Density = $\frac{M}{V} = M \div L^3$ (4)
Velocity = $L \div T$ (5)
Acceleration = $L \div T^2$ (6)
Momentum or Impulse = $M \times L \div T$ (7)
Force = $M \times L \div T^2$ (8)
Work or Energy = $M \times L^2 \div T^2$ (9)
Power or Rate of Doing Work = $M \times L^2 \div T^3$ (10)

Ex. 1. A force is represented by 32·18 when a second, a foot, and a pound are the units of time, length, and mass respectively: what will be the measure of the force when the unit of length is the centimètre, the unit of mass the gramme, the unit of time remaining the same; it being given that a gramme = 15·432 grains, and a centimètre = ·03281 feet?

Let m, l, t be the new units, Q the new measure force; then from (8)

$$Q = m \times l \div t^2, \text{ then}$$

$$m = \frac{\text{pound}}{\text{gramme}} . M = \frac{7000}{15{\cdot}432} . M$$

$$l = \frac{\text{foot}}{\text{cent.}} . L = \frac{1}{{\cdot}03281} L$$

$$t = T;$$

$$\therefore\ Q = \frac{7000}{15{\cdot}432} \times \frac{1}{{\cdot}03281} \times M . L \div T^2 = \frac{7000 \times 32{\cdot}18}{15{\cdot}432 \times {\cdot}03281}$$

$$= 444893.$$

Ex. 2. Joule found that the mechanical equivalent of heat was 772 foot-pounds, the unit of heat being that which would raise 1 lb. of water 1° F.: what will be the value of Joule's equivalent in kilogramme mètres when the unit of heat is the amount necessary to raise 1 kilogramme of water 1° C.?

When a weight W falls through a space L, the heat equivalent of work done is

$$H = \frac{WL}{J};$$

$$\therefore\ J = \frac{WL}{H}.$$

Let w, h, l be the new units, and j the value of Joule's equivalent; then

$$w = \frac{\text{pound}}{\text{kilog.}} . W = \frac{1}{2{\cdot}05} . W$$

$$l = \frac{\text{foot}}{\text{mètre}} . L = \frac{1}{3{\cdot}25} . L$$

$$h = \frac{\text{F. }1^\circ}{\text{C}} \times \frac{\text{pound}}{\text{kilog.}} . H = \frac{5}{9} \times \frac{1}{2{\cdot}05} . H = \frac{1}{3{\cdot}69} . H;$$

$$\therefore\ j = \frac{1}{2{\cdot}05} \times \frac{1}{3{\cdot}25} \div \frac{1}{3{\cdot}69}\ \frac{WL}{H}$$

$$= {\cdot}548 \times J$$

$$= 423 \text{ kilogramme mètres.}$$

Ex. 3. An *erg* is the work done in moving a gramme through a centimètre against a unit of acceleration. The English standard of working power is called the 'horse-power,' and is defined as the work done in raising 33000 lbs. one foot in one minute: convert this into ergs.

1 centimètre = ·3937079 inch; 1 gramme = 15·43235 grains; 1 erg = 981 cm.-gm.;

$$\therefore\ 1\text{ ft.} = \frac{12}{\cdot 3937079}\text{cms.} \qquad 1\text{ lb. avd.} = 7000\text{ grains troy}$$

$$= \frac{7000}{15{\cdot}43235}\text{gms.};$$

$$\therefore\ 1\text{ ft.-lb.} = \frac{12 \times 7000}{15{\cdot}43235 \times {\cdot}3937079}\text{cm.-gms.}$$

$$= \frac{12 \times 7000 \times 981}{15{\cdot}43235 \times 3937079}\text{ergs.};$$

$$\therefore\ 1\text{ h.-p.} = 33000\text{ ft.-lb. per minute}$$

$$= \frac{33000}{60} \quad \text{,,} \quad \text{second}$$

$$= \frac{33000 \times 12 \times 7000 \times 981}{60 \times 15{\cdot}43235 \times {\cdot}3937079}\text{ergs}$$

$$= 7460\text{ million ergs.}$$

Exercise 136.

(1) Given that 617 A = 100 B, 11 B = 57 C, 17 C = 3 D, 69 D = 64 E, 32 R = 5 S: find how many of S = 3760 A.

(2) If the rent of land in France is 150 francs per hectare, the rent per acre is £2. 10*s*. 7·287*d*., and 100 hectares being equal to 247 acres, what is the value of a franc?

(3) Express one mile in kilomètres, 1 kilomètre = 1,000 mètres, 64 mètres = 70 yards.

(4) £1. in London = 25 francs 19$\frac{7}{47}$ cents. in Paris, 35$\frac{1}{4}$ florins at Amsterdam = 40 marks banco at Hamburg, and 100 marks banco = 185 francs in Paris: how many florins in Amsterdam = £1. in London?

(5) How many tons of sugar at £2. 7*s*. 6*d*. per cwt. must be given in exchange for 860 lbs. of tea at 2*s*. 4$\frac{1}{2}$*d*. a lb.?

(6) 14 thalers are coined from 3609 grains of fine silver: find the value of an ounce of English standard silver, of which 37 parts out of 40 are fine, when the value of a thaler is 2*s.* 11½·51689*d.*

(7) £116. 17*s.* 5¼·6*d.* is the rent of 87 ac. 3 r. 25 p.: how many francs must I pay for 45 hectares, 75 centiares, when 100 hectares = 247 acres, and 25 francs = £1.?

(8) One cubic mètre = 1000 litres, one mètre = 39·4 inches: find the number of cubic inches in one litre.

(9) If 12 cwt. 3 qrs. 21 lbs. of coal cost £12. 14*s.* 4½*d.*, how many cwt. can be bought for £203. 10*s.*?

(10) Express 32 Spanish piastres in French money, exchange being at the rate of 25⅓ francs for £1., and £3. 7*s.* 6*d.* for 20 piastres.

(11) The weight of a cubic inch of water is 253·17 grains, that of a cubic inch of air ·31 grains: find to three places of decimals how many cubic inches of water are equal in weight to one cubic foot of air.

(12) Convert ·321875 of a ton into kilogrammes, assuming a cubic decimètre of distilled water weighs 15432·35 grains.

(13) A vessel steams 16 knots per hour: how many mètres per second is this equivalent to, taking a mètre as 3·2809 feet, and a knot as 6086·5 feet?

(14) A gallon of fresh water measures 277·271 c. in. and weighs 10 lbs. avoirdupois. A ton of sea water measures 35 c. ft. What is the weight of a gallon of sea water in pounds?

(15) Convert 77 feet per second into nautical miles per hour. One nautical mile = 6086·5 feet.

(16) An acre = ·40467 hectares and £1. = 25·25 francs. An estate measuring 1927 hectares is sold for 10100000 francs. What is the selling price per acre in £. *s.* *d.*?

(17) The rainfall at a particular place is 29·2 inches in a year, or 662475 gallons per acre: express the volume of a gallon in feet.

(18) Given that in any circle an arc : the circumference :: angle subtended by the arc : 360°, and that the circumference of a circle = 3⅐ × diameter, find to 4 places of decimals in inches the length of an arc which subtends an angle of one minute at a distance of 100 yards.

(19) 10 grades = 9 degrees. The angle which is subtended by an arc equal to the radius is 206264·8 seconds. Reduce this to grades, and from (18) deduce the value to 6 places of the quotient of the diameter by the circumference of a circle.

(20) Find the length in degrees, minutes, and seconds of an arc of a circle equal to twelve times the radius, having given radius = circumference × ·159155.

(21) Find the number of grades in an arc which is equal in length to the radius of the circle.

(22) Given that diameter = circumference × ·31831, find to the nearest minute the angle whose arc equals three times the radius.

(23) An imperial gallon = 277·274 c. in.; a Winchester bushel = 2150·42 c. in.: how many Winchester bushels = 100 imperial bushels?

(24) A mètre = 3·2809 ft.: how many square mètres are there in 1000 sq. yds.?

(25) A cubic foot of copper weighs 560 lbs: it is rolled into a square bar 40 feet long: an exact cube is cut from the bar: what is its weight to 4 places of decimals?

(26) 1 mètre = 39·3708 inches. How many centimètres are there in a foot? How many sq. cms. are there in a sq. ft.? How many cub. cms. are there in a cub. ft.?

(27) Express a velocity of 33527·7 cm. per second in feet and seconds.

(28) How many yards per minute and miles per hour are described by a body which travels at the rate of 1000 cms. per second?

(29) The acceleration of gravity is measured by 981 in the metrical system: find its numerical value in feet and seconds.

(30) What is the volume in cub. cms. of 1·5 kilogrammes of iron whose density is 7·25?

(31) Compare the momenta of a cannon ball of 600 lbs. moving at the rate of 1000 ft. per second, and that of an express train of 100 tons moving at the rate of 40 miles per hour.

(32) Compare also the work done in stopping the cannon ball and train in the preceding question.

(33) A weight of $1\frac{1}{2}$ tons is to be raised from a depth of 50 fathoms in a minute: determine the horse-power of the engine capable of doing the work.

(34) An engine is required to raise a weight of 13 cwts. from a depth of 140 fathoms in 3 minutes: determine its horse-power.

(35) How many cubic feet of water would an engine working at 110 horse-power raise per minute from a depth of 25 fathoms?

(**36**) There is a mine with three shafts, which are respectively 300, 450, and 500 feet deep; it is required to raise from the 1st 80, from the 2nd 60, from the 3rd 40 cubic feet of water per minute: what must be the horse-power of the engine?

(**37**) The equatorial diameter of the earth is 41847000 feet, and the earth makes one revolution in 86164 seconds: determine the velocity of a point on the earth's equator.

(**38**) A body moves with a velocity 12: how many miles will it pass over in one hour? What would be its velocity if we used yards and minutes as units instead of feet and seconds?

(**39**) Through how many inches would a body falling freely from rest descend in one minute?

(**40**) Feet and seconds being units, the numerical value of the accelerating force of gravity is 32: what would this become if the unit of space were a fathom, and the unit of time 15 seconds?

(**41**) Given that the accelerating force of gravity at the distance of the moon equals $\frac{1}{112}$ in feet and seconds: find its value in miles and hours.

(**42**) Given that the velocity generated in one second by gravity in a body falling freely is 32 feet per second, and the unit of mass being one ton, the unit of force one hundredweight, and the unit of velocity a velocity of 8 feet per second: find the unit of time and length.

(**43**) If an Armstrong shaft of 700 pounds moving with a velocity of 1200 feet per second strike an iron target, and be brought to rest, what will be the measure of the impulse when one hundredweight is the unit of mass, a yard the unit of length, and a minute the unit of time?

XX. ALLIGATION.

208. *Alligation*, or the mixing of things of the same kind, but of different qualities, will be understood from the consideration of the following examples:—

Ex. 1. A grocer mixes 5 lbs. of tea at 2*s*. 3*d*. with 3 lbs. at 2*s*. 7*d*.: find the price of the mixture.

$$\begin{array}{rrr} & s. & d. \\ \text{5 lbs. at } 2s.\ 3d. = & 11 & 3 \\ \text{3 lbs. at } 2s.\ 7d. = & \underline{7} & \underline{9} \\ \therefore\ \text{8 lbs. of mixture} = & 19 & 0 \end{array}$$

$$\therefore\ \text{1 lb. of mixture} = \frac{19s.\ 0d.}{8} = 2s.\ 4\tfrac{1}{2}d.$$

N.B.—Here we are given the quantity and price of each of the elements of the mixture to find the price of the mixture.

209. Next, given the prices of each of the elements, to find what quantity must be taken of each to make a mixture of a given price.

Ex. 1. How must a wine merchant mix sherry at 24*s.* a gallon and 32*s.* a gallon to make a mixture worth 26*s.* a gallon?

In making the mixture at 26*s.* a gallon
1 gallon at 24*s.* gives a gain of 2*s.*
1 „ 32*s.* „ loss of 6*s.*;

∴ that the gain may be equal to the loss for every one gallon of the latter, he must take 3 gallons of the former; *i.e.* he must mix them in the ratio 3 : 1, which is the inverse ratio of 2 : 6. The mixture, therefore, must be made in the inverse ratio of the differences of the prices and the mean price.

The work may be more conveniently arranged thus:

Place the prices under one another in order, write the mean price on the left.

$$26\left|\begin{array}{ll}24 & 6\\ 32 & 2\end{array}\right.$$

Place the difference between the mean price and the lowest price opposite the highest price, and the difference between the highest price and the mean price opposite the lowest price, and the mixture will be in proportion to the two remainders, *i.e.* in this case 3 : 1.

Ex. 2. If spirits at 16*s.*, 18*s.*, and 23*s.* per gallon be mixed so that the mixture is worth 20*s.* per gallon, what quantity of each must be taken?

Since two of the given prices are below the mean price, mix each of them separately with the highest price as in Ex. 1. Thus—

$$20\left|\begin{array}{ll}16 & 3\\ 23 & 4\end{array}\right. \qquad\qquad 20\left|\begin{array}{ll}18 & 3\\ 23 & 2\end{array}\right.$$

∴ the mixture must be made in the proportion 4 + 2 or 6 : 3 : 3.

Ex. 3. Mix wine at 24*s.*, 26*s.*, and 32*s.* a dozen respectively, so that the price of the mixture may be 27*s.* 8*d.* a dozen.

Reduce prices to fourpenny pieces, then the work may be arranged

$$83\left|\begin{array}{ll}72 & 13\\ 96 & 11\end{array}\right. \qquad 83\left|\begin{array}{ll}78 & 13\\ 96 & 5\end{array}\right.$$

Therefore, the proportion is 11 + 5 or 16 : 13 : 13.

N.B.—This is not the only solution; another way in which the mixture can be made is by taking 6 of the first, 5 of the second, and 7 of the third.

Ex. 4. How much brandy at 7*s.* and at 4*s.* must be mixed with 80 gallons of brandy at 12*s.* a gallon to make a mixture worth 8*s.* a gallon?

$$8\left|\begin{array}{ll}7 & 4\\ 12 & 1\end{array}\right. \qquad 8\left|\begin{array}{ll}4 & 4\\ 12 & 4\end{array}\right.$$

∴ the mixture must be : : 5 : 4 : 4;

and since there are 80 gallons at 12*s.*, there must be 64 gallons of each of the others.

Ex. 5. Mix tea at 2*s.* 9*d.*, 3*s.* 1*d.*, and 3*s.* 4*d.*, to form a mixture worth 2*s.* 10*d.* a lb.

Since here only one price is below the mean price, combine it with the others separately.

$$34\left|\begin{array}{ll}33 & 3\\ 37 & 1\end{array}\right. \qquad 34\left|\begin{array}{ll}33 & 6\\ 40 & 1\end{array}\right.$$

∴ the mixture must be made of 9 lbs. at 2*s.* 9*d.*, 1 at 3*s.* 1*d.*, 1 at 3*s.* 4*d.*

Ex. 6. Combine four sorts of gold of 24, 22, 20, and 15 carats fine respectively to make a mixture of 63 ounces of 17 carats.

Since there is only one value below the mean, combine it with the others separately.

$$17\left|\begin{array}{ll}15 & 3\\ 20 & 2\end{array}\right. \qquad 17\left|\begin{array}{ll}15 & 5\\ 22 & 2\end{array}\right. \qquad 17\left|\begin{array}{ll}15 & 7\\ 24 & 2\end{array}\right.$$

∴ the mixture must be made in the : : 3 + 5 + 7 or 15 : 2 : 2 : 2.

And since the whole weight is 63 oz., each part must be 45 oz., 6 oz., 6 oz, 6 oz.

Ex. 7. Combine tea at 2*s.* 8*d.*, 3*s.* 4*d.*, 3*s.* 8*d.*, and 4*s.* to make a mixture worth 3*s.* 6*d.* per lb. Mix each of the prices below the mean with one of the prices above it.

$$42\left|\begin{array}{ll}32 & 6\\ 48 & 10\end{array}\right.\qquad\qquad 42\left|\begin{array}{ll}40 & 2\\ 44 & 2\end{array}\right.$$

∴ we have to take in : : 3 : 1 : 1 : 5,

or $$42\left|\begin{array}{ll}32 & 2\\ 44 & 10\end{array}\right.\qquad\qquad 42\left|\begin{array}{ll}40 & 6\\ 48 & 2\end{array}\right.$$

or we may take them in : : 1 : 3 : 5 : 1.

Exercise 137.

(1) How much wine at 6*s.* per gallon and at 4*s.* per gallon must be mixed together to produce a compound worth 5*s.* per gallon?

(2) I buy 3 pounds of tea at 6*s.* per pound, and 13 pounds at 4*s.* per pound: at what price must I sell the mixture to clear one-sixth of my outlay?

(3) I have 21 ounces of gold worth £4. 5*s.* an ounce: how much silver at 4*s.* an ounce, and copper, which is of no value, must I mix with it to make a compound worth £2. 15*s.* an ounce?

(4) How many gallons of water must be mixed with wine at 5*s.* a gallon so as to fill a vessel of 50 gallons, and be sold at 4*s.* a gallon?

(5) I bought a certain number of apples at 8*d.* a dozen, 7*d.* a dozen, and 1*d.* a dozen; 2 dozen were given me; if I sell the lot at 5*d.* a dozen I just cover my expenses: how many had I altogether?

(6) Combine teas worth 3*s.* 6*d.*, 4*s.* 9*d.*, 3*s.* 9*d.*, and 5*s.* 6*d.* a pound, so that the mixture may be worth 4*s.* 6*d.* a pound.

(7) A wine merchant buys 80 bottles of wine, some at 5*s.*, some at 3*s.* 6*d.* a bottle, and after adding 20 bottles of water, he sells the mixture for £19.: how many bottles of 5*s.* wine did he buy?

(8) A cricket team all scored in a match, and their scores were 32, 35, and 39, the average score was 36: how many scored 39?

(9) On the books of a village school there are entered 23 boys, 78 girls: what should be the number of boys daily attending the school that the average total attendances for 33 days should be 58?

(10) In an examination the average of the candidates who were placed in the 1st class obtained 80%, those in the 2nd 74%, those in the 3rd 66%, in the 4th 60% of marks. If the average marks of them all was 68%, and there were 6 in the 2nd class, how many were there in the 1st?

(11) Divide 11*s.* into 2 parts, so that 13 times one added to 19 times the other shall amount to £8. 19*s.*

(12) Mix spirits 15 o. p., 8 u. p. to make a compound of strength 8 o. p. (15 o. p. means 100 gallons of it equals in strength 115 gallons of proof spirit; 8 u. p. means 100 gallons of it equals 92 gallons of proof spirit.)

XXI. PERCENTAGE.

210. We will now give some illustration of Percentages where no special reference is made to time.

All business houses regulate their transactions and calculate their profits and losses with reference to 100 as a standard.

211. Every question in Percentage can be solved by Cause and Effect at once.

The different cases that arise are—

Case (i.) Given the rate per cent. of a given quantity, find the value of it.

Ex. 1. My income is £32500., and taxes are 7½%: what do the taxes amount to?

£7½. is the first effect, 100 is the cause, the second effect is required, the second cause is £32500.;

$$\therefore 100 : 7\tfrac{1}{2} :: 32500 : x;$$

$$\therefore x = \frac{32500 \times 7\frac{1}{2}}{100} = £2637.\ 10s.$$

Hence the rule—

'Multiply the quantity by the rate per cent. and divide by 100.'

Ex. 2. Calculate 8¾ per cent. on £345. 15*s.*

£345. 15*s.* = £345·75.;

$$\therefore\ 4)\ \underline{345\cdot75 \times (9-\tfrac{1}{4})}$$
$$\begin{array}{r} 9 \\ \hline 311175 \\ 8643 \\ \hline 30\cdot2532 \end{array}$$

$\therefore$ Percentage = £30. 5s. 0·768d.

Case (ii.) Find how much per cent. one quantity is of another quantity.

Ex. 1. If out of a population of 7840, 245 cannot read, how many is that per cent.?

This means if there are 245 out of 7840, how many are there out of 100?

$$\therefore\ 245 : 7840 :: x : 100;$$
$$\therefore\ x = \frac{345 \times 100}{7840} = 7\tfrac{21}{32}.\quad \textit{Ans.}$$

Hence the rule—

'Multiply the part by 100, and divide by the whole quantity.'

Ex. 2. How much per cent. is 2d. in the shilling?

$$2 : 12 :: x : 100;$$
$$\therefore\ x = \frac{\cancel{2} \times 100}{\underset{6}{\cancel{12}}} = 16\tfrac{2}{3}.$$

Case (iii.) Given the percentage on a quantity, find the rate %.

Ex. 1. The percentage on £320. 17s. 6d. is £14. 8s. 9·45d.: what is the rate %?

£320. 17s. 6d. = £320·875; £14. 8s. 9·45d. = £14·439375;

$\therefore$ if the rate on £320. be £14, what is it on 100?

$$\therefore\ 320\cdot875 : 14\cdot439375 :: 100 : x;$$
$$\therefore\ x = \frac{14\cdot439375 \times 100}{320\cdot875} \qquad \begin{array}{r} 320875)14439375(45 \\ 1604375 \\ 0 \end{array}$$
$$= 4\cdot5 = 4\tfrac{1}{2}\%.$$

Rule—

'Multiply the percentage by 100, and divide by the quantity.'

Case (iv.) The percentage and rate being given, find the quantity.

Ex. 1. The population of England between the years 1841–1851 increased 2626290, *i.e.* 14·2%: what was it in the former year?

It increased 14·2 in 100: on what number is the increase 2626290.

$$14{\cdot}2 : 2626290 :: 100 : x;$$

$$\therefore\ x = \frac{2626290 \times 100}{14{\cdot}2} = 18495000.$$

Rule—

'Multiply the percentage by 100, and divide by the rate %.'

Ex. 2. If a merchant's debts exceed his assets by $60\frac{1}{10}$%, and his creditors lose £3211.: what do his debts amount to?

$$x = \frac{3211 \times 100}{60\frac{1}{10}} = \frac{3211000}{601} = £5342.\ \text{nearly.}$$

I. COMMISSION, BROKERAGE, INSURANCE.

212. *Commission* is the charge made by an agent for buying or selling goods, and is a percentage on the value of the goods bought or sold.

213. *Brokerage* is the charge made by a broker for buying or selling stocks, bills of exchange, ships, cargoes, &c., and is calculated on the whole amount of the transaction.

214. *Insurance* is a contract by which A undertakes to pay B a certain sum on a particular event, B paying A year by year or at once a percentage of that sum.

The instrument containing the contract is called the *policy*; what B pays A is called the *premium*.

When a man insures so as to recover all his property as well as the premium and other expenses of insurance, it is said to be *covered*.

215. *Discount*, without reference to time, is an allowance made by tradesmen to their customers for payment of ready money.

Exercise 138.

(1) Find the commission on £33. 17*s.* 9*d.* at 5%.
(2) " " £957. 19*s.* 4*d.* " $4\frac{1}{2}$%.
(3) " " £274. 19*s.* 6*d.* " $3\frac{3}{8}$%.
(4) " " £278. 14*s.* 9*d.* " $4\frac{7}{8}$%.

(5) Calculate the discount on £715. 1*s.* 8*d.* at 2⅜%.

(6) " " £55. 16*s.* 8*d.* " 3⅞%.

(7) " " £142. 10*s.* 0*d.* " 4¼%.

(8) " " £345. 15*s.* 0*d.* " 8¾%.

(9) I paid my agent £9. 11*s.* 0·99*d.* for purchasing goods to the amount of £272. 19*s.* 6*d.*: what was his commission?

(10) The premium on a policy assurance at £2⅜%. is £166. 17*s.* 2½*d.*: what is the sum assured?

(11) The premium on a policy assurance for £3125. is £87. 2*s.* 2¼*d.*: what is the rate %?

(12) If I sell goods at the rate of 10*d.* a pound, and buy them at the rate of £4. 1*s.* 8*d.* a hundredweight, what do I gain per cent.?

(13) From the following summary find the average acreage of each holder, the average rental of each holder, and the average rent per acre in the 4 divisions of Great Britain.

	Landholders No.	Extent of Lands Acres	Rental £
England .	921,316	29,179,622	94,443,929
Wales . .	51,520	3,833,968	4,908,372
Scotland .	132,131	18,946,694	18,698,804
Ireland . .	68,757	20,159,677	13,419,255

and then find the same averages for the whole kingdom.

(14) In the following tables are statistics for the years 1840, 1881:—

	England and Wales			Scotland		
Year	Comm.	Convc.	Acqt.	Comm.	Convc.	Acqt.
1840 .	27,187	19,927	7,235	3,872	2,909	920
1881 .	14,786	11,353	3,401	2,444	1,832	581

	Ireland.			United Kingdom.		
Year	Comm.	Convc.	Acqt.	Comm.	Convc.	Acqt.
1840 .	23,833	11,194	12,621			
1881 .	5,311	2,698	2,443			

Fill up the vacant columns for the United Kingdom: find the reduction per cent. of committals in the United Kingdom in 1881 as compared with 1840; the proportions of convictions to committals in 1840, 1881, in England and Wales, Scotland, Ireland, and United Kingdom.

(15) For the whole period of 42 years the convictions were 672264, or 75 per cent.; 109547, or 75 per cent.; 252832, or 51 per cent.: find the committals for each portion of the king-

dom, and the percentage of convictions to committals for the whole kingdom.

(16) In 1840 the value of goods imported into England was £62004000., in 1881 it was £397022489.: find the increase in value per cent.

(17) In 1840, if the values of the imports and exports of the United Kingdom had been distributed between the population, the proportion per head would have been £2. 6*s.* 9$\frac{3}{8}$*d.* and £1. 18*s.* 9*d.* respectively; the value of the exports was £51308740.: find the value of the imports.

(18) The value of British produce and manufactures exported rose from £51308740. in 1840 to £234022678. in 1881: find the increase per cent.

(19) In 1881 the proportion per head of the imports was £11. 7*s.* 4*d.*: find the proportion per head of the exports. (See Questions 16, 18.)

(20) The following table shows the proportionate real value and the duty thereon of the quantities of imports retained for home consumption in 1881–82.

	Real Value £	Revenue £	Percentage of Duty on Value
Cocoa and chocolate	375,008	53,444	
Coffee . . .	1,071,370	189,887	
Currants . . .	1,278,054	324,722	
Raisins . . .	770,628	142,562	
Spirits . . .	1,771,096	4,223,752	
Tea	8,550,707	3,974,487	
Tobacco . . .	1,894,720	8,838,176	
Wine . . .	5,392,095	1,366,121	

Fill in the 4th column.

(21) From the above table, calculate how much out of every shilling paid over the counter for the following articles goes to the Government:—Cocoa, coffee, currants, raisins, tea.

(22) What effect has the duty on the price of a nominal shilling's *worth* of spirits and of tobacco?

(23)

Year	Imports £	Exports £
1840	62,004,000	110,128,716
1850	152,329,053	115,821,092
1860	210,530,873	164,521,351
1880	411,229,565	286,414,466

Find the increase per cent. in the first 10 years, in 20 years, and in 40 years.

(i.) of the Imports; (ii.) of the Exports; (iii.) of the Imports and Exports.

(**24**) From the following account of the tonnage of British and foreign vessels, sailing and steam, entering and clearing, with cargoes and in ballast, at ports in the United Kingdom from and to foreign countries and British possessions, calculate the increase per cent. in British and foreign tonnage in 1881, compared with 1840.

Year	British	Foreign
1840	6,490,485	2,949,182
1881	41,543,259	16,406,286

(**25**) On examination of the statements of Income and Expenditure for the financial year ended March 31, 1882, it will be found that to the net Revenue from all sources in that year (viz. £85921532.), Customs, Excise, and Stamps contributed £58794216. What percentage of the whole is this? Whilst of the total Expenditure (£85569785.), Debt and War cost £58936983. What percentage of the whole is this?

(**26**) In the following table fill in the 4th column and the totals.

Area of Cultivation and of Irrigation in certain Provinces of India.

Provinces	Cultivated Acres	Irrigated Acres	Percentage of I. to C.
Punjab . . .	21,000,000	5,500,000	
N.-W. Provinces and Oudh . . .	36,000,000	11,500,000	
Bengal . . .	54,500,000	1,000,000	
Central . . .	15,500,000	770,000	
Behar . . .	6,500,000	100,000	
Bombay . . .	24,500,000	450,000	
Sind . . .	2,250,000	1,800,000	
Madras . . .	32,000,000	7,300,000	
Mysore . . .	5,000,000	800,000	
Total . .			

II. PROFIT AND LOSS.

216. If I sell a horse for £107. and gave £100 for him, I gain £7. on the outlay of £100., or, in other words, I gain 7 *per cent.* Conversely, if I buy a horse for £100., and wish to gain £7., or 7 per cent., I must sell him for £107.

Again, if I sell a horse for £92. and gave £100. for him, I lose £8. on the outlay of £100., or I lose 8 *per cent.* Conversely, if I buy a horse for £100. and sell him at a loss of £8., or 8 per cent., I must sell him for £92.

The following examples will illustrate how questions on Profit and Loss may be solved in accordance with the principles of Cause and Effect.

Ex. 1. I sell for 10*s.* that for which I gave 8*s.*: what do I gain per cent.?

I gain 2*s.* on 8*s.*: what do I gain on 100?
The first cause is 8*s.*, the first effect is 2*s.*;
The second cause is 100, the second effect is required;

$$\therefore 8 : 2 :: 100 : x,$$
$$\text{or} \quad \frac{2}{8} = \frac{x}{100};$$
$$\therefore x = 25.$$

∴ *Answer* is 25%.

Ex. 2. I bought a horse for £80.: at what price must I sell him so as to gain 20%?

I sell for £120. that which costs me £100.: at what price must I sell that which cost me £80.?
The first cause is £100., the first effect is £120.;
The second cause is £80., the second effect is required;

$$\therefore 100 : 120 :: 80 : x$$
$$\text{or} \quad x = \frac{120 \times 80}{100} = 96.$$

∴ *Answer* = £96.

Ex. 3. By selling an article for 12*s.* I lost 25 per cent.: what did it cost me?

I sell for 75*s.* that which costs me 100*s.*: what did I give for what I sell for 12*s.*?
The first effect is 75*s.*, the first cause is 100*s.*;
The second effect is 12*s.*, the second cause is required.;

$$\therefore 75 : 100 :: 12 : x;$$

$$x = \frac{100 \times 12}{75} = 16s.$$

$\therefore$ *Ans.* = 16 shillings.

EX. 4. If a hundredweight of tea cost £30. 16*s.*, at what price per pound must it be sold to gain £21. 4*s.* $2\frac{10}{11}d.$ per cent.?

The first cause is £100., the first effect is £121. 4*s.* $2\frac{10}{11}d.$;

The second cause is £30. 16*s.*, the second effect is £112. × cost per pound;

$$\therefore £100. : £121.\ 4s.\ 2\tfrac{10}{11}d. :: £30.\ 16s. : 112 \times x,$$

$$\text{or}\quad 100 \times 20 \times 12 \times 11 : 320000 :: 616 : 112 \times x.$$

$$\therefore x = \frac{\overset{\overset{\overset{20}{\cancel{80}}}{\cancel{160}}}{\cancel{320000}} \times \overset{56}{\cancel{616}}}{\cancel{100} \times \cancel{20} \times \underset{3}{\cancel{12}} \times \cancel{11} \times \underset{2}{\cancel{112}}} = \frac{20}{3} = 6s.\ 8d.$$

$\therefore$ *Ans.* = 6*s.* 8*d.* per pound.

EX. 5. If by selling an article for 5*s.* I lose 20 per cent., what must I sell it for to gain 20 per cent.?

1st Method. Find the cost price. Thus—

$$80 : 100 :: 60d. : x;$$

$$\therefore x = \frac{\overset{25}{\cancel{100}} \times \overset{3}{\cancel{60}}}{\underset{1}{\cancel{80}}} = 75d. = 6s.\ 3d.;$$

and since we have to gain 20 per cent. on this,

$$\therefore 100 : 120 :: 75 : x;$$

$$\therefore x = \frac{\overset{15}{\cancel{75}} \times \overset{6}{\cancel{120}}}{\underset{1}{\cancel{100}}} = 90d. = 7s.\ 6d. \quad \textit{Ans.}$$

2nd Method. 80, 120 are the two selling prices of which 100 is the prime cost.

60*d.*, x are the two selling prices of the cost of the article;

$$\therefore 80 : 120 :: 60 : x;$$

$$\therefore x = \frac{120 \times 60}{80} = 90d. = 7s.\ 6d. \quad \textit{Ans.}$$

Ex. 6. If 8 per cent. be gained by selling 218 yards of cloth for £92. 13*s.*, what will be the gain or loss per cent. by selling them at 9*s.* 2½*d.* a yard?

Here the two selling prices of the cloth are £92. 13*s.* and 218 × 9*s.* 2½*d.*, and the two selling prices of what cost £100. are £108. and £*x*;

$$\therefore £92.\ 13s. : 218 \times 9s.\ 2\tfrac{1}{2}d. :: 108 : x.$$

Now £92. 13*s.* = 1853 × 12 × 2 halfpence, and 9*s.* 2½*d.* = 221 halfpence;

$$\therefore\ 1853 \times 12 \times 2 : 218 \times 221 :: 108 : x;$$

$$\therefore\ x = \frac{\overset{109}{\cancel{218}} \times \overset{13}{\cancel{221}} \times \overset{9}{\cancel{108}}}{\underset{17}{\cancel{1853}} \times \cancel{12} \times \cancel{2}} = £117.$$

∴ the gain is 17 per cent.

Ex. 7. If I spend 12*s.* in buying 12 dozen articles at 13 to the dozen, and sell them at a penny each, what do I gain per cent.?

1st Method. I spend 144 pence, I realise 156 pence; ∴ I gain 12 pence on 144 pence;

$$\therefore\ 144 : 100 :: 12 : x;$$

$$\therefore\ x = \frac{\overset{25}{\cancel{100}} \times \cancel{12}}{\underset{\underset{3}{\cancel{12}}}{\cancel{144}}} = 8\tfrac{1}{3}.$$

Ans. = 8⅓ per cent.

2nd Method. I gain 1 article on every dozen I buy; ∴ I gain $\frac{1}{12}$th of outlay;

$$\therefore \text{gain per cent} = \tfrac{1}{12} \times 100 = 8\tfrac{1}{3} \text{ as before.}$$

Ex. 8. A grocer bought tea intending to gain 8 per cent. by its sale. When he had sold two-thirds of it, he was compelled to reduce the price 6*d.* per lb., and so only gained half as much as he had intended. What was the original cost per lb. of the tea?

Since he sold one-third of a lb. at 6*d.* a lb. less than he intended, he gained twopence less on the lb., which by the question is 4 per cent. less than he expected; *i.e.* 100 is the sum on which he gains 4: what is the sum on which he gains 2*d.*?

$$4 : 100 :: 2 : x;$$

$$\therefore\ x = 50d.$$

Ans. = 4*s.* 2*d.* a lb.

Exercise 139.

(1) I sell for 4*s*. 8*d*. what costs me 3*s*. $7\frac{3}{4}$*d*.: what do I gain per cent.?

(2) If a person sell 22 articles for the same money which he paid for 36, what does he gain per cent.?

(3) A house was sold for £2875. at a profit of 15 per cent.: what did it cost?

(4) At what price must I sell an article which cost £3. 14*s*. to gain 16 per cent.?

(5) By selling a certain number of yards at 3*s*. $4\frac{1}{2}$*d*. per yard a draper gains 6*s*. 6*d*., or 8 per cent.: how many yards did he buy?

(6) If 1 cwt. 3 qrs. cost £27. 10*s*., what would be the gain per cent. by selling them at 2*s*. $11\frac{2707}{8000}$*d*. a lb.?

(7) I gain 10 per cent. by selling tea at 6*s*. per lb.: what must I sell it for per lb. to gain $6\frac{17}{18}$?

(8) If 4 per cent. be lost by selling linen at 2*s*. 9*d*. a yard, what would be the loss or gain per cent. by selling it at 3*s*. $1\frac{13}{16}$*d*.?

(9) I exchange $\frac{1}{2}$ cwt. of sugar at $4\frac{1}{2}$*d*. per lb. for tea at 3*s*. 6*d*. per lb., and thereby gain $16\frac{2}{3}$ per cent.: how many lbs. of tea did I buy?

(10) By selling a carriage for £88. 11*s*. I gain 15 per cent.: at what price should I sell it to lose 5 per cent.?

(11) If I gain 10 per cent. by selling tea at 4*s*. per lb., what would be the selling price if I gain $19\frac{1}{8}$ per cent.?

(12) If a tea-dealer buy 3 cwt. of tea and sell it at 4*s*. $1\frac{1}{2}$*d*. per lb., making a profit of 10 per cent., what was his outlay?

(13) If goods sold at a profit of $12\frac{1}{2}\%$ realised £360., find the cost price.

(14) If by selling a horse for £80. I lost 4%, what should I have sold him for to gain 15%?

(15) By selling a horse for £75. I gained per cent. the cost price of the horse: what was it?

(16) What is the cost of an article which, after paying 10% for collection, and making a profit of 12%, a tradesman sells for £4. 16*s*.?

(17) The invoice price of an article is £12. 10*s*.; the allow-

ance to agents is 12%, and for bad debts 22%; the tradesman's profit 30%: what is the cost of the article?

(**18**) A tradesman buys an article for £4. and sells it for £6. 6*s.* 3¼*d.*, paying his collector 10% and allowing 12% for bad debts: what profit per cent. did he make?

(**19**) A grocer has a stock of tea which he must sell at 3*s.* 4*d.* per lb. in order to gain 33⅓ per cent.: in what proportion must he mix it with inferior tea at 1*s.* 9*d.* a lb. that he may make the same percentage of profit as he would by selling the mixture at the rate of 15 lbs. for £2. 7*s.*?

(**20**) A dishonest farmer mixes 5 per cent. of water with milk, which he sells to a milkman, who retails it, after adding 5 per cent. of water to what he receives: what per cent. of water is there in the milk retailed?

(**21**) If eggs be bought at the rate of 5 for 1*d.*, how many should be sold for 7*d.* to gain 40 per cent.?

(**22**) If 5 per cent. be gained by selling butter at £5. 5*s.* 6*d.* per cwt., what will be the gain per cent. by selling it at 1*s.* 3*d.* per lb.?

(**23**) A grocer buys coffee at £5. 12*s.* 6*d.* per cwt. and chicory at £2. 5*s.* 5*d.*: in what proportion must he mix them so that by retailing the mixture at 1*s.* 3*d.* per lb. he may gain 50 per cent.?

(**24**) A farm is let for £96. and 30 quarters of wheat. Find the price of wheat a quarter, when the whole rent is 15 per cent. lower than when it is 56*s.* a quarter.

(**25**) In what proportion must I mix gin at 15*s.* 6*d.* a gallon with gin at 17*s.* 1*d.* a gallon, so that by selling the mixture at 18*s.* a gallon I may clear 10 per cent.?

(**26**) If I buy 1350 oranges at 5 for a penny, and an equal number at 3 for a penny, at what price per dozen must I sell them to gain 25 per cent.?

(**27**) By the sale of his goods a bankrupt should have paid his creditors 16*s.* 3*d.* in the pound; but $\frac{3}{5}$ were sold at 17·5 per cent., and the remainder at 23·75 per cent. below their value. The sale realised £780.: how much did he owe?

(**28**) I bought 15 horses and 12 cows for £345. I sell the horses at a profit of 10 per cent., and the cows at a loss of 1½ per cent., and receive in all £365. 14*s.*: what did I give for a horse and a cow?

(**29**) How much tea worth 4*s.* per lb. must I mix with 25 lbs. at 4*s.* 8*d.*, so that by selling the mixture at 5*s.* 4*d.* per lb. I may clear $21\frac{83}{277}$ per cent.?

(**30**) I draw off 5 gallons from a 20-gallon cask of brandy and fill up the cask with water. This I do three times. What is the percentage of brandy left?

(**31**) A merchant sells 49 quarters of corn at a profit of 7 per cent. and 84 quarters at a profit of 11 per cent.; and if he had sold the whole at a profit of 9 per cent. he would have received £2. 10*s*. 9*d*. less than he actually did: what was the price paid for the corn?

(**32**) A person sells an article at 5 per cent. profit; if he had bought it at 5 per cent. less, and sold it for one shilling less, he would have gained 10 per cent.: what was the cost price?

(**33**) A. bought apples at the rate of 5 for 1*d*., and an equal number at the rate of 7 for 1*d*., and sold the whole at the rate of 2*d*. a dozen: what did he lose per cent.?

(**34**) A wine merchant buys wine at 72*s*. a dozen and at 54*s*. a dozen, and mixes them, and selling the mixture at 63*s*. a dozen, he clears 8¼ per cent.: how much of each sort did he buy?

(**35**) A farmer bought 296 sheep at £1. 7*s*. 6*d*. each, and after paying for their keep exchanges them for 37 cows, which he then sold at £15. per head, thereby gaining $15\frac{5}{13}$ per cent. on his expenditure: what fraction of their cost was the keep of the sheep?

(**36**) If I borrow £365. 4*s*. 8¼*d*. at simple interest for 3 years at 4 per cent., and let it out at compound interest for the same time at the same rate, find to 4 places of decimals my gain per cent.

(**37**) What does a banker gain per cent. on a bill for £105. due 15 months hence at 4 per cent. simple interest?

(**38**) My charges are 12% above cost price; I gain 5% on a bill for which I allow £1. 6*s*. 3*d*. discount: what is its amount?

(**39**) In what proportions must wines at 15*s*., 20*s*., 26*s*., and 30*s*. a gallon respectively be mixed so as to give a profit of $14\frac{2}{7}$ per cent. on the cost when sold for 24*s*. a gallon?

(**40**) If I borrow £1760. at simple interest and let it out at compound interest at 5% for 2½ years, interest being payable half-yearly, what per cent. profit do I make?

III. STOCKS AND SHARES.

217. The Stocks or Public Funds are the State Debts, arising from money borrowed from individuals, on the guarantee of a certain fixed rate of interest for every £100. Thus, if A.

has £100. in the Funds, he has his name entered on the Bank books as the person to whom that amount of stock belongs, and to whom the interest upon it is payable half-yearly. This interest is called a *dividend*.

218. The price of £100. stock varies from time to time; when it is exactly £100. it is said to be at *par*, if below £100. at a *discount*, and if above £100. at a *premium*.

219. Consols, or Consolidated Annuities, and other stocks are quoted on the Exchange between two prices, the first being the buyer's and the second the seller's price; thus—

3 Per Cent. Consols . . .	103 to 103¼
Great Western Railway .	150 to 150¾

and the stock is sold at one or other of these prices or at an intermediate one.

Every transaction in stock is made through a broker, who charges £⅛. or 2*s*. 6*d*. per cent. for every £100. stock or share he buys or sells; so that if A. buys Consols at 92⅝ and sells them at 92¾, he will have to pay his broker 92⅝ + ⅛ or 92¾ in the one case, and receive 92¾ − ⅛ or 92⅝ in the other.

N.B.—(1) All questions wherein brokerage is allowed will be followed by the letter B. (2) Carefully distinguish between *stock* money and *sterling* value; the former is the amount on which the interest is paid, the latter is the market value per cent.

If the principal is *in* the Funds it is *stock* money, if it is *not in* the Funds it is *sterling* money.

Questions on Stocks can all be solved by Cause and Effect.

EX. 1. Find the value of £1500. in the 3 per cents at 93¼.

This means that the price of £100. *in* the Funds, or £100. stock, is 93¼, and the value of £1500. stock is required.

The first effect is 93¼, the first cause is £100. stock; the second effect is required, the second cause is £1500. stock.

$$\therefore \pounds 100. : \pounds 1500. :: 93\tfrac{1}{4} : x.$$

$$\therefore \mathit{Ans.} = 93\tfrac{1}{4} \times 15 = \pounds 1398.\ 15s.$$

EX. 1*. Find the value of £P. *in* the r per cents at p.

$$\mathit{Ans.} = \frac{P \times p}{100}.$$

EX. 2. Find the quantity of stock purchased by investing £1686. 11*s*. 3*d*. in the 3½ per cents at 96¼. B.

This means that £100. stock, or one *share*, can be bought for $96\frac{1}{4} + \frac{1}{8}$ or $96\frac{3}{8}$, and the number of shares, or quantity of stock, that may be purchased with £1686. 11*s.* 3*d.* is required.

The first cause is £100. stock, the first effect is £$96\frac{3}{8}$.; the second cause is required, the second effect is £1686. 11*s.* 3*d.*;

$$\therefore £96\tfrac{3}{8}. : £1686.\ 11s.\ 3d. :: £100.\ \text{stock} : x;$$

$$\therefore Ans. = \frac{£1686.\ 11s.\ 3d. \times 100}{£96\frac{3}{8}.} = \frac{134925 \times \cancel{8} \times \overset{10}{\cancel{100}}}{771 \times \cancel{4} \times \cancel{20}} = £1750.\ \text{stock.}$$

Ex. 2*. Find how much stock can be purchased by investing £P. in the r per cents at p.

$$Ans. = \frac{P \times 100}{p}.$$

Ex. 3. Find how much I lose by buying £80000. stock at $91\frac{3}{4}$ and selling them at $90\frac{1}{8}$. B.

For each £100. stock or share I give £$91\frac{3}{4}. + \frac{1}{8}$, or £$91\frac{7}{8}$., and get £$90\frac{1}{8}. - \frac{1}{8}$, or £90.; ∴ I lose on each share £$1\frac{5}{8}$.;

∴ first effect is £$1\frac{5}{8}$., the first cause is £100. stock; the second effect is required, the second cause is £80000. stock;

$$\therefore £100. : £80000. :: £1\tfrac{5}{8}. : x;$$

$$\therefore Ans. = \frac{80000 \times 13}{8 \times 100} = £1300.$$

Ex. 3*. Find how much is lost by buying £P. stock at p_1 and selling them at p_2.

$$Ans. = \frac{P \times (p_1 - p_2)}{100}.$$

Ex. 4. Find the income obtained by investing £3963. 15*s.* in the 3 per cents at $94\frac{1}{4}$. B.

This means that £3. is the income produced by £100. stock, or one share, which can be purchased for £$94\frac{1}{4}. + \frac{1}{8}$, or £$94\frac{3}{8}$., or that an outlay of £$94\frac{3}{8}$. realises an income of £3.: what income, therefore, will be obtained by an outlay of £3963. 15*s.*?

The first cause is £$94\frac{3}{8}$.; the first effect is £3.

The second effect is required; the second cause is £3963. 15*s.*, or £3963·75;

$$\therefore £94{\cdot}375 : £3963{\cdot}75 :: £3. : x;$$

$$\therefore x = \frac{3 \times 3963{\cdot}75}{94{\cdot}375},$$

$$\text{or } x = \frac{3 \times \overset{42}{\cancel{3963750}}}{\cancel{94375}} = £126.$$

Ex. 4*. Find the income produced by investing £P. in the r per cents at p;

$$\therefore \textit{Ans.} = \frac{P \times r}{p} = I.$$

Ex. 5. Find the income obtained from £3321. in the 4 per cents at $100\frac{1}{4}$.

This means that £4. is the income from holding £100. stock; what would be the income from holding £3321. stock?

The first cause is £100. stock; the first effect is £4.

The second cause is £3321. stock; the second effect is required;

$$\therefore £100. : £3321. :: £4. : x;$$

$$\therefore x = \frac{3321 \times 4}{100} = £132{\cdot}84. \qquad \begin{array}{r} 3321 \\ 4 \\ \hline 132{\cdot}84 \end{array}$$

$$\therefore \textit{Ans.} = £132.\ 16s.\ 9{\cdot}6d.$$

Ex. 5*. Find the income obtained from £P. in the r per cents at p.

$$\textit{Ans.} = \frac{P \times r}{100} = I.$$

Ex. 6. Find the difference in income arising from the investment of £8736. in the 3 per cents at 91, and the 4 per cents at 96.

$$\text{The 3 per cents gives } \frac{\overset{\overset{96}{\cancel{1248}}}{\cancel{8736}} \times 3}{\underset{\cancel{13}}{\cancel{91}}} = £288. \text{ (see Ex. 4*).}$$

$$\text{The 4 per cents gives } \frac{\overset{\overset{364}{\cancel{728}}}{\cancel{8736}} \times \cancel{4}}{\underset{\underset{\cancel{1}}{\cancel{24}}}{\cancel{96}}} = £364.;$$

$\therefore$ he would gain £76. by investing in the 4 per cents.

2nd Method.—One pound in the 3 per cents realises an income of $£\frac{3}{91}$.

One pound in the 4 per cents realises an income of $£\frac{4}{96}$, or $£\frac{1}{24}$;

$\therefore$ on one pound there is a gain of $\frac{1}{24} - \frac{3}{91} = \frac{91-72}{24 \times 91} = \frac{19}{24 \times 91}$;

$$\therefore \text{whole gain} = \frac{\overset{\overset{4}{\cancel{96}}}{\cancel{8736}} \times 19}{\cancel{24} \times \cancel{91}} = £76., \text{ as before.}$$

Ex. 6*. Find the difference in income arising from the investment of £P. in the r_1 per cents at p_1, and the r_2 per cents at p_r.

$$Ans. = \frac{P\,(r_1 p_2 \sim r_2 p_1)}{p_1 p_2}.$$

Ex. 7. How much must I invest in the 4 per cents at $100\frac{5}{8}$ to obtain an income of £61. 6s. $9\frac{237}{403}$d.? B.

Here the first cause is $100\frac{5}{8} + \frac{1}{8}$, or $100\frac{3}{4}$; the first effect is £4.

The second cause is required; the second effect is £61. 6s. $9\frac{237}{403}$d.;

$$\therefore £4 : £61.\ 6s.\ 9\tfrac{237}{403}d. :: 100\tfrac{3}{4} : x;$$

$$\therefore x = \frac{£61.\ 6s.\ 9\frac{237}{403}d. \times 403}{16} = £1545.$$

Ex. 7*. How much must I invest in the r per cents at p to obtain an income I?

$$P = \frac{I \times p}{r} \text{ (cf. Ex. 4*).}$$

Ex. 8. What rate per cent. is obtained on money invested in the 3 per cents at $94\frac{1}{2}$? B.

£3. is obtained on an outlay of £$94\frac{1}{2}$.; how much is that on £100.?

The first effect is £3.; the first cause is £$94\frac{1}{2}$.

The second effect is required; the second cause is £100.;

$$\therefore £94\tfrac{1}{2}. : £3. :: £100. : x;$$

$$\therefore x = \frac{3 \times 100}{94\frac{1}{2}} = \frac{\overset{200}{\cancel{600}}}{\underset{63}{\cancel{189}}} = 3\tfrac{11}{63}.\ Ans.$$

Ex. 8*. What rate of interest is obtained on money invested in the r per cents at p?

$$Ans. = \frac{100 \times r}{p}.$$

Ex. 9. What is the better investment, the $3\frac{1}{2}$ per cents at 94 or the $4\frac{3}{4}$ per cents at 91?

From Ex. 4 it is evident we have to compare the fractions $\frac{3\frac{1}{2}}{94}$, $\frac{4\frac{3}{4}}{91}$.

Now $\frac{3\frac{1}{2}}{94}=\frac{7}{188}$, $\frac{4\frac{3}{4}}{91}=\frac{19}{364}$, and to compare these we may bring them to a common numerator, 133; the two fractions may be written $\frac{133}{3572}$, $\frac{133}{2548}$. And since the latter fraction has the smaller denominator, it is the greater fraction;

∴ it would be better to invest in the $4\frac{3}{4}$ per cents at 91.

Ex. 10. A man invested £3565. in the $2\frac{1}{2}$ per cents at $62\frac{3}{4}$, and when they had fallen to 58 he sold out and invested the money in the 4 per cents at $96\frac{7}{8}$: find the alteration in his income.

From Ex. 2. The number of shares he purchased at first

$$=\frac{3565}{62\frac{3}{4}}=\frac{14260}{251};$$

$$\therefore \text{ his first income } = \frac{\overset{7130}{\cancel{14260}}}{251}\times\frac{5}{\cancel{2}}.$$

He next sells out these shares, and realises by the sale

$$\frac{14260}{251}\times\frac{58}{1}.$$

This money he invests in the 4 per cents at $96\frac{7}{8}$;

$$\therefore \text{ his second income} = \frac{14260}{251}\times\frac{58}{775}\times\frac{8}{1}\times\frac{4}{1};$$

$$\therefore \text{ the difference in his income } = \frac{14260}{251}\left\{\frac{5}{2}-\frac{32\times58}{775}\right\}$$

$$=\frac{14260}{251}\times\frac{(3875-3712)}{2\times775}=\frac{\overset{46}{\cancel{1426}}\times163}{251\times\underset{\substack{\cancel{155}\\5}}{\cancel{755}}}$$

$$=\frac{7498}{1255}=\text{£}5.\ 19s.\ 5\tfrac{221}{251}d.\ \textit{Ans.}$$

Ex. 10*. I invest £P. in the r_1 per cents at p_1, and when they are at p_2 I sell out and invest the money in the r_3 per cents at p_3: find the alteration in my income.

$$\textit{Ans.}=\frac{\text{P}}{p_1p_3}\,(r_1p_3\sim r_3p_2).$$

220. *Shares.* Calculations on stock are, with slight alterations, applicable to shares. Thus, shares issued at £25., fully paid up, and sold at a discount of £7.—*i.e.* for £18.—may be compared to £100. stock at £72. by multiplying share and price by 4.

Ex. 11. The shares in a company are issued at 35, bought at 3 premium, and yield a six-monthly dividend of £1. 15*s.*; find (i.) the sterling value of the shares per cent., (ii.) the rate of interest, and (iii.) the income derived from investing £1900. in them.

$$\text{(i.)}\qquad \frac{x}{100}=\frac{38}{35}\qquad \therefore\ \textit{Ans.}=£108\tfrac{4}{7}.$$

$$\text{(ii.)}\qquad \frac{x}{100}=\frac{2\times 1\cdot 75}{38}\qquad \therefore\ \textit{Ans.}=£9\tfrac{4}{19}\%.$$

$$\text{(iii.)}\qquad \frac{x}{1900}=\frac{2\times 1\cdot 75}{38}\qquad \therefore\ \textit{Ans.}=£175.$$

The following exercise is arranged to illustrate the examples here worked out.

Exercise 140.

(i.) Find the value in sterling money of

(1) £1500. in the 3 per cents at 82.
(2) £4500. " 3¼ " " 90.
(3) £4000. " 3¼ " " 97¼. B.
(4) £4310. bank stock at 188⅞. B.
(5) £8350. 3% consols at 94½. B.
(6) £937. 12*s.* 11*d.* 3% consols at 90½. B.
(7) £768. 17*s.* 11*d.* 4½ per cents at 88. B.
(8) £27468. 10*s.* 5 per cents at 89¼. B.

Find the price of the stock, when

(9) £2600. in the 4 per cents is worth £2418.
(10) £1920. in the 3 per cents is worth £1488.
(11) £2240. in the 3¼ per cents is worth £1834.
(12) £1743 in the 3¼ per cents is worth £1444. 10*s.* 2½*d.* ¾*q.*

(ii.) Find the quantity of stock purchased by investing

(1) 2000 guineas in the 3½ per cents at 94.
(2) £500. in the 3½ per cents at 94.

(3) £4311. 8*s.* 9*d.* in the 3½ per cents at 85⅜.
(4) 400 guineas in the 4 per cents at 94.
(5) £1606. in the 4 per cents at 100¼.
(6) £588. 5*s.* in the 3¼ per cents at 90½.
(7) £3097. in the 5 per cents at 105⅜. B.
(8) 1000 guineas in the 3¼ per cents at 99¼. B.

Find the price of the stock when

(9) £2153. 10*s.* bank stock can be purchased for £4064. 14*s.* 7½*d.* B.

(10) £901. 9*s.* 10*d.* bank stock can be purchased for £725. 14*s.*

(11) £587. 15*s.* $1\frac{11}{48}$*d.* bank stock can be purchased for £540.

(12) £1500. bank stock can be purchased for £1350. B.

(iii.) Find the gain or loss on

(1) £1000. stock bought at 97⅛, sold at 83½.
(2) £500. stock bought at 98¾, sold at 103.
(3) £2400. stock bought at 79½, sold at 86. B.

Find the quantity of stock held when

(4) £45. was gained by buying at 75 and selling at 77.
(5) £75. was lost by buying at 80 and selling at 75.
(6) £172. was gained by buying at 86¼ and selling at 89½. B.

Find the selling price of the stock when

(7) £5000. bought at 93 was sold at a loss of £25.
(8) £80000. bought at 91⅝ was sold at a loss of £1500. B.
(9) £17500. bought at 88⅞ was sold at a gain of £175. B.

Find the buying price of the stock when

(10) £1200. sold at £100⅚. produced a gain of £100.
(11) £16000. sold at 94⅛ produced a gain of £40. B.

(12) 25000 fr. in the French Rentes sold at 66·9 produced a loss of 1275 fr. B.

(iv.) Find the income obtained by investing

(1) £2250. in the 4 per cents at 90.

(2) £3220. in the 3½ per cents at 80½.
(3) £4537. 10*s*. in the 3 per cents at 90⅝. B.
(4) £3500. in the 3 per cents at 94⅛. B.
(5) 5000 guineas in the 3½ per cents at 102⅜. B.
(6) 5000 guineas in the 3½ per cents at 102⅜. B.
(7) £3003. in the Dutch 2½ per cents at 49½.
(8) £3995. in the 3½ per cents at 99⅞.
(9) £5011. 10*s*. in the 3½ per cents at 96⅜.
(10) £32850. in the 3 per cents at 91⅛. B.
(11) £1274. in the 3½ per cents at 91.
(12) £434. 12*s*. 6*d*. in the 3 per cents at 91½.

(v.) Find the income obtained from
(1) £1500. in the 4 per cents at 75.
(2) £2000. in the 3 per cents at 95.
(3) £2000. in the 4½ per cents at 89¾.
(4) £531. 18*s*. 3⅓*d*. in the 3½ per cents at 94.
(5) £446. 16*s*. $2\frac{2}{47}$*d*. in the 3½ per cents at 85⅜.
(6) £650. in the 4 per cents at 94.
(7) £5050. in the 4 per cents at 100¼.
(8) £1601. 19*s*. 10¾*d*. in the 3¼ per cents at 90½.
(9) £2935. 10*s*. 10¾*d*. in the 5 per cents at 105⅜. B.
(10) £439. 12*s*. 6*d*. in the 3½ per cents at 92⅞.
(11) £2153. 10*s*. in the 4½ per cents at 188⅞. B.
(12) £3622. 10*s*. in the 3 per cents at 92¼.

(vi.) Find the difference of income arising from the investment of
(1) £1600. in the 3 per cents at 80, and the 4 per cents at 75.
(2) £4725. " 6 " 126, " 9 " 210.
(3) £3600. " 3 " 72, " 4 " 90.
(4) £1725. " 4 " 85, " 5 " 86.
(5) £3550. " 5 " 90, " 3 " 79.

The difference arising from investing in the following funds is given; find the sum invested.

(6) The difference is £95.—4 per cents at 75, the 3 per cents at 80.

(7) The difference is £15.—3 per cents at $92\frac{7}{8}$, the 4 per cents at $108\frac{3}{8}$.

(8) The difference is £2. 10*s.*—4 per cents at 90, the 3 per cents at 72.

(9) The difference is £5. 10*s.* $3\frac{87}{131}$*d.*—3 per cents at $89\frac{3}{8}$, the $3\frac{1}{2}$ per cents at $98\frac{1}{4}$.

(10) The difference is $10\frac{5}{37}$*d.*—$3\frac{1}{2}$ per cents at 96, the 4 per cents at 111.

(vii.) What sums of money must be invested in the following stocks in order to produce the under-mentioned incomes?

(1) £36. in the 3 per cents at 84.
(2) £240. in the 4 per cents at 93.
(3) £55. 6 fl. 8 c. $7\frac{1}{2}$ m. in the $4\frac{1}{2}$ per cents at 92.
(4) £159. 12*s.* in the $3\frac{1}{2}$ per cents at 105.
(5) £111. 8*s.* $1\frac{151}{377}$*d.* in the 3 per cents at $94\frac{1}{8}$. B.
(6) £178. 15*s.* $2\frac{766}{2467}$*d.* in the $3\frac{1}{2}$ per cents at $102\frac{2}{3}$. B.

(viii.) At what rate per cent. will a person receive interest who invests

(1) In the $3\frac{1}{2}$ per cents at $101\frac{1}{3}$?
(2) " 3 " $93\frac{3}{4}$?
(3) " $3\frac{1}{2}$ " $77\frac{7}{9}$?
(4) " 6 " 144?
(5) " 3 " $85\frac{5}{7}$?
(6) " 5 " $102\frac{7}{8}$. B?

(ix.) Which is the more profitable—

(1) The $3\frac{1}{4}$ per cents at $92\frac{7}{8}$, or the $3\frac{1}{2}$ per cents at 96?
(2) " 5 " $105\frac{3}{8}$, " 4 " 93?
(3) " $3\frac{1}{2}$ " 96, " 3 " $89\frac{3}{8}$?
(4) " $3\frac{1}{4}$ " $90\frac{1}{2}$, " 4 , $108\frac{3}{8}$?
(5) " $3\frac{1}{2}$ " 105, " 5 " $102\frac{2}{3}$?
(6) " 4 " 85, " 5 " 86?

(x.) Find the change of income arising from the following transactions:—

(1) Investing £1037. 10*s.* in the 3 per cents at 83, selling out at 84, and re-investing in the 4 per cents at 96.

(2) Investing £18150. in the 3 per cents at 90¾, selling out at 91, and re-investing in the 3½ per cents at 97½.

(3) Investing £5000. in the 6 per cents at 80, selling out at 84, and re-investing in the 4½ per cents at 90.

(4) Investing £3220. in the 3½ per cents at 80½, selling out at 92, and re-investing in the 4 per cents at £98. 2*s.* 8*d.*

(5) Investing £5151. in the 5 per cents at 101, selling out at 105, and re-investing in the 4¾ per cents at 102.

(6) Investing £9075. in the 3 per cents at 90¾, selling out at 91, and re-investing in the 3½ per cents at 97½.

Exercise 141.

(1) Find the capital required to buy £2500. stock in the 3 per cents at 92⅛.

(2) Find the price of the stock when £800. stock can be purchased for £740.

(3) A certain sum is invested in the 5 per cent. stock at 101; the stock rising to 105, it is sold out, and the proceeds invested in the 4¾ per cents at 102; thereby a change of income to the amount of £5. 12*s.* 6*d.* is effected. What is the sum invested?

(4) A person transfers £5000. stock from the 3 per cents at 72, and invests the proceeds in a 4 per cent. stock, thus increasing his income by £10.: what was the price of the 4 per cent. stock?

(5) How much stock must I hold in the 3 per cents at 80½, so that if I transfer it to the 4 per cents at 98 I may increase my income by £8. 8*s.* $9\frac{3}{5}$*d.*?

(6) A person sells out £1475. consols at 75¼, and invests this sum in railway stock paying 5 per cent. at 110⅝, and his income is increased by £5. 18*s.* 4*d.*: what was the rate per cent. of the consols?

(7) A person sells £8000. 3 per cent. consols at 92⅞, and invests the proceeds in railway shares at 102, thus increasing his income by £124. 4*s.* $3\frac{13}{17}$*d.*: how much per cent. did the railway shares pay?

(8) By investing £2681. 5*s.* in the 3 per cents, selling out therefrom, and re-investing in the 3½ per cents at 98¼, I gain £5. 10*s.* $3\frac{87}{131}$*d.* in income: what was the price of the 3 per cents?

(9) I invest £100. in the 3½ per cents at 96, sell out, and by

buying 4 per cent. stock increase my income by $10\frac{5}{37}$*d.* : what did I give for the 4 per cent. stock?

(**10**) What sum must I invest in the 3 per cents at 83, so that by selling out when they have risen 1 per cent., and re-investing in the 4 per cents at 96, I may increase my income by £6. 5*s.*?

(**11**) I invest £18150. in consols at 90¾; when they rise to 91 I sell out, and re-invest in shares at 97½ paying 3½ per cent., thus increasing my income by £56. 6*s.* 8*d.* : what was the rate per cent. of the consols?

(**12**) A person invested £5000. in the 6 per cents, sold out when they reached 84, and re-invested in the 4½ per cents at 90, thereby increasing his income by £112. 10*s.* : what did he give for the 6 per cents?

(**13**) If I invest £3220. in the 3½ per cents at 80½, at what price must I sell out, so that by re-investing in the 4 per cents at £98. 2*s.* 8*d.* my income may increase by £10.?

(**14**) I invest £5151. in the 5 per cents at 101; on their rising 4 per cent. I sell out, and buy shares at 102, and increase my income by £5. 12*s.* 6*d.* : what rate per cent. did the shares pay?

(**15**) A person invested £9075. in the 3 per cents at 90¾, and when they had risen to 91 he sold out, and bought shares paying 3½ per cent., and increased his income by £26. 13*s.* 4*d.* : what did he give for the shares?

(**16**) A person sells out of the 3 per cents at 86¼, and gets £3400.; if he invests one-half of this sum in the 4 per cents at 97, and the remainder in the 3½ per cents at 94, find the alteration in his income.

(**17**) A company, whose capital is £2500000., pays 6½ per cent. per annum to its shareholders, clear of income-tax at 4*d.* in the £. : find the total annual receipts, the expenses being 60 per cent. of the gross receipts.

(**18**) If the 3½ per cents be at 91, how much must I invest in them in order to have a yearly income of £932., after paying 7*d.* in the £. income-tax?

(**19**) A person has £500. a year from shares in water works which pay 10 per cent. dividend; he sells so many of these shares at a premium of 50 per cent. as when the proceeds are invested in the 3 per cents at 90 bring him £200. a year: what income has he from the remainder of his shares?

(**20**) The receipts of a railway company are apportioned as follows:—48 per cent. for working expenses, 10 per cent. for the reserve fund, a guaranteed dividend of 5 per cent. on one-fifth of the capital, and the remainder, £32000., for division amongst the holders of the rest of the stock, being a dividend at the rate of 4 per cent. per annum: find the capital and the receipts.

(21) A person who has £2954. in the 3 per cents at 80¼, transfers his capital into the 4 per cents, thereby increasing his income by £8. 8*s.* 9⅗*d.*: what was the price of 4 per cent. stock?

(22) The price of consols is 88⅜: how many railway shares, the market price of which is £8. 15*s.*, can be purchased for £1000. consols?

(23) A person sells out of the 3 per cents, and invests his money in railway 5 per cent. stock at par: his income is thus increased £63⅓. per cent.: what was the price of the 3 per cents?

(24) A person sells out of the 3 per cents at 96, and invests his money in railway 5 per cent. stock at par: find how much per cent. his income is increased.

(25) What is the price of the railway 5 per cent. stock, so that a man who sold out of 3 per cents at 96, and invested in it, increased his income by £50. per cent.?

(26) When the 3 per cents are at 91¾, find how much can be bought for £540.

(27) I sell out of the 3 per cent. consols at 99, and invest in exchequer bills bearing interest at the rate of 2½*d.* a day per cent., when the bills are at a premium of 7*s.* 6*d.*: what effect has this on my income?

(28) 2 persons have each a capital of £12000.; one invests it in the 3 per cent. consols at 90⅝, the other in railway shares paying 5 per cent. at 103¾: find by how much the income of the one exceeds that of the other.

(29) By a reduction of the interest on exchequer bills from 2½*d.* to 1½*d.* per cent. per day a person loses at the rate of £152. 7*s.* 9*d.* per annum: what amount of exchequer bills does he hold?

(30) What sum must I invest in the 3¼ per cents at 97½ to obtain an income of £884. 16*s.* 3*d.*?

(31) Which is the better stock for investment, the 3¼ per cents at 92⅝ or the 3½ per cents at par?

(32) If a person invest in 4 per cent. stock at 77½, at what rate per cent. will he receive interest for his money?

(**33**) When a 3¾ per cent. stock is at 93, find what price a 4½ per cent. stock must be at in order that an investment may be made with equal advantage in either stock.

(**34**) Compare the income arising from £9855. invested in 3 per cents standing at 90, and £10201. invested in 3¼ per cents standing at 101.

(**35**) How much money will be required to purchase £8621. 10*s*. 5*d*. stock in the 3 per cents standing at 87⅛, and what interest will the purchaser receive?

(**36**) A speculator buys shares when they are 19½, £15. having been paid up, and sells them again at £32. 9*s*. when £25. has been paid up: how much per cent. does he gain or lose?

(**37**) A man invests £9600. in the 4 per cents at 80, and at the end of each year invests the dividend which becomes due in the same stock: find his dividend at the end of the third year, the funds remaining all the time at 80.

(**38**) The value of money increases from 4 per cent. to 5 per cent. Supposing this to have a corresponding influence on the funds, how much ought the 3 per cent. consols to fall?

(**39**) I bought into the 3 per cents standing at 92¼, and sold out again when they had risen to 96¾, improving my income by investing the proceeds in the 5 per cent. stock at par to the extent of £183. 15*s*. per annum. What amount of stock did I purchase in the 3 per cents, and what had I to pay for it?

(**40**) What sum must a person have invested in the 3 per cents at 90⅝ if a transfer of ⅗ of his capital to the 4 per cents at 115 would increase his income £7.?

(**41**) If a person transfer from the 3 per cent. stock which he has at 73⅛ to the 3½ per cents at 81¼, he would increase his income £4. 10*s*.: how much stock has he in the 3 per cents?

(**42**) Find value of 4 per cent. stock that will give 5 per cent. interest after paying 5*d*. in the £. income tax.

(**43**) A person invests half of £1500. in 3 per cent. consols at 90, and the other half in Chilian 6 per cent. bonds at 75. Find his average rate of interest; and if he sell the consols at par and the bonds at 37½, find the reduction in his capital.

(**44**) How much must I invest in the 3 per cent. stock at 90 in order that when I sell £1000. stock at 93½ and the remainder at 84¼ I may gain £6. 5*s*.; and if I invest the proceeds in the 4 per cents at par, find the difference in my income.

(**45**) A certain sum invested in the 3 per cents gives an income annually of £75. ; the same sum in a 5¼ per cent. stock at par would produce an income of £126. 13*s*. 1½*d*. : what was the sum and the price of the 3 per cents?

(**46**) A person sells consols at 95⅞, and on their falling he sells the same quantity at 92⅜, but on their again rising he buys back the whole at 94¼ : what does he gain or lose by the transaction?

(**47**) How much Midland stock, paying 5¼ per cent., at 105⅜, must I sell so that by purchasing with the product South Eastern stock, paying 4 per cent., at 78¾, I may gain £3. 5*s*. $2\frac{6}{7}$*d*. in income?

(**48**) The 3 per cents are at 87½, and the 5 per cents at 131¼ : which is the better investment? and what amount invested would produce a difference of £50. in a man's income?

(**49**) What must be the market value of the 4 per cent. stock in order that, after deducting an income tax of 4*d*. in the £., it may yield 5 per cent. interest?

(**50**) What must be the value of 5 per cent. stock that, after paying an income tax of 10*d*. in the £., it may yield 5¾ per cent. interest?

(**51**) A person buys 5 shares in a concern at 40, next year the price is 45, and every succeeding year there is a fall of 4. Each year he sells out 1 share, and finds, at the end, he has neither gained nor lost. What interest did the concern pay?

(**52**) A person has £5000. stock in the 3 per cents, which he invests in the 3½ per cents at 87½, thus increasing his income by £5. : find the price of the 3 per cents.

(**53**) Find the sum which must be invested in the 3 per cents. at 90, at simple interest payable half-yearly, to amount in 23½ years to £2317. sterling, the price of the funds remaining unchanged. If the funds rose to 96, in how many years sooner could the required amount be realised?

(**54**) By investing ⅓ of a sum in the 3 per cents at 90, ⅕ of it in 4 per cent. preference shares at par, ¼ of it on mortgage at 4¼ per cent., and the remainder in New Zealand 5 per cent. stock at 104, a person has an income of £481. 16*s*. 8*d*. : find the sum invested.

(**55**) If the 3 per cents are at 95, and Government offer to receive a loan of £5000000., the lender to receive five millions in the 3 per cents, together with a certain sum in the 3¼ per cent. stock, what ought this sum to be?

(**56**) A speculator bought Spanish bonds (yielding no interest) at 18, 18 Honduras £100. bonds paying 10 per cent. interest at 35, and 27 Turkish £100. bonds paying 5 per cent. at 45. At the end of 2 years he sells the Honduras stock at 15, and the following year he sold the Spanish at 25, and the Turkish at 46. Including the interest, he gained £1146. : what number of Spanish bonds did he buy?

(**57**) A man invested a sum of money in the 3 per cent. consols at 88, and at the end of 4½ months, after receiving a half-year's dividend, sold out at 87⅞ : at what rate per cent. per annum did he receive interest on his capital?

(**58**) How much must I invest in the 5 per cents at 117½ to yield £29. more income than the same sum invested in the 3 per cents at 92¼?

(**59**) A person has a certain capital, half of which he invests in the 3 per cents at 90, and the other half in the 5 per cents at 110 : what was his capital if his total income (from both sources) is £6883. 10*s.*?

(**60**) A man has stock in the 3 per cents, which brings him in £240. a year. He sells out ¼ of the stock at 87¼, and invests the proceeds in railway stock at 174½: what dividend per cent. ought the railway stock to pay, so that he may increase his income £40. by the transaction?

XXII. EXCHANGE.

221. *Exchange* is the reduction of a sum of money in the coinage of one country into its equivalent value in the coinage of another country at a given rate.

The standard rate of exchange, depending only on the intrinsic value of the coinage of two countries, is called the *par* of exchange.

The actual rate of exchange at any time, depending both on the intrinsic value of the coinage and on the state of trade between the two countries at that time, is called the *course* of exchange.

Arbitration of exchange is the determination of the rate of exchange between places by knowing their rates with intermediate countries.

It will be seen from the following examples that all such reduction can be effected by Chain Rule (**206**).

Ex. 1. Exchange £1000. into francs, the rate of exchange being 25 francs 37½ centimes per £. sterling.

1 franc = 100 centimes
£1. = 25·375 francs.

∴ £1000. = 25375 francs.

Ex. 2. Exchange 3000 dollars into English money at the rate of 4*s.* 6*d.* per dollar, when English money is at a premium of 10 per cent. in America.

Here it is clear that £100. sterling would purchase £110. worth of American dollars.

Therefore if x be the number of pounds sterling required to purchase 3000 dollars,

£x. = 3000 dollars
1 dollar = 9 sixpences
40 sixpences = £1.
11 = 10.

∴ $11x$ = £6750.
or x = £613. 12*s.* $8\frac{8}{11}$*d.*

Ex. 3. I buy 1000 dollars U.S. bonds, 6 per cents, at $86\frac{3}{4}$: find what they cost me, brokerage $\frac{1}{4}$ per cent., and the nominal par of exchange 4*s.* 6*d.* per dollar; find also my income, the course of exchange being 4*s.* $0\frac{1}{2}$*d.* per dollar.

Here, to buy 1000 dollars U.S. bonds, I must spend $1000 \times 87 \div 100$ dollars = 870; *i.e.* $870 \times 4\frac{1}{2}$*s.* = £199. 15*s*

And my income is

$$\frac{1000 \times 6}{100} \times 4s.\ 0\tfrac{1}{2}d. = 4s.\ 0\tfrac{1}{2}d. \times 60 = £12.\ 2s.\ 6d.$$

Ex. 4. Wheat costs at Paris 22 francs 50 centimes the hectolitre; the same quantity at London costs £2. 18*s.* 6*d.* the quarter. If 1 quarter = 2·9078 hectolitre and £1 = 25·25 francs, in which place is wheat the dearer?

x francs at London = 1 hectol.
2·9078 hectol. = 1 qr.
1 qr. = 702*d.*
240*d.* = £1.
£1. = 25·25 fr.

$$\therefore x = \frac{702 \times 25·25}{240 \times 2·9078} = 25·4.$$

∴ wheat is 2 francs 90 centimes dearer at London than at Paris; or 12·89 per cent. dearer.

Exercise 142.

(1) If £500. is to be transmitted from London to a person in St. Petersburg: will he gain or lose, and how much, by having it sent to Paris, the exchange being 25·74 francs for £1., 3 roubles for 13 francs, and 5·92 roubles for £1.?

(2) If 98 Spanish reals = 25 French francs, 22 francs = 9 Frankfort florins, 30 florins = 49 Hamburgh marks, and 192 marks = 55 Norwegian dollars: how many Norwegian dollars are equivalent to 2048 Spanish reals?

(3) Cotton is quoted in New Orleans 42 cents per lb. (exchange 156¼), freight and all charges are ¾*d.* per lb.: what will be its cost price in Liverpool?

(4) Cotton is quoted in New Orleans 40 cents per lb. (exchange 130), freight and all charges are ¾*d.* per lb.: what will be its cost price in Liverpool?

(5) If I sell 434 sovereigns at 25·5 francs each, and buy Spanish piastres at 5·4 francs each, selling the latter at 5·6 francs each, and buying sovereigns at 26·35 francs each: find the gain or loss.

(6) If £3. = 20 thalers, 25 thalers = 93 francs, 27 francs = 5 scudi, 62 scudi = 135 gulden: how many gulden in £2.?

(7) What is the rate of exchange between London and Paris when securities, which are sold at Hamburgh for 13 marks 10*s.* per £1. sterling, and bought at Amsterdam at 35·05 florins for 40 marks, and which will purchase French securities at the rate of 58 florins for 120 francs? (16*s.* = 1 mark.)

(8) A person on leaving England exchanged his money for French at the rate of 25 francs for 1 sovereign, and on arriving at Munich received 135 Bavarian gulden for 300 francs: what was his loss (1) in English money, (2) in French money, supposing a gulden = 1*s.* 8½*d.*?

(9) A merchant in Hamburgh delivers goods to a merchant in London at 5 m. 16 pf. per pfund, a pfund = 1·068 lbs. The London merchant remits payment at 2052 pf. per £1., and sells the goods at £27. for 100 lbs.: what is his gain or loss per cent.?

(10) When British standard silver is valued at 5*s.* 1⅜*d.* an ounce, find how many francs are equal to 20*s.*, having given that 1 lb. troy standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings; 1 kilog. French standard silver, $\frac{9}{10}$ fine, is coined into 200 francs; and 1 kilog. = 15432 grains.

XXIII. THE METRIC SYSTEM.

222. The *Metric System* of weights and measures is now in use in many countries in Europe.

The basis of all measurement is the *metre*, the ten-millionth part of the distance from the North Pole to the Equator; or equivalent to about 39·37 inches in English Measure. In the formation of the multiples and submultiples the decimal system is followed; the Greek prefixes to any unit denoting the multiples, and the Latin prefixes the submultiples. Then the metre being the unit of length—

Its multiples are—

deca-metre	denoting	10	metres
hecto-metre	„	100	„
kilo-metre	„	1000	„
myria-metre	„	10000	„

Its submultiples are—

deci-metre	denoting	$\frac{1}{10}$	metre
centi-metre	„	$\frac{1}{100}$	„
milli-metre	„	$\frac{1}{1000}$	„

223. The units of Metric Measure are—

(1) *Length*—the metre.

(2) *Surface*—the are = 100 square metres.

(3) *Solidity*—the stère = 1 cubic metre.

(4) *Capacity*—the litre = the cube of the tenth part of a metre.

(5) *Weight*—the gramme = the weight of distilled water which fills the cube of the hundredth part of a metre.

(6) *Money*—the franc—a coin weighing 5 grammes, 9 parts silver, 1 part copper.

224. The following are rough approximates in English measures—

The kilogramme is	about	2$\frac{1}{5}$ lbs. avoird.
The litre	„	1$\frac{3}{4}$ pints
The kilometre	„	5 furlongs
The hectare	„	2$\frac{1}{2}$ acres
25 francs	„	£1.

225. From these data we will show how a complete table of weights and measures, with their English equivalents, may be constructed.

Ex. 1. Given that one metre = 39·37079 inches, express one hectare in acres.

3)39·37079 inches
12)13·123563
1·09363 yards

∴ one square metre = $(1·09363)^2$ square yards = 1·19601 square yards.

1·09363
363901
109363
9842
328
65
3

48400)119601(2·47114
22801
3441
53
5
1

∴ one hectare = 10000 square metres = 2·47114 acres.

Ex. 2. One gramme = 15·43234874 grains, express *one tonne* in tons.

One tonne = 1000000 grammes
= 15432348·74 grains; this divided by 7000
= 2204·62124 lbs. avoird.; this divided by 2240
= ·984206 tons.

226. The following table contains the weights and measures chiefly used, with their English equivalents—

1 yard = ·91438347 metre
1 foot = ·30479449 metre
1 inch = ·02539954 metre
1 mile = 1·6093149072 kilometre
1 grain troy = ·06479895 gramme
1 pound avoird. = ·4535926525 kilogramme

1 ton = 1·0160475416 tonne
1 acre = ·4046711 hectare
1 gallon = 4·54102 litres
1 metre = 1·09363307 yard
1 metre = 3·2808992 feet
1 metre = 39·3707904 inches
1 kilometre = ·621383 mile
1 gramme = 15·432348 74 grains
1 kilogramme = 2·20462125 lbs. avoird.
1 hectare = 2·47114 acres
1 litre = ·220215 gallon.

We will now work out a few examples.

Ex. 1. Express 5·60723 hectares in acres, roods, poles.

We see from the above table that one hectare = 2·4711 acres.

∴ 5·60723 hectares = 5·60723 × 2·47114 acres.

560723
411742
1121446
224289
39251
561
56
22
13·85625 acres
4
3·425~~00~~
40
17·000

∴ *Ans.* = 13 acres 3 roods 17 poles.

Ex. 2. A litre of alcohol weighs 792 kilog.: find the weig of a cubic foot in lbs.

A litre = a cubic decimetre
One metre = 30·37079 inches.

∴ one decimetre = 3·937079 inches
= ·328089 foot.

∴ one cubic decimetre = $(\cdot 328089)^3$ foot.

328089
980823
984267
65617
26247
262
29
1076422
9808323
0322926
21528
8611
86
9
·0353162

∴ one litre = one cub. decim. = ·035316 cub. ft.
but one kilog. = 2·20462 lbs.

∴ one litre of alcohol = 2·20462 × ·792 lbs.
= 1·74605904 lbs.

∴ one cub. ft. of alcohol = 1·74605904 ÷ ·035316
= 49·44 lbs.

Ex. 3. Find the price per yard in English money at 7·85 francs per metre, £1. being equivalent to 25·75 francs, or the exchange being 25·75.

1 metre = 7·85 fr.
25·75 fr. = £1.
1·0936 yds. = 1 metre (from the table)
£x. = 1 yd.

$$\therefore x = \frac{£7\cdot 85}{25\cdot 75 \times 1\cdot 09363}$$

25·75
363901
25750
2318
77
15
1
28·161

28·161)7·850(·2787
2218 9
247
22

Ans. = 5*s.* 7*d.*

£y.

$$\frac{y}{x} = \frac{\cdot 9143837}{25\cdot 2215} = \cdot 036$$

25221

The reciprocal of this, 27·58
the price per yard into price

Exc

Assuming 1 metre = 39·3
lons ; 1 kilogramme = 1543
centimetre of distilled wate

Find a multiplier and d

(1) Yards into metres.
(2) Miles into kilometre
(3) Square yards into c
(4) Acres into he
(5) Gallons in
(6) Bushel
(7) Pound

(**13**) 14·25 metres cost 195·3 francs : find the price per yard in shillings.

(**14**) The weight of one tonne of vin de Bordeaux is 357·804 kilogs., its density being ·9939 : find the weight in tons of the same volume of water.

(**15**) If 4·654 kilogs. of paint are spread over 65·85 square metres : how many square feet are covered by one kilogramme of paint?

(**16**) 5·75 litres of wine are mixed with 12·4 litres of water : express in pints the quantity of water in a litre of the mixture.

(**17**) 4·5 seconds elapsed between the *flash* of a gun and the report being perceived by a person : express in inches his distance from the gun.

(**18**) 15·75 kilogs. of sugar cost 42·50 francs : find the price per pound.

(**19**) 1·85 litre of wine cost 1 franc : express in pounds the cost of 47·36 litres.

(**20**) A pedestrian, who travels 5 leagues a day, starts from Fontainebleau to reach Lyons ; 2 days afterwards a courier, who travels 15 leagues a day, starts from Paris also for Lyons, and overtakes him in 3 days : express in miles the distance from Paris to Fontainebleau.

(**21**) A battle lasted 10 hours ; there were 30 pieces of cannon, each fired 7 times in the hour, the charge for each piece was 4 kilogs. of powder costing 1·8 francs each : what was the whole expense in pounds?

(**22**) A merchant bought 5 tonnes of wine, each containing 200 bottles. He paid for 3 of them at the rate of 150 francs a piece ; to the 5 tonnes he adds 100 bottles of water, and by selling the mixture at ·75 franc a bottle, he gains 175 francs : what was the price of the two last tonnes?

(**23**) In a college which numbers 360 pupils, 4 pupils consume daily a litre of wine : what is the annual wine bill, if a tonne cost 60 francs and contains 200 bottles, and 4 bottles contain 3 litres?

(**24**) A man has 32 teeth. A pauper agrees to have all his teeth drawn, provided he receive 2 francs for the first tooth, 4 for the second, 8 for the third, and so on, and return 3 francs for the first tooth he refuses to have drawn, 9 for the second, 27 for the third, and so on. After having lost 19 teeth, pain obliges him to discontinue the operation : what sum of money, besides his 19 teeth, does he lose?

(**27**) A long pole
one-fifth of its length
sixth green, and the
in yards the length of

(**28**) If I add the tr
a sum to 5000 francs,
pounds had I?

(**29**) In travelling 7
turn 1170 times, the fo
inches their circumferen

(**30**) If 35 centiares a
per acre?

(**31**) 35 hectares 5 a
metre: find the whole ar
and the cost per acre.

(**32**) A forest 3 squar
950 francs per hectare: w
acre?

(**33**) The price of 5% re
francs: what should be the
the 3%, now at 63·35 francs,

(**34**) If I buy 3500 francs
them at 96·4 francs, how muc

(**35**) I buy 5400 francs 3%
them at 67·80: what do I lose

(**36**) A reservoir 15 metre
metres high is filled with wat
does it contain?

(**37**)

(**40**) A vessel contains 703 litres of liquid which cost 5·2 francs a kilogramme: how much will the liquid cost if it weigh 1½ as much as the same quantity of water?

(**41**) The number of kilogrammes to the litre expresses the specific gravity referred to water as unity: find the weight in pounds of a cubic foot of the following:—

	Sp. gr.		Sp. gr.
Air . . .	1	Pure water .	1
Carbonic Acid .	1·53	Sea water .	1·025
Hydrogen .	·07	Alcohol, proof .	·916
Oxygen . .	1·16	Sulphuric acid .	1·8
Nitrogen . .	·97	Mercury . .	13·6
Steam at 212° .	·47		

(**42**) Given the weight in pounds of a cubic foot of the following substances, find their specific gravity:—

	Weight		Weight
Iron, cast .	444	Tin . . .	460
Iron, wrought .	480	Zinc . . .	437
Steel . . .	490	Silver . . .	655
Copper . .	550	Gold . . .	1190
Lead . . .	712	Platinum . .	1344

Find a multiplier or divisor which gives the price in English money:—

(**43**) Per yard, given price per metre in francs.

(**44**) Per square yard, given price per square metre in francs.

(**45**) Per acre, given price per hectare in francs.

(**46**) Per gallon, given price per litre in francs.

(**47**) Per bushel, given price per hectolitre in francs.

(**48**) Per pound avoird., given price per kilog. in francs.

(**49**) Per cwt., given price per quintal in francs.

(**50**) Per ton, given price per millier in francs.

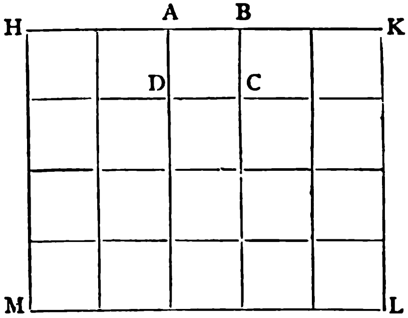
XXVI. MENSURATION.

I. AREAS.

227. If we take a line A B as our unit of length, and describe a square on it, this square may be taken as the unit of area; and the number of times this square is contained in any given area is the measure of that area.

For instance, construct a rectangle H K L M such that

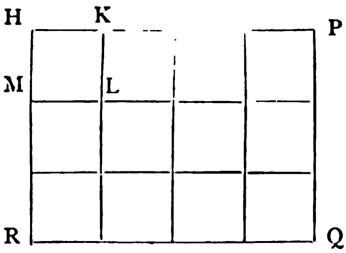
H K = 5 A B, K L = 4 A B, and let parallel lines be drawn through the points of section, as in figure.



Then A B is the unit of length, A B C D is the unit of area: H K contains A B 5 times. ∴ 5 is the *measure* of A B, and H K L M contains A B C D 20 times. ∴ 20 is the measure of the area H K L M.

∴ to find the area of a rectangle, we multiply the measure of the length by the measure of the breadth, and the product will be the measure of the area.

228. Next, suppose H K does not contain A B an exact number of times, so that H K = 3¾ × A B, then the measure of H K will be 3¾, and similarly, let the measure of H M be 2⅔: what will now be the measure of H K L M?



Produce H K to P, so that H P = 4 H K, and H M to R, so that H R = 3 H M. Then the measure of H P is 15, and that of H R is 8. Therefore the measure of H P Q R = 15 × 8 by the preceding case, and it contains H K L M 12 times.

$$\therefore \text{ the measure of H K L M } = \frac{15 \times 8}{12} = \frac{15}{4} \times \frac{8}{3}.$$

Thus, we have shown that, whether the measures of the length and breadth be integers or fractions, the measure of the area is their product.

Conversely, given the measure of the area and the measure of either the length or breadth, their quotient will be the measure required.

Ex. 1. Find the area of a rectangle 5 ft. 4 in. long, 4 ft. 3 in. broad.

If we take one inch as our unit, the measure of the length will be 64 and the measure of the breadth 51, and the measure of the area will be 64×51, or the area is 3264 sq. in., or 22 sq. ft. 96 sq. in.

If, however, we take one foot as the unit, the measure of the length will be $5\frac{1}{3}$, the measure of the breadth $4\frac{1}{4}$, and the measure of the area will be $5\frac{1}{3} \times 4\frac{1}{4}$, or $\frac{16}{3} \times \frac{17}{4}$, or $\frac{272}{12}$, or the area is $22\frac{2}{3}$ sq. ft., or 22 ft. 96 in., as before.

Ex. 2. The area of a rectangle is 104 sq. yds. 5 sq. ft. 60 sq. in., one of its sides is 79 ft.: what is the other?

Assuming one foot as the unit, the measure of the area is $941\frac{5}{12}$, the measure of the known side is 79. ∴ the measure of the side required is $941\frac{5}{12} \div 79$, or $\frac{11297}{12} \times \frac{1}{79} = 11\frac{869}{948} = 11\frac{11}{12}$. ∴ the length of the required side is 11 ft. 11 in.

Ex. 3. A room is 10 ft. high, 5 yds. 1 ft. long, and 3 yds. broad; it contains a door 8 ft. by 4 ft., two windows, each 5 ft. by 4 ft., and a fire-place 6 ft. by 4 ft. 6 in.: how much paper will be required to cover the walls?

The walls of a room may be considered as together equal to a rectangle whose length is the circumference of the room, and breadth the height of the room: thus, one foot being the unit the measure of the area of the walls is $10 \times 2\,(16 + 9) = 10 \times 2 \times 25 = 500$; the measure of the two windows $= 2 \times 5 \times 4 = 40$; of the fire-place $= 6 \times 4\frac{1}{2} = 27$; of the door $= 8 \times 4 = 32$. ∴ the measure of the surface to be papered is $500 - 40 - 27 - 32 = 401$, or the area is 401 sq. ft. or 44 sq. yds. 5 sq. ft., which is the quantity of paper required.

Ex. 4. The length, breadth, and height of a room are 7 yds. 1 ft. 3 in., 5 yds. 2 ft. 9 in., and 4 yds. 6 in., respectively: what will be the cost of the paper 2 ft. broad which will cover the walls at 9*d.* per yard?

Taking one yard as the unit, the circuit of the room is

$2 \times 13\frac{1}{3}$, the height is $4\frac{1}{6}$, ∴ its area is $2 \times 13\frac{1}{3} \times 4\frac{1}{6}$; the area of a piece of paper 2 ft. wide and one yard long is $\frac{2}{3} \times 1$; its cost is 9*d.*

∴ the 1st cause is $\frac{2}{3} \times 1$, the 2nd cause is $2 \times 13\frac{1}{3} \times 4\frac{1}{6}$;
And the 1st effect is 9*d.*, the 2nd effect is x.

$$\therefore \tfrac{2}{3} \times 1 : 2 \times 13\tfrac{1}{3} \times 4\tfrac{1}{6} :: 9d. : x.$$

$$\therefore x = \frac{\cancel{2}}{1} \times \frac{\overset{20}{\cancel{40}}}{\cancel{3}} \times \frac{25}{\underset{\cancel{3}}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{1} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{1}{1} = 1500d. = £6.\ 5s. \quad \textit{Ans.}$$

EX. 5. The length, breadth, and height of a room are 16 ft. 8 in., 15 ft. 9 in., and 14 ft., respectively; the cost of the paper at 3*s.* 6*d.* a yard is £30. 5*s.* $1\frac{1}{3}$*d.*: find the width of the paper.

Taking one foot as the unit, the circuit of the room is $2 \times 32\frac{5}{12}$, the height is 14; and if one penny be the unit of price, one piece of paper x ft. wide and 3 ft. long is 42, the whole cost of the paper is $7261\frac{1}{3}$.

$$\therefore x \times 3 : 2 \times 32\tfrac{5}{12} \times 14 :: 42 : 7261\tfrac{1}{3}.$$

$$\therefore x = \frac{2 \times 32\frac{5}{12} \times 14 \times 42}{3 \times 7261\frac{1}{3}} = \frac{\cancel{2}}{1} \times \frac{\cancel{389}}{\underset{6}{\cancel{12}}} \times \frac{\overset{\cancel{7}}{\cancel{42}}}{1} \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{4}{\underset{\cancel{1556}}{\underset{\cancel{3112}}{\cancel{21784}}}}}$$

$$= \frac{7}{4} = 1 \text{ ft. } 9 \text{ in.}$$

∴ *Ans.* = 1 ft. 9 in.

EX. 6. Find the cost of carpeting a floor 23 ft. 4 in. long and 16 ft. 9 in. wide at 6*s.* 3*d.* a sq. yd.

Taking as our unit of length one foot, the area of the floor is $23\frac{1}{3} \times 16\frac{3}{4}$, the area of a piece of carpet 3×3; and if one shilling be the unit of price,

The 1st cause is 3×3, the 2nd cause $23\frac{1}{3} \times 16\frac{3}{4}$.
The 1st effect is $6\frac{1}{4}$, the 2nd effect is x.

$$\therefore 3 \times 3 : 23\tfrac{1}{3} \times 16\tfrac{3}{4} :: 6\tfrac{1}{4} : x.$$

$$\therefore x = \frac{70}{3} \times \frac{67}{4} \times \frac{25}{4} \times \frac{1}{3 \times 3} = \frac{469000}{12 \times 12 \times 12}.$$

$$\begin{array}{r@{}l} 12\,)\,\underline{469000} & \\ 12\,)\,\underline{39083} & —4d. \\ 12\,)\,\underline{3256} & —11\tfrac{1}{3}d. \\ 271 & —4\tfrac{17}{18}d. \end{array}$$

$\therefore$ *Ans.* = £13. 11*s.* $4\frac{17}{18}$*d.*

EX. 7. The carpet for a room 21 ft. 8 in. by 16 ft. 6 in. at 3*s.* 4½*d.* a yard costs £8. 18*s.* 9*d.*: find the width of the carpet in inches.

Taking one inch as the unit of length, and one penny as the unit of price,

The 1st cause is $x \times 36$, the second cause is 260×198.
The 1st effect is $40\frac{1}{2}$, the second effect is 2145.

$$\therefore x \times 36 : 260 \times 198 :: 40\tfrac{1}{2} : 2145.$$

$$\therefore x = \frac{260 \times 198 \times 81}{36 \times 2145 \times 2} = \frac{\overset{\cancel{2}}{\cancel{4}} \times \cancel{65} \times \cancel{11} \times \cancel{18} \times 81}{\underset{\cancel{2}}{\cancel{36}} \times \cancel{11} \times \underset{3}{\cancel{195}} \times \cancel{2}}$$

$$= \frac{81}{3} = 27 \text{ inches}$$

229. (1) Taking one foot as the unit of length, one penny as the unit of price; if h, l, b be the height, length, and breadth of a room, w the width of the paper, and p its price per yard, and P the whole cost of the paper, then the relation between these quantities may be thus obtained—

The 1st cause is the area of a piece of paper $w \times 3$.
The 2nd cause is the paper on the 4 walls $= 2\,h\,(l + b)$.
The 1st effect is p, the 2nd effect is P.

$$\therefore 3 \times w : 2\,h\,(l + b) :: p : P,$$
$$\text{or } P \times 3 \times w = 2\,h\,(l + b)\,p.$$

From this result, any 5 of the quantities being given, the sixth can be obtained.

(2) Similarly, for the carpet of such a room—

The 1st cause is $3 \times w$, the 2nd cause is $l \times b$.
The 1st effect is p, the second effect is P.

$$\therefore 3 \times w : l \times b :: p : P,$$
$$\text{or } 3 \times w \times P = l \times b \times p.$$

From this result, any 4 of these quantities being given, the fifth can be obtained.

what is its width?

(3) The area of a squa
what is its length?

(4) A rectangular floor co
measures 18 ft. 9 in.: what is

(5) A piece of carpet 16 ft
area is 396 ft. 60 in.: how lon

(6) 221$\frac{11}{27}$ yds. of carpet ar
long by 24 ft. 6 in. broad: wh

(7) 26 yds. 4 in. of paper $\frac{3}{4}$
cover a wall 15 ft. 8 in. long: l

(8) Find the breadth of a ro
and area 377 ft. 125$\frac{5}{8}$ in.

(9) What length of paper $\frac{7}{8}$
quired to cover the walls of a r
breadth 14 ft. 7 in., and height 1

(10) The area of a square i
a rectangle is 142 sq. yds. 90 in.
3 ft. 4 in. less, how much is the
the square?

(11) If 759 tiles 7 in. square
19 ft. 3 in. long: what is its wid

(12) How many pieces of ca
5 ft. 3 in. long will be requir
29 yds. 1 ft. 72 in.?

(13) How many pieces of pa
broad will cover a wall containi

12 ft. 6 in. wide, is £6. 10*s.* 2½*d.* : if the carpet is ¾ yard wide, what is its price per yard?

(**17**) What is the width of paper required for the walls of a room 19 ft. 6 in. long, 17 ft. 3 in. wide, and 10 ft. high, which, at 2¼*d.* per yard, costs £1. 7*s.* 6¾*d.*?

(**18**) The cost of a cornice all round a room at 3*s.* 6*d.* per yard is £2. 15*s.* 1½*d.*; the room is 29 ft. long, 18 ft. 3 in. wide, and 15 ft. 6 in. high: how deep is the cornice?

(**19**) The length of a room is 24 ft. 3 in., the height is 11 ft. 6 in., and the cost of painting the four walls at 4*s.* per square foot is £183. 12*s.* 4*d.* : what is the breadth?

(**20**) A path is made across a field at one end and down one side; the field is 79 yds. long and 64 yds. wide; the area of the path : area of field :: 141 : 2528; what is the width of the path?

(**21**) If the cost of the carpet for a room 20 ft. 3 in. long by 16 ft. wide at 3*s.* 4*d.* a yard be £8., how wide is it?

(**22**) The cost of papering a room 14 ft. 6 in. long, 10 ft. 6 in. broad, and 8 ft. high, with paper 2 ft. wide is £2. 1*s.* 8*d.* : what is the price per yard?

(**23**) A quadrangle is 50 ft. long by 40 ft. broad; it is crossed in each direction by a path, and the remainder has to be turfed; it requires 1600 strips of turf 1½ ft. long and 6 in. broad: how broad is the path?

(**24**) The cost of papering a room 20 ft. long, 12 ft. broad, 10 ft. high, with paper 2⅔ ft. broad at 4½*d.* per yard was £1. 13*s.* 4*d.* : how much per yard was charged for hanging?

(**25**) 136 yds. 2 ft. 8 in. of paper are required for a room 27 ft. 5 in. long, 14 ft. 7 in. wide, and 12 ft. 10 in. high: how wide is the paper?

(**26**) The cost of papering a room 12 ft. wide and 10½ ft. high, with paper 21 inches wide at 3½*d.* per yard, is £1. 12*s.* 8*d.*: how long is the room?

(**27**) A carpet 20 ft. 6 in. long by 12 ft. 9 in. costs £30. 9*s.* 10½*d.* : find the price per square yard.

(**28**) The cost of paper for a room 21 ft. long, 10¼ ft. high, at 7½*d.* per square yard, is £2. 13*s.* 4⅝*d.* : how wide is it?

(**29**) A room 10 ft. high, 20 ft. long, and 16 ft. wide contains a door 8 ft. high by 4 feet. wide, and windows each 5 ft. by 3 ft.: if 73⅑ sq. yds. of paper are required for the walls, how many windows were there?

(**30**) A room is 10 ft. high, 5 yds. 1 ft. long, and 3 yds. broad;

it contains a door 8 ft. by 4, two windows each 5 ft. by 4, and a fire-place 6 ft. long: if it require 44 sq. yds. 5 sq. ft. of paper for the walls, how wide was the fire-place?

(31) The cost of carpet for a room 16 ft. wide is £13. 1*s.* 4*d.*; the carpet is $\frac{3}{4}$ yard wide, and costs 5*s.* 3*d.* a yard: how long is the room?

(32) The cost of painting a room 22 ft. 5 in. long, 18 ft. 4 in. broad, and 10 ft. 9 in. high, is £29. 4*s.* 1*d.*: how much is that per square foot?

(33) A room 11 ft. high and 17 ft. long requires 88 yds. of paper to cover its walls, and $24\frac{5}{9}$ sq. yds. to cover its floor: how wide is the paper?

(34) The cost of papering a room 10 ft. 8 in. wide, $9\frac{1}{2}$ ft. high, with paper 2 ft. wide at $2\frac{1}{2}$*d.* a yard, is 19*s.* $9\frac{1}{2}$*d.*: how long is it?

(35) A plot of ground 50 yds. long and 40 yds. wide is laid with sods 6 in. broad; the sods cost £36. at 3*s.* per hundred: what is the length of a sod?

(36) What quantity of Turkey carpet is required for a room 20 ft. 4 in. long and 18 ft. 8 in. broad, allowing a margin of 2 ft. 8 in. all round? Find also the number of tiles, each 8 in. by 4, which will be needed to fill this margin.

(37) Find the price per yard of paper 2 ft. wide required for a room 17 ft. 4 in. long, 12 ft. 8 in. broad, and 12 ft. high, when the whole cost is £3. 15*s.*

(38) The cost of carpet $\frac{3}{4}$ yard wide, at 5*s.* 6*d.* per yard, for a room 19 ft. 6 in. long is £12. 2*s.* $3\frac{2}{3}$*d.*: what is the width of the room?

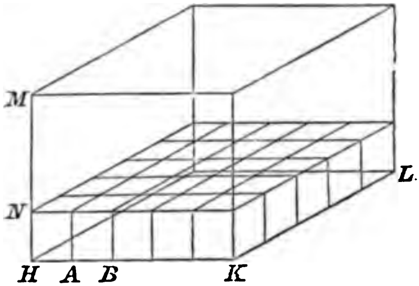
(39) It costs £643. 10*s.* 9*d.* to level and turf a square cricket-ground at 9*d.* per square yard; it costs £196. 10*s.* to inclose it with iron railing: what did the railing cost per yard?

(40) The area of a square cricket-field is 9 acres 3 roods 8·16 poles. A running-path of the uniform width of 3·9 yards is constructed close to the boundary of the field at a cost of 4*d.* per square yard, and the remainder of the field is laid down in turf at a cost of 5*s.* 6*d.* per hundred square yards: find the total cost of preparing the field.

(41) A room is 24 ft. long, $13\frac{1}{2}$ wide, and $9\frac{1}{2}$ high; the ceiling is whitewashed at 7*d.* the dozen square yards; the walls are papered at $\frac{3}{4}$*d.* the square foot (deducting 16 per cent. for windows, doors, etc.), and the floor is carpeted at 3*s.* 7*d.* the square yard: find the whole cost.

II. SOLIDS.

230. To find the volume of a retcangular parallelopiped, we proceed thus :—



Let A B be our unit of length, and let H K contain A B 5 times ; then its measure is 5. Similarly, let the measure of K L be 5, and the measure of M H be 4. Now, if we divide M H into 4 parts, H K into 5 parts, and K L into 5 parts, and draw through these points planes parallel to the sides, we obtain 4 × 5 × 5 solids, each side of which equals A B, and therefore the measure of the whole volume = 4 × 5 × 5.

Whence we deduce the rule—

Express the length, breadth, and depth or height, in units of the same denomination, and their product will give the measure of the volume in cubit units of that denomination.

231. Generally we may state the above thus : Let V be the measure of the volume, h, l, b, that of the height, length, breadth, respectively ; then

$$V = h \times l \times b ;$$

whence, gîven any 3 of these quantities, we can easily find the 4th.

Ex. 1. Find the cubical content of a room 40 ft. 10½ in. long, 25 ft. 8 in. broad, and 16 ft. 9 in. high.

Taking one foot as the unit of measurement,

$$l = 40\tfrac{7}{8},\ b = 25\tfrac{2}{3},\ h = 16\tfrac{3}{4}.$$

$$\therefore V = 40\tfrac{7}{8} \times 25\tfrac{2}{3} \times 16\tfrac{3}{4} = \frac{\overset{109}{\cancel{327}}}{8} \times \frac{77}{\cancel{3}} \times \frac{67}{4} = 17572\tfrac{27}{32}.$$

Ex. 2. What is the length of a block of marble 5 ft. 3 in. broad, 4 ft. 7 in. thick, which contains 248 cub. ft. 1116 in

Taking one foot as the unit of measurement,

$$V = 248\tfrac{31}{48},\ b = 5\tfrac{1}{4},\ l = 4\tfrac{7}{12}.$$

$$\therefore l = \frac{\overset{\overset{217}{\cancel{2387}}}{\cancel{11935}}}{\cancel{48}} \times \frac{\cancel{4}}{21} \times \frac{\cancel{12}}{\underset{\cancel{11}}{\cancel{55}}} = \frac{31}{3} = 10 \text{ ft. } 4 \text{ in.}$$

III. DUODECIMALS.

232. The following example will explain the method of cross multiplication or Duodecimals, by which rule practical men generally work.

Multiply by duodecimals 8 feet 2 in. 4 parts by 4 feet 6 inches 9 parts, and the product by 3 feet 7 inches, and express the result in cubic feet cubic inches.

Arrange the work thus—

8	2	4			
4	6	9			
32	9	4			(1)
4	1	2	0		(2)
	6	1	9	0	(3)
37	4	7	9	0	(4)
3	7				
112	1	11	3		(5)
21	9	8	6	3	(6)
133	11	5	9	3	(7)

Multiply 1st by 4 feet; 4×4 parts $= 16'' = 1$ inch 4 parts $= 1'\ 4''$; place $4''$ under the parts and carry 1; 4×2 inches $= 4 \times 2' = 8'$ and $1'$ to carry $= 9'$; $4 \times 8 = 32$. Hence (1) is $32\ 9'\ 4''$.

To obtain (2) multiply by $6'$; $4'' \times 6' = 24''' = 2''$; put down $0'''$, carry $2''$; $6' \times 2' = 12''$ and $2''$ to carry $= 14'' = 1'\ 2''$; put down $2''$, carry $1'$; $8 \times 6' = 48'$ and $1' = 49' = 4$ ft. $1'$.

To obtain (3) $4'' \times 9'' = 36'''' = 3'''$; $9'' \times 2' = 18'''$, and $3''' = 21''' = 1''$ and $9'''$ over; $8 \times 9'' = 72''$ and $1''$ to carry $= 73'' = 6'\ 1''$.

(4) is obtained by adding (1), (2), (3).

To obtain (5) multiply (4) by 3 feet; $3 \times 9''' = 27''' = 2''\ 3'''$; $3 \times 7'' = 21''$ and $2'' = 23'' = 1'\ 11''$; $3 \times 4' = 12'$ and $1' = 13' = 1\ 1'$.

To obtain (6) multiply (4) by 7′; 9‴ × 7′ = 63⁗ = 5‴ 3⁗; 7′ × 7″ = 49‴ and 5‴ = 54‴ = 4″ 6‴; 7′ × 4′ = 28″ and 4″ = 32″ = 2′ 8″; 37 × 7′ = 259′ and 2′ = 261′ = 21 9′.

(7) is obtained by adding (5) and (6).

N.B.—12⁗ (fourths) = 1‴ (third); 12‴ (thirds) = 1″ (second or part); 12″ (seconds or parts) = 1′ (prime or inch); 12′ (primes or inches) = 1 foot. In all cases the product will be the denomination, shown by the sum of the indices.

The answer, therefore, to the above sum in feet and inches might be written

$$133 + \frac{11}{12} + \frac{5}{12^2} + \frac{9}{12^3} + \frac{3}{12^4} \text{ cubic feet, or}$$

133 cubic feet 1677¼ cubic inches.

Exercise 145.

Find by duodecimals the value of—

(1) 15 ft. 1′ 3″ × 8 ft. 7′ 4″ × 6 ft. 3′.
(2) 6 ft. 1′ 3″ × 8 ft. 7′ 2″ × 8 ft. 7′.
(3) 8 ft. 7′ 2″ × 5 ft. 7′ 11″ × 3 ft. 7′.
(4) 36 ft. 8′ × 7 ft. 11′ × 6 ft. 5′.
(5) 13 ft. 7′ × 9 ft. 3′ × 2 ft. 5′.
(6) 5 ft. 6′ 4″ × 2 ft. 7′ 3″ × 6 ft. 5′.
(7) 7 ft. 3′ 5″ × 5 ft. 7′ 4″ × 4 ft. 2′.
(8) 36 ft. 7′ × 7 ft. 11′ × 6 ft. 10′.
(9) 9 ft. 7′ 3″ × 2 ft. 7′ 5″ × 5′ 7″.
(10) 6 ft. 3′ 8″ × 7 ft. 4′ 8″ × 8 ft. 5′.

For hints as to solutions of the following Exercise the student is referred to pp. xxxv. and xxxvi.

Exercise 146.

(1) Find the side of an equilateral triangle whose area cost as much paving at 9*d.* per foot as palisading the 3 sides did at 15*s.* a yard.

(2) A triangular field 738 links and 538 links in the perpendicular produces an income of £12. a year: at how much an acre is it let?

(3) How much will the turfing a round plot cost at 4*d.* per square yard if it be 130 feet round?

(4) The paving of a semicircular alcove with marble at 2*s.* 6*d.* a foot came to £10.: what was the length of the semicircular arc?

(5) A field in the form of an equilateral triangle contains half an acre: what must be the length of a tether fixed at one of its angles and to a horse's nose to enable him to graze exactly half of it?

(6) A hexagonal prism is 25¼ feet long, and the central diagonal of its base is 2¼ feet: find the whole surface and solid content.

(7) Find the whole surface and volume of the frustum of an octagonal pyramid whose perpendicular height is 6 feet, and each side of the two ends 4 feet and 5 feet respectively.

(8) The diameter of a well is 3 ft. 9 in. and its depth 45 ft.: what did the excavation cost at 7*s.* 3*d.* per cubic yard?

(9) If a heavy sphere 4 inches in diameter be placed in a conical glass full of water, whose diameter is 5 and altitude 6 inches: find how much water will run over.

(10) An ale glass in the form of a conic frustum is 3⅜ inches in depth, the diameter of the mouth is 2⅜ inches, and that of the bottom is 1 inch: find its content and determine how many of such glasses an imperial gallon would be equivalent to.

(11) A cask in the form of 2 conic frustums joined at the bases, has the diameter at the head 20 inches and at the bung 25 inches; also the length is 3 ft. 4 in.: find the weight of water required to fill it, supposing that a cubic foot of water weighs 1000 ozs.

(12) A right cone and a hemisphere lie on opposite sides of a common base of 2 ft. diameter, and the cone is right-angled at the vertex; if a cylinder circumscribe them in this position, how much additional space is thereby enclosed?

(13) Divide a cone into 3 equal parts by sections parallel to the base, and find the altitudes of the three parts, the height of the cone being 20 inches.

(14) What quantity of canvas is necessary for a conical tent whose perpendicular height is 8 feet and the radius at the bottom 6½ feet?

(15) If a cubic foot of brass be drawn into wire of $\frac{1}{40}$ inch diameter, what will be the length of the wire, supposing no loss of metal in working?

(16) A heavy sphere whose diameter is 4 inches, is let fall into a conical glass, $\frac{1}{5}$ full of water, whose diameter is 5 and altitude 6 inches: what part of the axis of the sphere is immersed in the water?

MISCELLANEOUS EXAMPLES.

(1) Write in words: 2003; 4070; 3005; 1800.

(2) Find the sum of the numbers 35, 58, 92, 203, 282, 27, 108, 572.

(3) Subtract 83578073 from 143800912.

(4) What is the remainder when 1078362584 is divided by 70873?

(5) Find the difference between £362. 3*s.* $2\frac{1}{2}$*d.*, and £224. 5*s.* $3\frac{3}{4}$*d.*

(6) If I travel 63 miles on Monday, 51 miles on Tuesday, 34 miles on Wednesday, 211 miles on Thursday, 29 miles on Friday, and 39 miles on Saturday, how far do I travel in the week; and how far per day on the average?

(7) What is a half, a third, a fourth, a tenth, a fifteenth of £200.?

(8) Say how many shillings there are in 37 pence, 53 pence, 96 pence, 62 pence, 71 pence, 31 pence.

(9) If A has £611. 1*s.* 1*d*, and B has £480. 5*s.* $8\frac{3}{4}$*d.* less than A, how much has B?

(10) Add together £7. 8*s.* $0\frac{1}{2}$*d.*, £71. 13*s.* $5\frac{3}{4}$*d.*, £109. 19*s.* $2\frac{1}{2}$*d.*, £830. 7*s.* $10\frac{1}{4}$*d.*, £93. 11*s.* $11\frac{3}{4}$*d.*, £18. 18*s.* $8\frac{3}{4}$*d.*

(11) Add together a hundred and seven, three thousand nine hundred and ten, eleven thousand and nineteen. Multiply the result by a hundred and nine.

(12) If a clock loses a second a day, how much will it lose in four years? How long will it be before it loses twelve hours?

(13) From 3 tons 1 qr. 18 lbs. 10 oz., subtract 1 ton 2 qrs. and 15 oz. Multiply the remainder by 7.

(14) Add $1\frac{3}{8}+7\frac{1}{16}+5\frac{7}{12}$. Multiply $2\frac{9}{16}$ by $4\frac{2}{5}$.

(15) What would 3 lbs. 7 oz. Troy cost, at the rate of £1. 14*s.* for every 17 dwts.?

(16) Express $\frac{3}{16}$ and $\frac{7}{11}$ as decimals; and ·5625 as a vulgar fraction.

(17) Find the cost of 2358 articles at £1. 11*s.* 2*d.* each.

(18) Divide 425 tons 2 qrs. 21 lbs. 8 oz. by 90.

(19) If the annual cost of a cavalry soldier be £52. 11*s.* 3¾*d.*, and of an infantry soldier £26. 3*s.* 5¾*d.*, what is the difference of the cost per annum of a cavalry regiment of 400 men, and of an infantry regiment of 800?

(20) If 8 fires, kept burning 9 hours a day for 28 days, consume 3⅕ tons of coal, how many tons will be consumed by 12 fires that are kept burning 7 hours a day for 15 days?

(21) What is the cost of carpeting a room 25 feet 9 inches long, by 22 feet 6 inches wide, at the price of 5*s.* 4*d.* per square yard?

(22) Find the simple interest on £396. 13*s.* 4*d.* for 4 years at 3¼ per cent. per annum.

(23) What number multiplied by 86 will give the same product as 163 × 430?

(24) Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}$.

(25) Find, by Practice, the value of 6573 articles at £5. 18*s.* 10¾*d.* each.

(26) At what rate per cent. will £250. become £275. in 2½ years?

(27) Find the discount on a bill of £366. 19*s.* drawn June 18, at 3 months, paid August 13, at 5 per cent.

(28) Find the change in income when a person transfers £2616. 5*s.* from the 5 per cents. at 95⅜ to the 4 per cents. at 83. Brokerage as usual.

(29) A cistern is filled by two pipes in 20 and 24 minutes respectively, and emptied by a tap in 30 minutes: what part will be filled in 15 minutes when all are opened?

(30) An article is sold so as to gain 5 per cent. If it had been bought at 5 per cent. less, and sold for 1*s.* less, the gain would have been 10 per cent. Find cost price.

(31) Divide £26. 15*s.* 11¼*d.* by 49, and multiply the result by 498.

(32) Reduce 114 acres 2 roods 8 perches 4 yards to feet.

(33) If a dollar be worth 4*s.* 2*d.* and a thaler worth 2*s.* 10*d.*, how many dollars should be given for 1350 thalers?

(34) Find the value of

$$\frac{5}{8}+\frac{11}{21}-\frac{17}{24}+\frac{16\frac{1}{4}}{126}-\frac{\frac{1}{4}}{4\frac{4}{5}} \text{ and of } \frac{5-\dfrac{1}{5-\frac{1}{5}}}{3-\dfrac{1}{3-\frac{1}{3}}}\times\frac{9}{23}\div\frac{1}{7}.$$

(35) What fraction of a guinea is 10*s*. 4½*d*. Find the value of the same fraction of £13. 6*s*.

(36) State the rule for the division of decimals. Divide ·00314388 by 21·3, and 31438·8 by ·0213.

(37) Find, by Practice, the value of 837 things at £3. 8*s*. 10½*d*. each; and of 3 tons 2 cwts. 1 qr. 13 lbs. at 5¼*d*. a lb.

(38) If the rent of 520 acres 2 roods of land be £381. 14*s*., find the rent of 168 acres 2 roods 7 perches 8¼ yards at the same rate.

(39) If 14 men working 9 hours a day complete a piece of work in 71½ days, and if 2 men do as much in an hour as 3 women, how long a day must 13 women work in order to complete the same work in 105 days?

(40) After deducting one tenth of his gross income for trade expenses and £37. for life insurance, a merchant pays as income tax on the remainder, 3*d*. in the pound, £16. 8*s*. 3*d*.: find his gross income.

(41) Add together $2\frac{1}{20}+13\frac{5}{6}+11\frac{8}{15}$, and express the result as the fraction of $32\frac{11}{18}-5\frac{7}{36}$.

(42) A share of $\frac{3}{16}$ of a ship was sold for £1260.: find the price of the ship, and of a share of $\frac{7}{64}$ of the ship.

(43) Reduce to its simplest form the fraction

$$\frac{2\left(18\frac{1}{2}+13\frac{1}{4}\right)\times 11\frac{1}{8}}{7\frac{1}{9}\times 3}\times 3\frac{1}{2}.$$

(44) Find the value of 3·024 × ·027, ·00365 ÷ 2·92, and of 180 ÷ 3·1416, to 4 places of decimals.

(45) Find the value as a decimal of $\cdot 0\dot{3}+2\cdot 0\dot{2}\dot{1}+1\cdot\dot{4}1\dot{7}$.

(46) Express £7. 3*s*. 6¾*d*. as the decimal of £1.

(47) Find the cost of covering the outside of a box 5 ft. long, 4 ft. wide, 3 ft. high, with paper 6 inches wide, at 1*d*. a foot.

(48) Find the rate on £1387. at 3*s*. 7½*d*. in the £.

(49) Find the cost of 54 tons 16 cwts. 2 qrs. at £1. 3*s*. 4*d*. a ton.

(50) If 3 oz. 6 drs. of zinc, Avoirdupois, cost 8*s.* 9*d.*, what would 3 oz. 15 dwt. Troy cost?

(51) If money is lent at the rate of 2*d.* a week foi every pound lent, what is the rate of profit per cent. per annum made by the lender?

(52) Find the simple interest on £345. 7*s.* 2*d.* in 2 years at $3\frac{3}{4}$ per cent.

(53) A sum of £28561. in the $3\frac{1}{2}$ per cents. is allowed to accumulate for 2 years, the interest being reinvested each half-year at 91: find the amount of stock at the end of one year, and of two years.

(54) Write down 109570063 in words; also subtract eighty-seven millions three hundred and forty-seven thousand eight hundred and forty-one from it.

(55) Divide 470940232 by 100, and also by 499, and multiply the latter result by 99.

(56) Reduce 235602 farthings to £ *s.* *d.*, and 19 tons 2 cwt. 2 qrs. 2 lbs. 4 oz. to ounces.

(57) Simplify—

(1) $1\frac{3}{8}+\frac{22}{9}-2\frac{3}{4}+\frac{5}{16}+\frac{23}{18}+\frac{11}{48}$.

(2) $\frac{2}{3}$ of $3\frac{1}{8}\times\frac{15}{29}$ of $\frac{11}{30}\div\frac{1}{6}$.

(3) $1\frac{1}{3}$ of $1\frac{5}{8}$ of £3. 12*s.* 6*d.*

(58) What number multiplied by $\frac{2}{5}$ will give $\frac{7}{8}$ for product?

(59) Reduce £1. 14*s.* $7\frac{1}{2}$*d.* to a fraction of £2. 2*s.*

(60) Find the G.C.M. of 952 and 5544, and the L.C.M. of 2, 3, 4, 6, 8, 12, 14.

(61) Divide 42·5 by 100 and reduce the result to a vulgar fraction in its lowest terms.

(62) Reduce $\frac{1001}{2464}$ to a decimal; reduce £1. 5*s.* $8\frac{1}{2}$*d.* to a decimal of £100.

(63) What number added to fifty-three thousand and four will make a million; and what is the difference between the thousandth part of a million and the twenty-fifth part of twenty thousand?

(64) A merchant gained £12205. in 6 years and 3 months: what was his average gain per annum?

(65) Find the G.C.M. of 5610, 11781, and 13651.

(66) Add together $59\frac{1}{7}+(\frac{2}{3}\times\frac{3}{4})+\frac{1}{2}$ of $5\frac{1}{4}$.

(67) Find the value of $\dfrac{6\frac{7}{9}-2\frac{2}{9}}{\frac{1}{9}\text{ of }18\frac{1}{2}}$ and $\dfrac{\frac{2}{7}\left(\frac{64}{3}+19\frac{2}{3}\right)}{\frac{8}{11}+\frac{12}{13}\text{ of }6\frac{1}{2}}$.

(68) Subtract $\frac{5}{7}$ of $\frac{1}{4}$ of £5. from 100 half-crowns.

(69) Divide ·001275 by ·17.

(70) Reduce 7*s.* 6*d.* to the decimal of £1.
If 1 oz. cost £·03125. how much will ·0625 lb. Avoirdupois cost?

(71) Find the cost of 24 tons 6 cwts. 2 qrs. 8 lbs. of coal at 17*s.* 6*d.* a ton.

(72) What will be the cost of papering a room 18 ft. 9 in. long, 13 ft. 6 in. broad, and 11 ft. 10 in. high, with paper 27 inches wide, at $5\frac{1}{2}$*d.* a yard?

(73) (1) Multiply £968. 10*s.* $10\frac{1}{2}$*d.* by 71.
(2) Divide £1240. 11*s.* 6*d.* by 113.

(74) Reduce 36 tons 2 cwt. 3 qrs. 16 lbs. to pounds; and bring back your answer to tons, cwts., qrs., and lbs.

(75) Find the G.C.M. of—

(1) 825 and 1375.
(2) 49, 91, and 105.

And find the L.C.M. of—

(1) 14, 21, 35, and 49.
(2) 34, 51, 17, and 85.

(76) Simplify the following:—

(1) $\frac{4}{15}+\frac{8}{9}+4\frac{5}{6}+6\frac{2}{3}$.
(2) $18\frac{4}{17}+16\frac{17}{18}+24\frac{19}{36}+27\frac{11}{34}$.
(3) $\frac{7}{8}\times 3\times\frac{4}{7}$ of $3\frac{1}{2}$ of 6.
(4) $2\frac{1}{2}\times 3\frac{1}{3}\times 4\frac{1}{4}\times 5\frac{1}{5}\times\frac{24}{25}\times\frac{11}{34}$.

(77) (1) Reduce to vulgar fractions the following: $2\cdot0\dot{5}4$, and $3\cdot\dot{5}1\dot{3}$. (2) Bring to decimals $\frac{4}{1001}$ and $2\frac{43}{74}$.

(78) Perform the following divisions to 4 places of decimals:—

(1) ·06÷5·1. (2) 6·5÷6·84.
(3) ·007284÷·0046

(79) Reduce—

(1) 3*s.* 4*d.* to the fraction of a half-guinea.
(2) $2\frac{1}{2}$ cwt. to the fraction of 2 tons 12 lbs.

(80) Find the value of—

(1) ·359375 of £2.
(2) ·046875 of 3 qrs. 12 lbs.

(**81**) Reduce—

(1) £3. 11*s.* 9¾*d.* to the decimal of £1.
(2) 4 days 18 hrs. to the decimal of a week.

(**82**) Find the value of—

(1) $\dfrac{\frac{1}{5}\left(\frac{1}{2}-\frac{1}{3}\right)-\frac{1}{3}\left(\frac{1}{5}-\frac{1}{6}\right)}{\frac{1}{7}\left(\frac{1}{3}-\frac{1}{4}\right)-\frac{1}{4}\left(\frac{1}{6}-\frac{1}{7}\right)}\div\dfrac{7}{13}$ of $\dfrac{1\frac{3}{5}}{3}$.

(2) $\frac{2}{7}$ of £12. + $\frac{1}{4}$ of $\dfrac{2\frac{1}{5}}{1\frac{4}{7}}$ of £2. 12*s.* − $\frac{1}{33}$ of £2. 13*s.* 3*d.*

(**83**) Divide ·90388 by ·0000295, and $\cdot 21\dot{3}\dot{5}$ by $\cdot 13\dot{8}$.

(**84**) There are two rooms to be papered with the same kind of paper: the dimensions of the one are 16 ft. by 12 ft., and 11 ft. in height; of the other, 18 ft. by 17 ft., and 12 ft. in height; the cost of papering the former is £2. 4*s.*: what is the cost of papering the other?

(**85**) Find the discount on £250. for 1 year 219 days at 4½ per cent.

(**86**) A coal merchant sold 40 tons of coal at a profit of 10 per cent., and 60 tons at a profit of 30 per cent., and gained on the average 3*s.* 3⅗*d.* per ton. He bought the 100 tons at a uniform price: what was it per ton?

(**87**) In a foot-race A starts 20 yards behind B; A takes 38 steps while B takes 35, but 18 of B's are equivalent to 19 of A's. A catches B in three minutes: what is A's rate of running?

(**88**) Simplify—

(1) 4·035 + 91·3754 + 1·00 + ·001 − ·5401.
(2) 3·653 × 64·25.
(3) 26·99178 ÷ ·057

(**89**) If the rent of 182 acres of land be £273., how many acres can be rented for £21.?

(**90**) If 14 acres of pasture will maintain 21 horses for 4 weeks, for how long will 5 acres keep 6 horses?

(**91**) Find the simple interest on £493. 16*s.* 8*d.* for 2 years and 1 month at 4⅘ per cent.

(**92**) How much 3½ per cent. stock at 92⅛ can be purchased for £12300., allowing ⅛ per cent. for brokerage?

(**93**) Find the present worth of £769. 2*s.* 6*d.* due 15½ years hence, at 3 per cent.

(**94**) Find the difference between the fractions $\frac{168}{169}$ and $\frac{142}{143}$; multiply that difference by ½ of 35¾.

(95) Express £1. 3*s.* 8*d.*—(1) as the fraction, (2) as the decimal, of £5. 18*s.* 4*d.*; and find the value of ·1253125 of £100.

(96) Two merchants form a partnership for conducting a business; one contributes a capital of £3000. and the other a capital of £5000.: at the end of a year the profit on the business is found to be £1880.; how much ought each to receive?

(97) Find the value of $(\cdot05)^3 \times (1\cdot04)^2$.

(98) Extract the square root of ·001296, and of 15 to 4 decimal places.

(99) Show that a common measure of any two numbers will divide their sum or difference, and also, any multiple of either of them.

Explain the difference between Interest and Discount.

(100) Find the side of a square floor containing 100·4004 square feet and the edge of a cube containing 138188·413 cubic inches.

(101) Allowing $24\frac{3}{4}$ square yards to each tree, how many pear-trees may be planted in an orchard of $4\frac{1}{2}$ acres?

(102) Simplify—

(1) $(\frac{4}{7} + 2\frac{1}{2} + \frac{5}{11} + \frac{13}{14}) \div (\frac{7}{8} - \frac{1}{3})$.

(2) $\cdot0047 + 8 - \cdot0004 + (\cdot007 \times 5\cdot6)$.

(103) Reduce $\frac{3}{13}$ to a decimal and find the value of $\cdot40\dot{6}$ of £13. 17*s.* $10\frac{1}{2}$*d.*

Find (by Practice) the value of 745 reams 7 quires 18 sheets of paper at 17*s.* 6*d.* per ream.

(104) Find the cost of papering a room 16 ft. 8 in. long, 12 ft. 10 in. broad, and 11 ft. $7\frac{1}{2}$ in. high, with paper $\frac{3}{4}$ yard wide, at 3*s.* 6*d.* per piece of twelve yards.

(105) Divide 37·475 by ·00075, ·00000875 by 15; and reduce £1. 10*s.* to the decimal of 5 guineas.

(106) Divide 4 tons 13 cwts. 2 qrs. 27 lbs. 7 oz. by 7 cwt. 1 qr. 27 lbs. 14 oz., and verify the result.

(107) Find the present value of £175. 10*s.* 6*d.* due 9 months hence, at $4\frac{1}{2}$ per cent. discount.

(108) A man enters France with 25 sovereigns, 1 half-sovereign, and 13 florins in his purse. He spends 250 francs, 74 centimes; how much English money has he left? (Rate of exchange = 25·20 per £ sterling.)

(109) If I invest £1000. in 3 per cent. consols at par, and sell out at $99\frac{1}{4}$, what do I lose? (Reckon brokerage as usual.)

(110) What is the difference between the simple and compound interest of £256. 10*s.* for 2½ years, at 4 per cent.?

(111) If a six-oared boat costs £45., what would an eight-oared boat cost, supposing the price of a boat to be proportionate to the square of the number of oars?

(112) I lend a friend £2560.; at the end of 2½ years he returns me £2761. 12*s.*: at what rate has the simple interest been calculated?

(113) After paying income-tax at 7*d.* in the pound, I have £825. 4*s.* 2*d.* remaining: what is my income?

(114) Divide 21420 by 315, and explain each step of the process.

(115) If each of 537 persons has £2. 15*s.* 7¼*d.*, how much have they altogether?

(116) If a German twenty-mark piece is worth 19*s.* 10¼*d.*, what is the smallest number of marks which can be paid with English gold coin?

(117) Prove that if the numerator and the denominator of a fraction be divided by the same number, the value of the fraction will not be altered.

Simplify $\dfrac{2}{3-\dfrac{1}{4-\frac{2}{5}}}$, and show that $\dfrac{111130}{2111109}-\dfrac{1}{19}=\cdot00000\dot{9}$.

(118) Divide 34·80246 by ·001023; and express £5. 17*s.* 9½*d.* as a decimal of 4*s.* 7*d.*

(119) Find the sum of $2\frac{3}{11}$ of 4*s.* 7*d.*, $\cdot2\dot{3}$ of 7*s.* 6*d.*, and ·01 of $1\frac{1}{8}$ of 1*s.* 8*d.* What fraction of this sum is £·0025.?

(120) Find the discount on £539. 10*s.* due 10 months hence at 4½ per cent.

(121) A man bought 15 ac. 1 rd. 20 po. of land for £1860. 7*s.* 6*d.* and sold a plot 150 yds. square for 50 per cent. more than it cost him: for how much did he sell it?

(122) Find to four places of decimals the square root of $7\frac{1}{3}$.

(123) The price per ton of coals in the winter half of the year is on the average thirty per cent. more than in the summer half of the year, and five tons are used by a person in the winter for every two tons in the summer. What would be the cost of coals for the summer half if the whole cost for the year were £16. 3*s.*?

(124) A man who has £3465. in Midland Railway Stock sells out at 140 and buys North-Eastern Consols at 165: what is the

change in his income if the Midland Stock pays six per cent. and the North-Eastern seven per cent.?

(**125**) The compound interest on a sum of money for three years at five per cent. is £331. 0*s.* 3*d.*: what is the simple interest?

(**126**) A multiplication sum was worked correctly, and then all the figures except those given below were rubbed out, and the lines show the places where the figures rubbed out were: find all the missing figures:

$$\begin{array}{ccccc} & & 4 & - & - \\ & & & 3 & - \\ \hline & 3 & 6 & - & - \\ - & - & 7 & - & \\ \hline - & - & 3 & - & - \end{array}$$

(**127**) How long would it require to count the National Debt (seven hundred and seventy-seven millions of pounds) at the rate of £100. a minute? Also find the weight (tons, cwt., &c.) in gold of the debt, having given that a thousand sovereigns weigh 123284 grains.

(**128**) Find the value of $\frac{2}{3}$ of $\frac{5}{7}$ of 13*s.* 4*d.*, and of ·7365 of 6*s.* 8*d.* What fraction of 13*s.* 2¼*d.* is 9*s.* 10½*d.*?

(**129**) If 7 men earn £9. 10*s.* 6*d.* in 10½ days, what sum will 28 men earn in 31½ days?

(**130**) If A can by himself perform a certain quantity of work in 5 days, B twice as much in 7, and C four times as much in 11 days, in what time can A, B, and C together perform three times the original work?

(**131**) The discount of £500. due 4 years hence is £83. 6*s.* 8*d.*: find the rate of interest.

(**132**) A boy has been doing a multiplication sum on a slate, and accidentally rubbed it out; all that remains is 47 in the product, there having been two figures preceding these and one after. He, however, remembers that the multiplicand was 792: find for him the multiplier.

(**133**) How many days of 8⅔ hours must 14 men work to earn £18. 4*s.*, when the wages of 12 men for 15 days of 9 hours each is £20. 5*s.*?

(**134**) What must I charge a customer for an article worth £4. 0*s.* 5*d.*, so as to be able to allow him 3½ per cent. discount?

(**135**) If by selling an article for £126. I lose 16 per cent., what should I sell it for to gain 4 per cent.?

(**136**) A puts 10 cattle in a field for 8 months, B 18 cattle for

6 months, C 11 cattle for 10 months, D 5 cattle for 12 months, and pays £6. : what did the rent of the field come to?

(**137**) I give 34 gallons of brandy in exchange for 26 cwt. 3 qrs. 18 lbs. of sugar at £1. 12*s.* 6*d.* per cwt., and lose 7 per cent. : what was the price of the brandy?

(**138**) I transfer £4275. stock from the 3 per cents. at 76 to the 4 per cents. and gain £23. 3*s.* in income : what is the price of the 4 per cent. stock?

(**139**) A room is 13 ft. 4 in. long, 10 ft. wide, and 10 ft. 6 in. high : what will it cost to cover its 4 walls with paper ¾ yd. wide, the piece of 20 yds. costing 3*s.* ; and the floor with carpet 1½ yd. wide at 2*s.* 3*d.* per yard?

(**140**) Value of 2·003125 of £8. ; 12 oz. 8 drs. as decimal of 2 cwt. 26 lbs.

(**141**) Value of 1 ton 5 cwt. 2 qrs. 7 lbs. at £3. per ton (by Practice).

(**142**) If the 4*d.* loaf weighs 10 oz. when wheat is 63*s.* a quarter, what would be the weight of the 2*d.* loaf when wheat is 54*s.* a quarter?

(**143**) Amount of £1485. in 3 years at 5 per cent. compound interest.

(**144**) From a bill of £3. 11*s.* 8*d.* due 18 months hence, a tradesman deducts 5*s.*, what is the rate per cent. at which the true discount is calculated?

(**145**) When the 3 per cents. are at 75, what is the price of £1000. stock? How much stock can be purchased for £1000.? What income is derived from investing £1000.? What is the rate of interest?

(**146**) A room is 34 feet long, 13 feet wide, and 10 feet high. It is papered with paper 2 feet wide at 6*d.* per yard, and carpeted with carpet ¾ yard wide, at 3*s.* 6*d.* per yard. Find (1) area of 4 walls ; (2) length of paper ; (3) cost of paper ; (4) area of floor ; (5) length of carpet ; (6) cost of carpet.

(**147**) A printer sets up 8500 letters in a day, and charges 5½*d.* for every 1000 : how much will he earn in a week?

(**148**) How much may a person spend in 73 days, if he wishes to lay by every year 50 guineas out of an income of £450.?

(**149**) Find the value of— $\dfrac{\frac{1}{2} \times \frac{1}{3} + \frac{1}{4}}{\frac{1}{2\frac{1}{3}} + \frac{1}{3\frac{1}{2}} + \frac{1}{4\frac{1}{3}}}$; ·0032 × 23·45 ; 181·3 ÷ ·00037 ; 1·46875 of an acre ; ·4097½ of a guinea.

(150) What is the height of a steeple whose shadow was 148 ft. 4 in. at the same time that the shadow of a staff, 6 ft. 4 in. long, was 5 ft. 3 in.?

(151) In what time will £732. 11*s*. amount to £1709. 7*s*. $7\frac{1}{3}$*d*. at $5\frac{1}{3}$ per cent.?

(152) A man walks towards a town 12 miles off at the rate of 4 miles an hour; 15 minutes afterwards a friend goes after him at the rate of $4\frac{1}{2}$ miles an hour: when and where will he meet him?

(153) Multiply two millions two hundred and one by ten thousand one hundred, and find by what number 2765 must be divided in order that the quotient may be 132 and the remainder 521.

(154) Subtract $\frac{5}{6}$ from $\frac{6}{7}$, $\frac{6}{7}$ from $\frac{7}{8}$, and $\frac{7}{8}$ from $1\frac{1}{9}$, and add together the results. Find the G.C.M. of 286720 and 1310309.

(155) Find the value of $\frac{5}{9}$ of a guinea + $\frac{3}{16}$ of £1. + $\frac{7}{10}$ of a crown + $\frac{3}{8}$ of a shilling; and express 19*s*. $8\frac{1}{4}$*d*. as a fraction of £5. 10*s*.

(156) The price of gold being £3. 17*s*. $10\frac{1}{2}$*d*. an ounce, find the weight of a nugget worth £341·0925.

(157) Simplify $\dfrac{2}{3+\dfrac{\cdot 2}{3+\dfrac{\cdot 02}{3\cdot 002}}}$, and subtract $\cdot 85416\dot{6}$ of 10*s*. from $\cdot 11\dot{7}\dot{2}$ of £13. 15*s*.

(158) A hollow rectangular vessel, formed of material one inch thick, whose external dimensions are 13 ft. 6 in., 6 ft. 8 in., and 6 ft. 11 in. respectively, weighs 1 cwt. 1 qr. 10 lbs. Find the weight of a solid mass of the same material and the same dimensions.

(159) Find the value in English money of 1572·4185 francs when the exchange is at 26·675 francs per £.

(160) Egyptian 7 per cents. being at 40, in how many years' time might the debt be repudiated without loss to an investor, reckoning only simple interest, and considering the interest on safe investments to be 4 per cent.?

(161) A person holding £2450. 3 per cent. consols sells out at $93\frac{1}{2}$ and invests the proceeds in Russian $4\frac{1}{2}$ per cents., whereby his income is increased by £31. 13*s*. 9*d*. At what price does he buy the Russian stock?

(162) If a manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 40 per cent., what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

(163) If 175 is the divisor, 1092 the quotient, and 157 the remainder, find the dividend.

(164) A Frenchman at his death leaves his field of 37 ac. 2 rds. 19 po. 17 yds. to be divided equally between his three sons: what is the share of each?

(165) An athlete walks 1500 miles in 1000 hours, 1½ miles in each hour; he began his task on August 26 at 2 A.M. and walked the last 1½ miles in 16 minutes: find the day and time of the completion of his walk.

(166) Find the value of the following fractions:—

(1) $\frac{1}{2}+\frac{5}{6}+\frac{7}{12}-\frac{7}{8}$. (2) $(\frac{1}{2}+\frac{5}{6})\times\frac{7}{12}-\frac{7}{8}$. (3) $\frac{1}{2}+\frac{5}{6}\times(\frac{7}{12}-\frac{7}{8})$.

(167) Gold is worth £3. 17*s.* 10*d.* per oz.: what is the value of a nugget of gold which weighs 15 lbs. 8 oz. 15 dwts. 14 grs.?

(168) In November last there fell 6·4 inches of rain. A cubic yard of water weighs 15 cwt.; find how many tons of rain fell on a field of 12 acres during the month.

(169) Forty thousand men have provisions calculated to last for two months (one month = 30 days). At the end of the first month they receive a reinforcement of 10000 men; but, finding themselves besieged, they reduce their rations to one-third. Their general determines to make a sortie with 25000 picked men when he has 100000 full rations left. When will the sortie take place?

(170) A bicycle has a diameter of 52 inches. How many revolutions will it make in traversing 2 miles $1\frac{8}{11}$ furlongs?

(171) Divide ·000125 by 5000, 2·5, and ·00025 respectively; and reduce an oz. Troy to the decimal of an oz. Avoirdupois.

(172) A, B, C, D travel together, A managing the common purse. At the end of their journey they settle accounts. It is then found that A, B, C, D have advanced £34. 2*s.* 10*d.*, £20., £20., £15., respectively, but that A has advanced on account of private expenses to himself £7. 16*s.* 6*d.*, to B £9. 5*s.* 6*d.*, to C £4. 19*s.* 9*d.*, to D £6. 7*s.* 5*d.* The balance in the common purse being £12. 14*s.* 9*d.*, show how their accounts must be settled.

(173) If 5 men mow 18 acres in 8 days, working 11 hours a day, in how many days will 8 men mow 54 acres, working 5½ hours a day?

(174) Find the discount on a bill of £4358. 14*s.* 6*d.* due 4 months hence, the rate of interest being 3½ per cent.

(175) A horsedealer buys two hunters for £100. apiece, and sells them at a gain and loss of 20 per cent. respectively: find his total gain or loss, and also his total gain or loss per cent.

(176) A man invests £17600. in the 3 per cents. at 88. When these have risen 5½ per cent. he sells out and invests in the 4 per cents. at 110. How much stock will he hold in the latter, and what will be the change in his income?

(177) A carpenter is ordered to make a cubical box, whose inner content is to be one cubic yard. He uses an inch plank eleven inches wide. What length of plank must be cut off?

(178) Make out the following bill:—

56 lbs. 9 oz. beef at 10½*d.*
75 lbs. 6 oz. mutton at 11½*d.*
18 lbs. 10 oz. lamb at 1*s.* 2*d.*
45 lbs. 8 oz. veal at 1*s.* 1½*d.*
14 lbs. 12 oz. suet at 8½*d.*

Discount for cash 5 per cent.

(179) Reduce $\frac{274}{271}$ and $\frac{252}{2710}$ to decimals, and add them. Reduce the answer to a vulgar fraction.

(180) The personal estate of a deceased person is divided in the following manner: Three-fourteenths to his eldest son, one-sixth to his second son, one-twelfth to his daughter, £100. to his executor, and the residue to his widow, who received £7685. What did his eldest son get?

(181) What decimal multiplier will convert feet per second into nautical miles per hour? Use it to convert a speed of 77 feet per second into nautical miles per hour. One nautical mile is 6086·5 feet. *Four decimals sufficient.*

(182) Find the amount at compound interest of £1785. at 10 per cent. for three years.

(183) What is the weight, to the nearest pound, of the plating of an iron tank open at the top, 27 feet square at the base, and 9 ft. high? The plating is $\frac{7}{16}$ of an inch thick, and 4½ per cent. is to be allowed for rivets and overlapping of plates. The iron is to be reckoned at 480 lbs. to the cubic foot.

(184) Multiply three millions one hundred and nine by ten thousand and one, and find by what number 27658 must be divided in order that the quotient may be 285 and the remainder 13.

(185) A farmer sells 24 sheep at £1. 9*s.* 2*d.* per head, and 20 at £1. 12*s.* 6*d.*: how many can he buy with the proceeds at £1. 13*s.* 9*d.* each?

(186) The distance from Dover to Calais is 1271160 inches, from Folkestone to Boulogne 9280 poles: which is the greater, and by how much?

(187) Reduce $\frac{1645}{2115}$, $\frac{2387}{2604}$ to lowest terms, and find their difference. Also divide ·0437 by 11·4.

(188) The price of gold being £3. 16*s*. 9*d*. per ounce, find the weight of a nugget worth £2837·875.

(189) Express $\frac{4}{13}$ of $\frac{9}{85}$ of $2\frac{5}{6}$ of 7*s*. 7*d*. + $\cdot 04\dot{3}$ of 15*s*. 6*d*. as the decimal of £41. 3*s*.

(190) Which, and by how much, is the greater, the simple interest on £540. 10*s*. for 3½ years at 4½ per cent., or on £270. 6*s*. for 6¾ years at 4⅔ per cent.?

(191) Simplify $\dfrac{\frac{22}{31}+\frac{3}{29}}{\frac{34}{41}+\frac{6}{19}} \div \dfrac{\frac{21}{37}+\frac{8}{23}}{\frac{20}{43}+\frac{13}{17}}$.

(192) If 11 men, working 10 hours a day, earned 643½ francs in 13 days, how many days would 25 men, working 9 hours a day, be in earning 2673 francs, when the pay for an hour's work had increased 20 per cent.?

(193) A man embarks his property in four successive ventures. In the first he clears 100 per cent., and in each of the others he loses 20 per cent.: what percentage of profit remains to him on his original capital?

(194) A person has stock in the 3 per cents. which brings him £240. a year. He sells out one-fourth of the stock at 87¼, and invests the proceeds in railway stock, when a £100. share is worth £174½. What dividend per cent. per annum ought the railway stock to pay, so that he may increase his income £40. a year by the transaction?

(195) A engages £15000., B £9000. in a business. The gross receipts for the first year are £5600., but of this 5 per cent. is paid for insurances, and one-seventh for other expenses; of the rest, B receives a certain sum for managing, and the remainder is divided between them in proportion to the capital employed. If A receives £2500., find B's salary as manager.

(196) Divide £274. 18*s*. 6¾*d*. by 72, both by long and short division.

(197) Find the value of $\frac{7}{8}+\frac{7}{12}+\frac{7}{16}+\frac{7}{18}$, $\dfrac{3\frac{3}{7} \text{ of } 2\frac{11}{12}}{\frac{1}{33} \text{ of } 8\frac{9}{11}}$, 2·7 × ·27 × ·027 × 270, 8 ÷ ·002, 37·5 ÷ 7·68, ·015625 ÷ 25, $\frac{7}{256}$ as a decimal, ·6 of £1. + ·3125 of 1*s*. + $\cdot\dot{2}$ of 1 guinea.

(198) If $4\frac{5}{9}$ oz. Avoirdupois cost $8\frac{31}{32}$*s*., what will $8\frac{13}{24}$ lbs. cost? If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of $12\frac{1}{2}$ sheets?

(**199**) Value of 2 cwt. 4 lbs. 12 oz. at £3. 1*s.* per cwt. Value of a piece of timber 123 ft. 9 in. by 2 ft. 4 in. by 2 ft. at $9\frac{1}{2}$*d.* per solid foot.

(**200**) A plate of gold 3 in. square and $\frac{1}{8}$ in. thick is hammered so as to cover 7 square yards: find its thickness.

(**201**) Find the difference between the simple and compound interest on £365. 4*s.* $8\frac{1}{4}$*d.* for 3 years at 4 per cent.

(**202**) Find the difference between the discount on £274. for $2\frac{1}{2}$ years at 3 per cent., and the interest on the same invested in 3 per cents. at $95\frac{1}{2}$ for the same time.

(**203**) How many herrings can be bought for 2*s.* 6*d.* at two for three halfpence; and what would be the value of 10000 at the same rate?

(**204**) How many days has a boy lived on December 13, 1878, who was born on April 1, 1863? How many of these were Sundays?

(**205**) Divide 4654355 by 471, and prove the truth of your result. If your dividend represents farthings find the value of your quotient.

(**206**) Multiply £215. 7*s.* $8\frac{3}{4}$*d.* by 196, and divide it by $2\frac{1}{2}$.

(**207**) Find the value of—

(1) $\frac{3}{4}$ of $\frac{1}{5} + 1\frac{4}{5} + \frac{3}{4} + 3\frac{7}{10} + 4\frac{1}{2}$.

(2) $\dfrac{4\frac{7}{26} - 2\frac{11}{39}}{\frac{9}{26} + \frac{25}{39}}$

(3) $\frac{2}{7}$ of a guinea + $\frac{5}{8}$ of a sovereign − $\frac{7}{8}$ of $\frac{2}{5}$ of a crown.

(**208**) A steamer crossed the Atlantic in 9 days of $24\frac{1}{2}$ hours each, burning 3 tons of coal per hour. Find, by Practice, the cost of the coal at 18*s.* $10\frac{3}{4}$*d.* per ton.

(**209**) Find the value of

$$1 + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \frac{1}{1.2.3.4.5}$$

accurately to three places of decimals, and divide 12·5 by ·0025, 2500, and 2·5 respectively in turn.

(**210**) Find the number of acres in a rectangular field whose sides are 165 yards and 110 yards respectively; and what must be cut off its length so that the slice may contain one acre?

(**211**) Gold weighs 18 times as much as water; a cubic foot of water weighs 1000 ounces; an ounce of gold is worth

£3. 17*s.* 6*d.* : find the cubical content of £558000., the gold paid for an ironclad.

(212) A grocer buys coffee at 2*s.* 4*d.*, and chicory at 6*d.* a lb.: how must he mix them so that on selling the mixture for 2*s.* a lb. he may clear 25 per cent. on his outlay?

(213) A minor, aged 18, receives a legacy of £1000., subject to a duty of 12 per cent. This is invested for him at $3\frac{1}{2}$ per cent.: what will it amount to when he comes of age three years afterwards?

(214) A gambler begins to play with £100. in his purse. If he wins and loses on alternate nights half the money which he has at the beginning of each night, find what he has gained or lost after six nights.

(215) A and B run a two-mile race. For half the distance A runs 44 yards whilst B runs 43, but when A has finished the first mile he only runs 43 yards to B's 44: which wins the race, and by how much?

(216) A person invests £9250. in the $3\frac{1}{2}$ per cents. at $92\frac{3}{8}$, and sells out when they have risen to $96\frac{1}{8}$. If he then invest in the 3 per cents. at $71\frac{7}{8}$, how much stock will he possess, and what will be the change in his income? (Brokerage is to be taken into account.)

(217) Define Greatest Common Measure and Least Common Multiple, and find the G.C.M. of 19527 and 23667.

(218) Simplify—

(1) $(\frac{1}{7} \div \frac{3}{10} + \frac{2}{3} \times \frac{2}{7} - \frac{5}{18} \text{ of } \frac{4}{7}) \times 1\frac{1}{8}$.

(2) $\dfrac{7\frac{1}{2} \times 7\frac{1}{2} \times 7\frac{1}{2} - 8}{7\frac{1}{2} \times 7\frac{1}{2} - 4}$.

(219) Multiply 3·04 by the sum of ·201 and ·0152, and divide ·10724 by ·003215.

(220) Extract the square root of 36180225 and of ·000144.

(221) Find the value of—

(1) 4 mi. 3 fur. 37 po. $4\frac{1}{8}$ yds. × $5\frac{5}{7}$.

(2) $\cdot 56\dot{6}1298\dot{7}$ of 11 guineas.

(222) Express $\frac{37}{77}$ as a decimal, and find the square root of ·00094249.

(223) If 11 cwt. can be carried 4 miles for 7*s.*, how far can 26 cwt. 23 lbs. be carried for 5 guineas?

(224) What is the rent of a farm of 356 ac. 3 rds. $39\frac{1}{2}$ po. at £2. 13*s.* 4*d.* per acre?

(225) A person investing in the $3\frac{1}{2}$ per cents. obtains $4\frac{1}{2}$ per cent. interest on his capital: what is the price of the stock?

Find the income produced by investing £1180. 3*s.* 4*d.* in the $3\frac{1}{8}$ per cents. at 97.

(226) Distinguish between interest and discount, and find the present worth of £405. 10*s.* due two years hence at 4 per cent., compound interest.

(227) What is the height of a room 15 ft. 4 in. by 12 ft. 8 in., when it takes 74 yards 2 feet of paper, 2 ft. 6 in. wide, to cover the walls?

(228) Divide 157931 by 165.

(229) Multiply £131. 17*s.* $8\frac{1}{2}$*d.* by 121; and divide 5 mi. 1030 yds. 1 ft. by 231.

(230) Simplify—

(1) $\frac{33423}{64275}$.

(2) $$\frac{7\frac{8}{9}}{(7\frac{8}{9}-4\frac{5}{6})(7\frac{8}{9}-1\frac{2}{3})}-\frac{4\frac{5}{6}}{(7\frac{8}{9}-4\frac{5}{6})(4\frac{5}{6}-1\frac{2}{3})}+\frac{1\frac{2}{3}}{(7\frac{8}{9}-1\frac{2}{3})(4\frac{5}{6}-1\frac{2}{3})}.$$

(231) Find the value of $\frac{5}{11}$ of £1. 4*s.* 9*d.* − $\frac{2}{7}$ of 19*s.* 3*d.* + $1\frac{4}{5}$ of £2. 3*s.* 9*d.*, and reduce the result to the fraction of £9. 15*s.*

(232) One cask contains 20 gallons of wine, and another 20 gallons of water; five gallons are drawn from each, and that which is drawn from one cask is put into the other. This process is repeated twice: find the number of gallons of wine in each cask after the third operation.

(233) Find the cost of papering the four walls of a room 16 ft. 5 in. long, 14 ft. 3 in. broad, and 10 ft. 8 in. high, with paper 23 inches wide at $4\frac{1}{2}$*d.* a yard.

(234) Multiply 1·63 by ·24, and also $1\cdot\dot{6}\dot{3}$ by $\cdot 2\dot{4}$, and find the difference between the products.

(235) Find the value of ·02575 of £41. 13*s.* 4*d.*

(236) Find the value of 111 lbs. 10 oz. 6 dwts. at £2. 7*s.* 6*d.* per lb.

(237) If 15 men do a piece of work in 18 days, working 9 hours a day, in how many days of 6 hours will 45 men do the same work?

(238) In what time will the simple interest on £1432. 9*s.* 2*d.* amount to £143. 4*s.* 11*d.* at $3\frac{3}{4}$ per cent.?

(239) Find the difference between the interest and discount on £518. 15*s.* for $1\frac{1}{2}$ years at $2\frac{1}{2}$ per cent.

(240) Add together three million three thousand five hundred and seventy one; thirty-nine thousand and eighteen; eight hundred thousand three hundred and ninety-one; five thousand and sixty; four hundred and ninety-five thousand and three.

From the sum take away one million five hundred and eighty-five thousand four hundred and seventy, and divide the remainder by thirty-nine.

(**241**) A tradesman has earned the following sums in six successive years:—£430. 19*s*. 6*d*., £617. 13*s*. 3*d*., £515. 3*s*. 2*d*., £390. 12*s*. 1*d*., £828. 17*s*. 9*d*., £699. 4*s*. 3*d*.: what has his annual income been?

(**242**) If a pendulum vibrates five times in two seconds, how many times will it vibrate in a week?

(**243**) Explain the process for finding the Greatest Common Measure of two quantities; and find the Greatest Common Measure of 10353 and 14877.

(**244**) Find, by Practice, the cost of 408 tons 15 cwts. 3 qrs. of metal at £5. 16*s*. 8*d*. per ton, and explain each step of the process.

(**245**) What is meant by reducing one quantity to the fraction of another? Add together $\frac{3}{5}$ of £1., $\frac{2}{3}$ of 1*s*., and $\frac{5}{8}$ of 16*s*. 4*d*., and express the result as a fraction of £2. 5*s*. 9*d*.

(**246**) State the rules for the multiplication and division of decimals. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46?

(**247**) If three men and four women can reap a field of 12⅓ acres in 3½ days, working 16 hours a day, in what time can two men and ten women reap a field of 15 acres, working 12 hours a day, supposing that one man's work is equivalent to that of two women?

(**248**) How much 3½ per cent. stock at 97¾ can be purchased for £7038., and what income would be thence derived?

(**249**) Distinguish between simple and compound interest. Find the amount of £880. for 1¼ years at 4½ per cent., simple interest.

(**250**) Add together two millions five hundred and seventy-one; ninety thousand and eighteen; five thousand and sixty; nine hundred and ninety-five thousand two hundred and fifty-seven; and divide the result by fifty-one.

(**251**) A shopkeeper paid 18 guineas for fourteen pieces of calico, each containing 81 yards, and gained £12. 19*s*. 10½*d*. by selling the whole at a uniform price: what was the selling price per yard?

(**252**) Taking a year as 365 days 6 hours, how many minutes are there in 10 years?

(**253**) Explain the process for finding the Least Common

Multiple of several quantities; and find the Least Common Multiple of 54, 81, 63, and 14.

(254) Find, by Practice, the cost of 13 cwts. 3 qrs. 4 lbs. of metal at £2. 18*s*. 4*d*. per cwt., and explain each step of the process.

(255) What is meant by reducing one quantity to the fraction of another? Add together $\frac{2}{3}$ of £1., $\frac{4}{5}$ of 2*s*. 6*d*., and $\frac{6}{7}$ of a guinea, and express the result as the fraction of £5.

(256) State the rules for the multiplication and division of decimals. What decimal multiplied by 125 will give the sum of $\frac{1}{16}$, $\frac{7}{32}$, $\frac{3}{8}$, and ·34375?

(257) If the gas for 5 burners, 5 hours every evening for 10 days, cost 4*s*. 8*d*., what will be the cost of 75 burners which are lighted four hours every evening for 15 days?

(258) £7200. $3\frac{1}{2}$ per cent. stock is purchased for £7038.: what is the price of the stock; and what income would be thence derived?

(259) Distinguish between interest and discount. Find the discount on £929. 10*s*. due $2\frac{1}{4}$ years hence at $2\frac{1}{2}$ per cent.

(260) Find the sum of the 21 odd numbers which follow 15432.

(261) Divide the difference of the squares of 9409 and 10609 by the sum of the squares of 97 and 103.

(262) Reduce $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}-\frac{13}{6}-\frac{1}{20}$ to a fraction in its lowest terms.

(263) The weights of a university eight, beginning with bow in order, are—

	st.	lbs.		st.	lbs.
bow	10	9	5	12	$8\frac{1}{2}$
2	11	$9\frac{3}{4}$	6	12	$10\frac{1}{2}$
3	12	6	7	12	$4\frac{1}{2}$
4	11	$13\frac{1}{2}$	stroke	10	$12\frac{3}{4}$

Find (in lbs.) the result obtained by halving the sum of the averages of the first three and last five.

(264) Find the value of 51 things, four of which cost £19. 3*s*. 1*d*.

(265) 10000000 acres are equivalent to 4046711 hectares: find to five places of decimals what fraction a hectare is of a square mile.

(266) Prove the rule for multiplication of decimals. Express the difference of $\cdot 2\dot{2}8571\dot{4}$ and $\cdot 42857\dot{1}$ as a vulgar fraction in its lowest terms.

(267) Express $\frac{1}{19}$ of £1. 2*s.* $6\frac{3}{4}$*d.* + ·125 of 11*s.* 8*d.* as a decimal of 1*s.* 9*d.*

(268) Extract the square roots of 1664·232025 and 4·38204.

(269) Find the amount of £100. in four years at $3\frac{1}{2}$ per cent., compound interest. (Give your result correct to a tenth of a penny.)

(270) Find the present value of £1. due two years hence, compound interest being reckoned half-yearly at 6 per cent. per annum. (Give your result as a decimal of a pound correct to five places.)

(271) A man has £4100. which he invests in 3 per cent. stock at 87, and 5 per cent. stock at 104: what sums must he invest in the respective stocks to make $3\frac{1}{2}$ per cent. on the whole?

(272) Find the sum of the 21 odd numbers which follow 14566.

(273) Divide the difference of the squares of 9604 and 10404 by the sum of the squares of 98 and 102.

(274) Reduce $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}-\frac{7}{6}-\frac{5}{8}-\frac{1}{10}+\frac{10}{12}$ to a fraction in its lowest terms.

(275) The weights of a university eight, beginning with bow in order, are—

	st.	lbs.		st.	lbs.
bow	10	9	5	12	$8\frac{1}{2}$
2	11	$9\frac{3}{4}$	6	12	$10\frac{1}{2}$
3	12	6	7	12	$4\frac{1}{2}$
4	11	$13\frac{1}{2}$	stroke	10	$12\frac{3}{4}$

Find (in lbs.) the result obtained by halving the sum of the averages of the first five and last three.

(276) Find the value of 29 things, seven of which cost £26. 18*s.* $10\frac{1}{4}$*d.*

(277) A square mile contains 258·9895 hectares: find to five decimal places how many acres there are in a hectare.

(278) Prove the rule for the division of decimals. Express the difference of $1{\cdot}5\dot{3}8461\dot{5}$ and ${\cdot}0\dot{7}692\dot{3}$ as a vulgar fraction in its lowest terms.

(279) Express $\frac{1}{17}$ of £1. 0*s.* $2\frac{1}{4}$*d.* + ·875 of 1*s.* 8*d.* as a decimal of 1*s.* 2*d.*

(280) Extract the square roots of 6877219041 and 1·361.

(281) Find the amount of 100 guineas in three years at $4\frac{1}{2}$ per cent., compound interest. (Give your result correct to a tenth of a penny.)

(**282**) Find the discount off a debt of £1. due four years hence, reckoning 3 per cent., compound interest. (Give your result as a decimal of a pound correct to five places.)

(**283**) A man has £3000. which he invests in 3 per cent. stock at 96, and 6 per cent. stock at 108 : what sums must he invest in the respective stocks to make 3½ per cent. on the whole?

(**284**) What sum will amount to £159. 12*s*. 9*d*. in 5½ years at 6¼ per cent., simple interest, and at what rate would it amount in the same time to £173. 5*s*.?

(**285**) If the discount on £396. 18*s*., due 2 years and 3 months hence, be £42. 10*s*. 6*d*., what is the rate per cent.?

(**286**) A person has £2400. in the three per cents. He sells out when they are 90 and buys 6 per cent. railway bonds, the price of the latter being £144. per bond of £200. What income does he derive from this investment, and what is the actual interest which he obtains upon his capital?

(**287**) In a certain contested election where there are three candidates, A, B, and C, B obtains one-third of the whole number of votes polled, and the number of votes given for him bears to the number given for C the ratio of 7 to 6. A obtains 2304 votes. How many votes are given for B and C respectively?

(**288**) Four towns, containing 13500, 24250, 32000, and 54450 inhabitants respectively, contribute to the Bengal Famine Fund in proportion to their population. If the whole amount of their contributions be £2732. 8*s*., how much does each town contribute?

(**289**) A boy buys a certain number of oranges at the rate of 3 for 2*d*. and one-third of that number at 2 for a penny: at what rate must he sell them so as to gain 20 per cent. upon his outlay? If his profit when all are sold be 5*s*. 4*d*., find the number bought.

(**290**) Three men and four boys can do a piece of work in 10 days, and four men and five boys can do it in 7⅔ days. Two men are set to work at it, and after they have been working four days three boys are sent to help them. In what time will the work be finished?

(**291**) If £5. sterling be worth 127 francs, and 20 francs be worth 22·5 lire in Italian currency, express £24. 16*s*. 8*d*. in lire and centesimi (1 lira = 100 centesimi).

(**292**) In what time will £279. 0*s*. 9*d*. amount to £348. 15*s*. 11¼*d*. at 3¾ per cent per annum, simple interest?

(**293**) If the discount on a bill of £226. 16*s.* 5½*d.* due 15 months hence be £10. 3*s.* 1½*d.*, what is the rate per cent. of simple interest?

(**294**) By selling tobacco at 1*s.* 3*d.* per lb. a tobacconist gains 35 per cent. What does his profit amount to on the sale of 4 cwt. 11 lbs.?

(**295**) A person invests £3636. in railway shares, paying 5½ per cent., the price of a £100. share being £120. He sells out when the shares are at 136, and invests the proceeds in the Moscow Jaroslay railway 5 per cent. debentures, the price being £101. per debenture of £100. Find the difference of the incomes derived from the two investments.

(**296**) A can do one-half as much again as B can do in the same time, and B can do one-third as much again as C. Working together they can do a certain piece of work in 5 days, but if after working 2 days A strikes, how long will B and C take to finish it?

(**297**) A sum of £52. 18*s.* is made up of half sovereigns, half-crowns, and sixpenny pieces, the numbers of the several coins being in the proportion of 3, 5, 7: how many are there of each?

(**298**) What sum will amount to £353. 12*s.* in 2 years and 4 months at 4½ per cent. per annum, simple interest?

(**299**) The present value of a bill of £169. 19*s.* 4½*d.* due one year and three months hence is £163. 6*s.* 8*d.*: at what rate per cent. is the simple interest calculated?

(**300**) A grocer has a stock of tea which he must sell at 3*s.* 4½*d.* per lb. in order to gain 35 per cent.: how much will he gain per cent. if he mixes it in the proportion of 5 to 2 with inferior tea, which cost him 2*s.* 1*d.* per pound, and sells the mixture at the rate of 14 lbs. for £2. 5*s.*?

(**301**) A person holds a certain amount of stock in the 3 per cents., he sells out when they are at 96, and invests the proceeds in railway shares which are at £112. per £100. share, and pay 5½ per cent. What amount of stock in the 3 per cents. did he originally hold if his income from the railway shares be £121.?

(**302**) A and B ride a race of 31 miles on bicycles, the circumference of the driving-wheels being 168 and 162 inches respectively. B's driving-wheel makes 3520 revolutions in an hour, while A's makes 3410. Which will win, and by how much?

(**303**) The price of wheat is such that if sold by the weight it is just twice the price of hay, which is £5. a ton. The hay crop is a ton and a half to the acre; what must be the size of a field bearing 5 quarters to the acre of wheat weighing 56 lbs. to the bushel, so that the produce of the field may be worth the hay crop of a field measuring 5 acres?

(**304**) If 14 men reap 70 acres of corn in 10 days, how many men can reap 20 acres in 4 days at the same rate of labour?

(**305**) A legacy is to be divided between two persons so that the sum which one shall have may bear the same proportion to the whole legacy as £626. 1*s.* 7*d.* bears to £2184. 7*s.* 6*d.*; show that the error for every £1000. of legacy, if the proportion be taken as 2 : 7, will be between 18*s.* and 18*s.* 1*d.*

(**306**) Find the value of ·125 of 10*s.* and ·375 of 13*s.* 4*d.*

(**307**) Find the square roots of $\frac{3}{5}$ and 3·026 to four places of decimals.

(**308**) The edges of a brick are $8\frac{3}{4}$, $4\frac{1}{4}$, $2\frac{1}{2}$ inches respectively; 500 of them are stacked together; how much does the space occupied by them fall short of one cubic yard?

(**309**) Distinguish between simple and compound interest. What is the amount of £108. 12*s.* 6*d.* in 15 years at $4\frac{1}{2}$ per cent. simple interest?

(**310**) In a turnip field the rows of turnips are 2 feet apart, and the turnips 9 inches apart in the rows: if the weight of the turnips average 1 lb. 1 oz. apiece, what will be the weight of the whole crop on an acre?

(**311**) What is the value of 41 qrs. 5 bls. 2 pks. of wheat at 5*s.* 6*d.* a bushel?

(**312**) If 22 men can dig a trench 420 yards long, 5 wide, and 3 deep, in 350 days of 9 hours each, in how many days of 11 hours each will 252 men dig a trench 210 yards long, 3 wide and 2 deep?

(**313**) What is the capacity of a vessel, out of which, when it is half full, 9 gallons being drawn, there remains one-fifth of the whole content?

(**314**) Reduce 6 days 13 hours 45 minutes 37·92 seconds to the decimal of a week.

(**315**) Extract the square roots of $\frac{5}{3}$ and 379·864 to four places of decimals.

(**316**) A room is 20 ft. 10 in. long and 16 ft. broad: what length of carpet 2 ft. wide will it require so as to leave a margin of 1 ft. in width uncarpeted all round the room?

(317) Which is the advantageous stock to invest in: the 3 per cents. at 94½ or the 2½ per cents. at 79?

(318) The gross receipts of all the railways in the United Kingdom for the year 1874 was £59255715., but the net receipts only £26643003.: what percentage of the gross receipts was consumed in working expenses?

(319) The length of the year is 365·242264 days. What provision is made in our Calendar to meet the difficulty of its not consisting of an exact number of days? In what years in this century will December 7 again fall on a Tuesday?

(320) What percentage is an income-tax of 3*d.* in the pound. And what income-tax would be paid by a capital sum of £1102. 15*s.* invested in 4 per cent. stock at 100¼?

(321) Find the compound interest on £150. in 4 years at 5½ per cent.

(322) If 6 men and 8 women can reap a field of 25 acres in 3½ days, working 16 hours a day, in what time can 4 men and 20 women reap a field of 30 acres, working 12 hours a day, supposing that one man's work is equivalent to that of two women?

(323) How much 3½ per cent. stock at 97¾ can be purchased for £7938., and what income would be thence derived?

(324) Find the amount of £880. for 2¼ years at 4½ per cent., simple interest.

(325) Define Notation. Write in figures (*a*) seven hundred and fifty-six thousand and thirty; (*b*) six hundred and fifty-five thousand two hundred and twenty-six. Find the sum of these numbers, also their difference, and divide the sum by the difference.

(326) A shopkeeper bought £2. worth of steel pens at 16*d.* per box, each containing 12 dozen, and retailed them at 2½*d.* per dozen: what did he gain on his outlay?

(327) If a train be moving uniformly at the rate of 22½ miles in an hour, what number of yards will it pass over in a second?

(328) A person distributes £4. 14*s.* 6*d.* among six men, eight women, and twelve boys. Each woman had 3 times as much as each boy, and each man half as much again as each woman. What did each receive?

(329) Find, by Practice, the cost of 10 ac. 3 rds. 26 po.

at £117. 15*s.* 6⅔*d.* per acre, and explain each step of the process.

(**330**) Define a fraction, and show that if its numerator and denominator are multiplied by the same quantity its value is not altered. Which is the greatest and which the least of the fractions $\frac{7}{15}$, $\frac{9}{17}$, $\frac{13}{25}$, $\frac{11}{21}$?

(**331**) What vulgar fractions may be expressed exactly as decimals, and what vulgar fractions cannot be so expressed? Give examples. Multiply ·002 by ·0004; and divide 9·0075 by 2·5. Reduce ⅜ of 2 qrs. 14 lbs. to the decimal of a cwt.

(**332**) A rate of 1*s.* 4½*d.* in the £. on the rental of a parish produces £352.: what is the total rental?

(**333**) Which is the more profitable investment, to buy £100. railway shares at 125, or £6. bank shares at 9¾, the railway paying 6¾ per cent. dividend, and the bank 7½ per cent.?

(**334**) Define simple interest, compound interest, and amount. At what rate per cent. will a sum of money treble itself at simple interest in 30 years? Find the amount of £1024. for 3 years at 6¼ per cent., compound interest.

(**335**) Under what circumstances can a vulgar fraction be expressed as a terminating or circulating decimal? Give examples of each. Find the product of $1\cdot\dot{3}$ and ·75.

(**336**) If 4 fowls are worth 3 ducks, 7 ducks worth 2 geese, and 9 geese worth 7 turkeys, what is the price of a fowl when a turkey is worth 18*s.*?

(**337**) A bankrupt's liabilities are £3758. 17*s.* 6*d.*: what are his assets if he can pay 13*s.* 7½*d.* in the £.?

(**338**) Find the compound interest on £2500. in four years, at 5 per cent. per annum.

(**339**) Distinguish between interest and discount. If the difference between the interest and discount for one year on a sum of money at 5 per cent. per annum be £5., what is the sum?

(**340**) When the income-tax was 2*d.* in the £. a person's nett income was £2380.: what is it now that the tax is 3*d.* in the £.?

(**341**) A person sells 16 Russian bonds of £100. each at 75⅞, and invests the proceeds in a railway stock at 120. The brokerage for selling bonds is ⅛ per cent. stock, and the brokerage, &c., on purchasing railway stock is 1 per cent. on the actual value. What amount of stock did he buy?

(342) If a cubic foot of water weigh 1000 oz., what weight of water will fall on an acre of ground during a rainfall of an inch?

(343) If a person give £255. 5*s.* in exchange for a bill of £280. 15*s.* 6*d.* due a year hence, at what rate per cent. is he discounting the bill?

(344) A wine merchant buys whisky at 17*s.* 6*d.* per gallon, and, after adding water to it in the proportion of 1 gallon to 6, retails it at 18*s.* per gallon: what percentage of profit does he make on his outlay?

(345) Two taps can separately fill a cistern in 10 and 12 minutes, and when the waste-pipe is open they can together fill it in 15 minutes: how long does the waste-pipe take to empty the cistern when the taps are not running?

(346) If the income-tax be 3*d.* in the £., what income-tax will be payable on the interest arising from the investment of a capital sum of £14700. in a 3 per cent. stock at $91\frac{7}{8}$?

(347) If the hands of a clock coincide every sixty-five minutes, how much does the clock lose or gain in a day?

(348) If A can give B 10 yards and C 21 yards start in a race of 120 yards, and B can give C $1\frac{1}{2}$ seconds start over the same distance, how long does each take to run 120 yards?

(349) Find the rent of 643 ac. 1 ro. 27 po. of land, at £1. 16*s.* 8*d.* per acre.

(350) How many cwt. of lead will be required to cover a roof 48 feet long and 32 feet wide, with a sheet of lead $\frac{1}{12}$th of an inch thick? (A cubic foot of lead weighs 12096 ounces.)

(351) A grocer has two sorts of tea which cost him 3*s.* and 3*s.* 6*d.* per pound respectively: in what ratio must he mix them so as to gain 10 per cent. by selling the mixture at 3*s.* 8*d.* per pound?

(352) What is the present worth of a bill for £396. 14*s.* 4*d.* due 30 days hence, at 4 per cent. per annum?

(353) How much 3 per cent. stock has been sold out at 89, if the owner's income be increased by £12. a year by investing the proceeds in 4 per cent. stock at 92?

(354) Find the difference between the simple and the compound interest of £526. 4*s.* for four years, at 3 per cent. per annum.

(355) Income-tax is at present levied on all incomes over £150., £120. being deducted from incomes under £400. before the tax is reckoned. A man observes that his tax this year at

5*d.* per £. will amount to the same as a tax at 2*d.* per £. upon his whole income: what is his income?

(**356**) A bankrupt who is paying 7*s.* 6*d.* in the £. divides among his creditors £1575.: what do his debts amount to?

(**357**) If 36 qrs. 3 bush. 1 peck of barley cost £67. 19*s.* 2*d.*, what will 150 qrs. 1 bush. 3 pecks cost?

(**358**) An undergraduate on a certain afternoon spent at his bookseller's two-sevenths of the money in his purse, and then at his grocer's two-fifths of the remainder. He afterwards went for a ride, paying for horse-hire two-thirds of what he had over, and then returned to his rooms, possessing 5*s.* How much did he spend in each way?

(**359**) At what rate per cent. per annum, simple interest, will £427. 7*s.* 6*d.* amount to £470. 2*s.* 3*d.* in two years and a half?

(**360**) Find what percentage of the contents of a cask of spirits must be replaced by water, so that a wine-merchant may gain 20 per cent. by retailing the mixture at the same price per gallon as he paid for the undiluted spirit.

(**361**) Which is the better investment, three per cent. stock at 81, or five per cent. stock at 127? What difference of annual income will it make whether a sum of £2571. 15*s.* be invested in the one or the other of these stocks?

(**362**) A square lawn is bordered by a path 4 feet 6 inches wide, the path and lawn together occupying one-tenth part of an acre. Find the cost of paving the path with bricks at 7*s.* 6*d.* per square yard.

(**363**) The true length of a year is 365 days 5 hrs. 48 mins. 48 secs. In every 400 successive years the calendar gives 303 ordinary years and 97 leap years. Find what the error introduced by this method of reckoning amounts to in that period.

(**364**) At what rate per cent. of simple interest will £511. 14*s.* $4\frac{1}{2}$*d.* amount to £603. 16*s.* $3\frac{3}{4}$*d.* in $4\frac{1}{2}$ years, and in what time would it amount at the same rate to £695. 18*s.* 9*d.*?

(**365**) The present value of a bill of £479. 6*s.* 6*d.* is £385., the discount being at the rate of $4\frac{1}{3}$ per cent. per annum. How long has the bill to run?

(**366**) A person has £2000. in 5 per cent. stock. He sells out when the stock is at $92\frac{1}{2}$, and buys 6 per cent. railway bonds, the price being £74. per bond of £100. What is the difference in his income, and what is the actual rate of interest which he now receives on the capital invested?

(**367**) In a certain contested election there are three candidates, A, B, and C. C obtains one-fourth of the whole number of votes polled, and the number of votes given for him bears to the number given for A the ratio of 4 to 7. B obtains 1260 votes. Find the number of votes polled by A and C respectively.

(368) Four towns contribute to the Bengal Famine Fund in proportion to their population. Their contributions are respectively £151. 17*s.* 6*d.*, £272. 16*s.* 3*d.*, £360., and £612. 11*s.* 3*d.* The total population of the four towns is 62,100. What is the population of each?

(369) A boy buys a certain number of apples at 2 for a penny and half that number at 4 for 3*d.* At what price must he sell them so as to gain 20 per cent. upon his outlay? If his profit when all are sold be 4*s.* 8*d.*, how many did he buy?

(370) Two men and three boys can do a piece of work in 15 days which five men and four boys can do in 8 days. Three men are set to work at it, and after they have been working seven days a certain number of boys are sent to help them, and the work is finished in $5\frac{8}{11}$ days more. How many boys are sent to help?

(371) If £4. sterling be worth 117 lire in Italian currency, and 23·4 lire be worth 21 francs, express £26. 3*s.* 4*d.* in francs and centimes (1 franc = 100 centimes).

(372) On what sum does the simple interest for 4½ years at 6 per cent. amount to £276. 6*s.* 6¾*d.*?

(373) If the discount on a bill of £157. 13*s.* 4*d.* be £40. 6*s.* 8*d.* simple interest being calculated at 5½ per cent. per annum, how long has the bill to run?

(374) A tobacconist makes a profit of £6. 11*s.* 3*d.* by the sale of 4 cwt. 2 lb. of tobacco at 1*s.* 2*d.* per lb: what is his gain per cent.?

(375) A person holds New Zealand 5 per cent. bonds to the amount of £2600. He sells out when they are at £104. per £100. bond, and invests the proceeds in shares in a mine paying 6 per cent. What must be the price of the latter if his income from them exceed his former income by £8. 13*s.* 4*d.*?

(376) A, B, and C can together do a piece of work in 5 days, but A strikes after working 2 days, and B and C finish it in 5 days more. C can do the whole work by himself in 15 days. In what time can A and B respectively do it?

(377) A French mètre contains 39·371 English inches: express to 3 decimal places an English mile in mètres. What is the least number of mètres that contains an exact number of inches?

(**378**) A contractor bought 250 sheep, and sold them for £532. 5*s.* 10*d.* at a gain of $16\frac{2}{3}$ per cent.: what was the cost price of each sheep?

(**379**) If the cost of paving a floor $12\frac{1}{2}$ yds. long by 6 yds. $4\frac{1}{2}$ ins. wide with tiles each 7 inches square be £70. 17*s.* 6*d.*, find the price of the tiles per 100.

(**380**) Find the value of $\sqrt{\cdot 121} - \cdot 04\dot{7}$ correct to three places of decimals.

(**381**) A sum of £38. 4*s.* is made up of half-sovereigns, half-crowns and threepenny pieces. There are twice as many half-crowns as half-sovereigns, and the number of half-sovereigns bears to the number of threepenny pieces the ratio of 3 to 11. How many are there of each coin?

(**382**) At what rate will £560. amount to £649. 12*s.* in 3 years at simple interest?

(**383**) On what amount due $4\frac{1}{2}$ years hence does the discount amount to £27. 6*s.* 6*d.*, simple interest being calculated at the rate of $5\frac{1}{4}$ per cent. per annum?

(**384**) A grocer has a stock of tea which he must sell at 3*s.* 4*d.* per lb., in order to gain $33\frac{1}{3}$ per cent.; but if he mixes it with inferior tea in the proportion of 4 to 1 he can gain the same percentage of profit by selling the mixture at the rate of 15 lbs. for £2. 7*s.* What did the inferior tea cost him?

(**385**) A person derives an income of £126. 13*s.* 4*d.* from money invested in the 3 per cent. consols. He sells out when they are at 93 and buys Russian $3\frac{1}{2}$ per cents. at 95. By how much is his income increased by the transfer?

(**386**) A and B ride a race of 16 miles on bicycles. The driving-wheel of A's bicycle makes 550 revolutions in 9 minutes, while that of B's bicycle makes 600. A accomplishes the distance in 1 hr. 42 min. 24 secs., and wins by 3 min. 12 secs. Find the circumference of the driving wheels.

(**387**) If the carriage of 41 cwt. 1 lb. for 49 miles cost £20. 9*s.* 6*d.*, what must be paid for the carriage of 13 cwt. 2 qrs. 19 lbs. for 35 miles?

(**388**) If $\frac{17}{36}$ of an estate be left to the elder of two children and the remainder to the younger, and the difference of their legacies be £725., what is the value of the whole estate?

(**389**) Find the value of ·628125 of £1. and ·93375 of £5.

(**390**) Extract the square roots of ·012 and 17 to four places of decimals.

(391) The edges of a brick are 8¾, 4¼, 2¾ inches respectively: how many can be stacked in a space equal to a cubic yard?

(392) Distinguish between interest and discount. Find the simple interest of £475. 18*s*. 6*d*. for 3 years and 3 months at 4¼ per cent.

(393) If 20 cannon, which fire 5 rounds in 6 minutes, kill 500 men in one hour, how many men will 10 cannon, firing 3 rounds in 5 minutes, kill in an hour and a half, at the same rate?

394) A cistern is filled by two spouts in 15 and 25 minutes respectively, and emptied by a tap in half an hour: what portion of it will be filled in ten minutes, when they are all left open together?

(395) What decimals of a pound are 9·34 pence and ·78 of a farthing?

(396) Extract the square roots of 2 and $\frac{1}{17}$ to four places of decimals.

(397) A room is 10 ft. 9 in. high, 20 ft. 10 in. long, and 16 ft. broad; there are two windows measuring 9 ft. by 3 ft. each, and one door measuring 7 ft. by 4 ft., and the fireplace and chimney-piece occupy 20 sq. ft. of wall: how many pieces of paper 2 ft. wide will it take to paper the room, and what will be the size of the remnant over? (A piece of paper is twelve yards long.)

(398) What income will be derived from investing £3496. 18*s*. 6*d*. in the 2½ per cents. at 79?

(399) Write down in words the difference between five hundred and forty-five thousand three hundred and eighty-four, and three hundred million two hundred.

(400) The whole population of British India is estimated at 190531440, of which 139421250 are Hindoos. Find the smallest numbers which represent the ratio of the Hindoos to the whole population.

(401) The marching stride of the English soldier is 30 in., and he takes 116 steps in a minute. The Italian soldier has a stride of only 29½ in., but takes 120 steps in a minute. If a regiment of each start together for an eight hours' march, what will be the distance between them at the end of that time?

(402) The path round the Botanic Garden is half a mile and 104 yards. Two men start back to back to run round; one runs at the rate of 10 miles an hour, the other at the rate of 10½; at what distance from the starting-place will they meet for the second time?

(**403**) Find the cost of 56 cwt. 2 qrs. 14 lbs., at £5. 9*s*. 6*d*. per cwt.

(**404**) What is a decimal? Can every vulgar fraction be expressed exactly as a decimal? Illustrate your answer by the fractions $\frac{1}{3}$, $\frac{4}{27}$.

(**405**) Add together $\frac{2}{5}$, $\frac{35}{80}$, $\frac{14}{100}$, $\frac{3}{140}$, and $\frac{3}{2800}$.

(**406**) Add together ·5 of a shilling, ·7 of a crown, and ·125 of £1.

(**407**) The gross receipts of all the railways in the United Kingdom for the year 1874 was £59255715., and the working expenses were £32612712. What percentage of the gross receipts remained as net profit?

(**408**) The length of the year is 365·242264 days. What provision is made in our calendar to meet the difficulty of its not consisting of an exact number of days? On what day of the week will Christmas Day fall in the year 1900?

(**409**) The income-tax is 2*d*. in the pound. What percentage is this? What income-tax will be paid by a capital sum of £4500. invested in the 3 per cents. at $96\frac{3}{8}$?

(**410**) Find the compound interest on £250. in 4 years at $4\frac{1}{2}$ per cent.

(**411**) If the gas for 5 burners, 5 hours every evening for 10 days, cost 4*s*. 3*d*., what will be the cost of 75 burners which are lighted 4 hours every evening for 15 days?

(**412**) I invest £10. in railway shares when they are at a premium of 10*s*. At the end of a year I receive a guinea per share. What interest do I get?

(**413**) If 81 bushels of wheat are consumed by 56 men in 5 days, how long will 16 men take to consume 28 bushels?

(**414**) A parish is rated at £2748. The parish is required to raise by rate a sum of £143. 2*s*. 6*d*. What is the rate per £1.?

(**415**) A person invests £6477. in the 3 per cent. consols at $95\frac{1}{4}$, and when they have risen to 96 he sells out and invests the proceeds in $2\frac{1}{4}$ per cent. stock at 60. Find the difference in his income.

(**416**) Define simple interest, compound interest, and amount. At what rate per cent. per annum will a sum of money double itself at simple interest in 20 years? Find the compound interest on £10000. for 3 years at $3\frac{1}{2}$ per cent.

(**417**) If 5 fowls are worth 3 ducks, 14 ducks worth 5 geese, and 3 geese worth 2 turkeys, what is the price of a fowl, when a turkey costs a guinea?

(**418**) A bankrupt's liabilities are £1879. 8*s*. 9*d*. : what are his assets if he can pay 6*s*. 9¾*d*. in the £.?

(**419**) Find the compound interest on £2000. in three years at 4 per cent. per annum.

(**420**) Now that the income-tax is 3*d*. in the £. a person's nett income is £2370. per annum: what will it be when the income-tax is raised to 7*d*.?

(**421**) A person sells 25 Egyptian £100. bonds at 50⅝, and invests the proceeds in a railway stock at 125; the brokerage for selling bonds is ⅛ per cent. on the stock, and the brokerage, &c., for buying railway stock is 1 per cent. on its actual value. What amount of stock did he buy?

(**422**) During a rainfall of an inch 100 tons of water fall on an acre of ground: how many ounces are there in a cubic foot of water?

(**423**) What is the discount on £286. 13*s*. payable two years hence, reckoning compound interest at 5 per cent. per annum?

(**424**) A wine-merchant buys gin at 12*s*. 6*d*. per gallon, and after adding water retails it at 13*s*. 4*d*. He makes 20 per cent. profit on his outlay: what proportion of water does he add to the gin?

(**425**) Two taps can separately fill a cistern in 9 and 12 minutes, and the waste-pipe can empty it in 8 minutes. How long will the two taps together take to fill the cistern when the waste-pipe is open?

(**426**) If the income-tax be 3*d*. in the £., what income-tax will be payable on the interest arising from the investment of a capital sum of £7000. in a 4 per cent. stock at 109⅜?

(**427**) If the hands of a clock coincide every sixty-six minutes, how much does the clock gain or lose in a day?

(**428**) A can give B 20 yards and C 41 yards start in a race of a quarter of a mile, and B can give C 3 seconds start over the same distance, how long does each take to run a quarter of a mile?

(**429**) Find the rent of 526 acres 3 roods 19 poles of land at £2. 3*s*. 4*d*. per acre.

(**430**) How many cwt. of lead will be required to cover a roof 40 feet long and 28 feet wide with a sheet of lead $\frac{1}{10}$th of an inch thick? (A cubic foot of lead weighs 12096 ounces.)

(**431**) A grocer has two sorts of tea which cost him 2*s*. and 2*s*. 6*d*. per pound respectively; in what ratio must he mix them so as to gain 12 per cent. by selling the mixture at 2*s.* 4*d.* per pound?

(**432**) What is the present worth of a bill for £214. 13*s.* 4*d.* due 60 days hence, at 5 per cent. per annum?

How much 3 per cent. stock has been sold out at 88, if when the proceeds are invested in 4 per cents. at 120 the owner's income be reduced by £2. a year?

Find the difference between the simple and the compound interest of £635. 12*s.* for four years at 3 per cent. per annum.

(**433**) Income-tax is at present levied on all incomes over £150., £120. being deducted from incomes under £400. before the tax is reckoned. A man observes that his tax this year, at 5*d.* per £., will amount to the same as a tax at 3*d.* per £. upon his whole income. What is his income?

(**434**) An undergraduate on a certain afternoon spent at his bookseller's two-ninths of the money in his purse, and then at the grocer's two-sevenths of the remainder. He afterwards went for a ride, paying for his horse hire two-fifths of what he had over, and then returned to his rooms possessing 15*s.* How much did he spend in each way?

(**435**) At what rate per cent. per annum simple interest will £560. 11*s.* 8*d.* amount to £664. 13*s.* 5*d.* in three years and nine months?

(**436**) Find what percentage of the contents of a cask of spirits must be replaced by water, so that a wine-merchant may gain 25 per cent. by retailing the mixture at the same price per gallon as he paid for the undiluted spirit.

(**437**) Which is the better investment—three per cent. stock at 87 or six per cent. stock at 168? What difference of annual income will it make whether a sum of £541. 6*s.* 8*d.* be invested in the one or the other of these stocks?

(**438**) A square field is bordered by a path three yards wide, the field and path together occupying two and a half acres. Find the cost of covering the path with gravel at 1*s.* 6*d.* per square yard.

(**439**) The true length of a year is 365 days 5 hrs. 48 mins. 48 secs. In every 400 successive years the calendar gives 303 ordinary years and 97 leap years. Find in how many years the error introduced by this method of reckoning will amount to one day.

(**440**) Convert $\frac{1}{128}$ to a decimal fraction, and find the vulgar fraction corresponding to the recurring decimal $\cdot 22\dot{2}9\dot{7}$.

(**441**) Reduce $\dfrac{\sqrt[3]{5{\cdot}12}+\sqrt[3]{{\cdot}03375}}{\sqrt[3]{80}-\sqrt[3]{{\cdot}01}}$ to a simple decimal.

(**442**) Reduce $\dfrac{\sqrt{1{\cdot}75}-\sqrt{{\cdot}63}}{\sqrt{3{\cdot}5}-\sqrt{2{\cdot}1}}$ to its simplest form.

(**443**) If a parcel 12 lbs. weight is carried 80 miles by rail for 2*s.* 4*d.*, and the rate for the distance over 50 miles is two-thirds of the rate for the first 50 miles, how far can a parcel of 8 lbs. be carried for 4*d.*?

(**444**) If the 3 per cents. are at $91\frac{1}{8}$, what interest does this give on £100.? (Omit brokerage and fractions of a penny.)

(**445**) How many lbs. in ·321875 of a ton weight? Convert it into kilogrammes (omitting fractions), assuming that a cubic decimetre of distilled water weighs 15432·35 grains.

(**446**) Reduce $\frac{499}{40}$ and $\frac{1}{3125}$ to decimal fractions; then multiply them together, and divide the product by ·0000075. Reduce the quotient to a vulgar fraction.

(**447**) A vessel steams 16 knots per hour. How many mètres per second is this equivalent to, taking a mètre as 3·2809 feet, and a knot as 6086·5 feet?

(**448**) A tile of copper 1 foot long, 9 inches wide, and five-eighths of an inch thick, is rolled into a plate 6 feet long and 4 feet wide. How thick will the plate be? Give your answer both as a vulgar fraction and in decimals of an inch exactly.

(**449**) I purchase $2\frac{1}{2}$ cwt. of tea at £13. 1*s.* 8*d.* per cwt., and $1\frac{1}{2}$ cwt. at £18. 12*s.* per cwt. I mix them, and sell the mixture at 3*s.* 6*d.* per lb. What is the profit per lb., the profit per cent. on the outlay, and the profit per cent. on the selling price? (Two decimal places sufficient.)

(**450**) A cargo steamer leaves port at the rate of $8\frac{1}{2}$ miles per hour at 3 A.M. on Monday. At midnight a despatch boat starts in pursuit, at the rate of 15 miles an hour. When will they be in sight of one another, supposing them to be visible 6 miles off?

(**451**) Find how much the square root of

$$9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{8}}} \text{ differs from } \frac{355}{113}.$$

(**452**) A ladder 41 feet 1 inch long is placed against a wall with its foot 12 feet 11 inches out from the wall. How high will it reach up the wall?

(**453**) Reduce (when necessary) the following amounts to the nearest decimal of a pound, not going beyond three figures of decimals, and add them all together:—£307. 2*s*. 9½*d*.; £5. 0*s*. 1¼*d*.; ·74 of a shilling; two hundred pounds; 3*s*. 3½*d*.; 2*s*. 6½*d*.; ·003 of a pound.

(**454**) Reduce ·064 and 15·625 to vulgar fractions; multiply them in that form, and then reduce the result to decimals. Prove by multiplying the decimals as they stand.

(**455**) The equatorial diameter of the earth being 41847662 English feet, and the polar axis 41707536 feet, by how much per cent. is the polar axis the shorter? By what length would the difference be represented in a globe 24 inches in diameter?

(**456**) A coal merchant has £30000. invested in rolling and other stock. He buys coal at 11*s*. per ton, and its freight averages 6*s*. 6*d*. The expenses in town are reckoned as equivalent to a dead-weight charge of £3000. a year plus a tonnage charge of 1*s*. 6*d*. a ton; how many tons must he sell annually to cover his expenses, and pay 5 per cent. upon his capital?

(**457**) An express train leaves London for Exeter (distance 174 miles) at 9.15 A.M. at the rate of 40 miles an hour. An ordinary train leaves Exeter for London at 11.20 A.M. at the rate of 24 miles per hour. At what time and at what distance from London will they meet (neglecting stoppages)?

(**458**) Find a decimal multiplier which will convert Troy ounces per inch into tons per mile. Use it to find the weight in tons per mile of wire, which weighs $\frac{3}{100}$ths of an ounce Troy to the inch run.

(**459**) 14 tons 3 cwt. copper at £92. 15*s*. per ton; 12 tons 12 cwt. spelter at 35*s*. 10*d*. per cwt.; 3 tons 5 cwt. tin at 78*s*. 9*d*. per cwt.; coal consumed, 14 tons at 14*s*. 3*d*.; loss of metal in casting $\frac{1}{30}$; labour equal to one man for 31 days at 4*s*. 9*d*.; other expenses reckoned at £125. What is the cost per ton to the nearest shilling?

(**460**) Reduce to its lowest terms the following fractional expression:—

$$\begin{array}{ccccc}\text{tons} & \text{cwt.} & \text{qrs.} & \text{lbs.} & \text{oz.} \\ 528 & 13 & 3 & 27 & 1 \\ \hline 674 & 6 & 1 & 2 & 1\end{array}.$$

(**461**) A passenger train going 41 miles an hour, and 431 feet long, overtakes a goods train on a parallel line of rails. The goods train is going 28 miles an hour, and is 713 feet long. How long does the passenger train take in passing the other?

(462) Find the square root of 6005320036.

(463) A, B, and C went into a joint-stock business, in which A invested £50., B £60., and C £80. B withdrew his money after 10 months; but A and C continued in the business for 6 months longer, and then wound it up. The total profit was found to be £335. How much should each receive?

(464) A gallon of fresh water measures 227·271 cubic inches, and weighs 10 lbs. Avoirdupois. A ton of sea-water measures 35 cubic feet. What is the weight of a gallon of sea-water in pounds and decimals?

(465) A tradesman's prices are 15 per cent. above cost price. If he allows a discount of £2. on a bill of £46., what profit does he make per cent.?

(466) Find the cost of plating the outside of a cubical box containing 37 cub. ft. 64 cub. in. at 3*s*. per sq. ft.

(467) A cistern is filled by a pipe in 3 hours and emptied by a second pipe in 4 hours. If both pipes be open together for 5 hours and the discharge pipe be then closed, how long will it take to fill the cistern?

(468) A man who has £40000. stock in the New Three per Cents. sells out at 94½, and invests in India 4 per Cents. at par. Find the alteration in his income.

(469) A wine merchant buys spirit at 17*s*. per gallon and mixes it with spirit which cost him 24*s*. a gallon, in the proportion of three gallons of the former to one of the latter. At what price per gallon must he sell the mixture in order to realise a profit of 12 per cent. on its cost?

(470) Reduce to its lowest terms the following fractional expression, and find its value to four decimal places:—

$$\frac{\frac{1}{2}+\frac{3}{5}+\frac{5}{7}}{\frac{5}{2}+\frac{4}{5}+\frac{1}{7}}.$$

(471) Find the square root of 4774671801.

(472) A sovereign consists of 22 parts by weight of pure gold to 2 parts of alloy, and it weighs 123·274 grains Troy. Neglecting the value of the alloy, what is the value of pure gold per ounce Troy to the nearest penny?

(473) A, B, and C run a race for a mile. B has one minute start, and C two minutes on A. The actual pace of each is— A, 10 miles an hour; B, 8½ miles an hour; C, 7½ miles an hour. Who wins, and how long after the winner does the last man pass the winning-post?

(**474**) A carriage-wheel is 8 feet 3 inches in circumference. How many turns does it make in driving 7 miles and 1331 yards?

(**475**) 400 calendar years are 146097 days. What would be the length of a calendar year in seconds, supposing all such years were alike?

(**476**) What decimal fraction is $2\frac{1}{4}$*d.* of 7*s.* 6*d.*; and what decimal of a cwt. is 64 lbs.?

(**477**) Find out which is the greater of $\frac{56}{143}$ and $\frac{72}{187}$, and divide $\frac{16}{11}$ by their difference.

(**478**) An acre is ·40467 hectares, and a pound sterling is 25·25 francs. An estate measuring 1927 hectares is sold for 10100000 francs. What is the selling price per acre in £. *s.* *d.*?

(**479**) A certain number of persons ventured equal shares in a business, so as to make up a total capital of £1719. 8*s.* 4*d.* The first dividend was $7\frac{1}{2}$ per cent. upon the capital, and amounted to £2. 14*s.* $10\frac{1}{2}$*d.* per share. What was the number of shares?

(**480**) Find the sum of 2·35 of 2*s.* 1*d.*, and $0{\cdot}\dot{0}\dot{3}$ of £6. 3*s.* 9*d.*, and express it as a vulgar fraction of £2. 13*s.* 6*d.*

(**481**) The rainfall at a particular place is 29·2 inches in a year. To how many gallons per acre is this equivalent, taking a gallon as 0·16 of a cubic foot of water?

(**482**) Extract the square roots of $8\frac{1}{3}$ and 0·144, in both cases to three places of decimals.

(**483**) A man pays a tenth of his income in rates and taxes and a twelfth in assurances; he has left £492. 13*s.* 1*d.*: what is his income?

(**484**) A person sells out £3965. from 3 per cent. stocks at 74, and re-invests them in $5\frac{1}{2}$ per cents. at 143: what is his gain or loss in annual income?

(**485**) A warehouse of five storeys is let in flats. Each flat (except the top one) lets for three-fourths of the rent charged for all the flats above it, and the rent of the whole warehouse is £2401. What is the rent of the middle flat?

(**486**) What is the meaning of the term 'average'? If a train move over 30, 24, 20, 16 miles in four successive hours, what is its average rate per hour? Show that the same principle is involved in 'alligation.'

(**487**) State the rules for the Least Common Multiple of two, and of three or more quantities, and apply the latter to the following case:—518003, 544231, 583573.

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(**487**) State the rules for the Least Common Multiple of two, and of three or more quantities, and apply the latter to the following case:—518003, 544231, 583573.

(488) Find the square roots of 52·9, 5·29, ·529, ·0529, and ·00529 to three figures of decimals.

(489) A certain district contains a rural population of 3789, and a town population of 2354. Among the rural population the births are 31 per thousand annually and the deaths 25 per thousand. Among the town population the births are 42 per thousand, and the deaths 30 per thousand. What is the annual increase or decrease in the whole population?

(490) A gallon of liquid, measuring 277·271 cubic inches, being poured into a certain square receptacle, fills it to a depth of 1·79 inches: find the side of the square to two places of decimals.

(491) 100 parts of zinc ore yield 55 parts of metallic zinc, and 100 parts of copper ore yield 25 parts of metallic copper. In making brass out of these, 5 per cent. of the metal is wasted, and the coals and labour cost as much as the metal. The zinc ore is worth £9. per ton delivered, and the copper ore £17. What will be the net cost per ton of brass consisting of a mixture of equal parts of the two metals?

(492) Simplify $\frac{4{\cdot}84 - 1\frac{7}{9}}{2{\cdot}2 - 1\frac{1}{3}}$, and $\frac{1{\cdot}\dot{5} \text{ of } 2s.\ 3d.}{1\frac{2}{9} \text{ of } \frac{10}{33} \text{ of } 5s.\ 7\frac{1}{2}d.}$

(493) Calculate to four places of decimals $\frac{4}{\sqrt{5}+1}$.

(494) Write down in order of magnitude $\frac{41}{6}$, $\frac{90}{11}$, $\frac{131}{16}$, and $\sqrt{67}$, and prove your answer.

(495) A cistern measures internally 9 ft. 4 in. long, 5 ft. 6 in. wide, and 3 ft. 4½ in. deep. It is lined with lead at 7 lbs. per square foot. What is the cost of the lining when lead is £2. per cwt.?

(496) A bridge has five equal spans, each of 100 feet measured from centre to centre of the piers, and a boat is moored exactly in a line with one of the two middle piers. The whole length of the bridge subtends a right angle as seen from the boat. Find the distance of the boat from the bridge, to the nearest foot.

(497) A bankrupt's debts amount to £3625. 16*s.* 3*d.*, and his assets to £1208. 12*s.* 1*d.*: how much can he pay in the £.?

(498) Reduce $\frac{5}{8}$ of a halfcrown to the fraction of a shilling and $\frac{2}{7}$ of a cwt. to the fraction of a lb.

(499) What will it cost to carpet a room 18¾ feet long and 17½ feet wide with carpet 3½ feet wide at £$\frac{1}{3}$. per yard?

(**500**) Add together 29·0146, 3146·5, 2109, ·62417, 14·16.

(**501**) Divide ·101 by ·5, ·05, 50, and 5000.

(**502**) Reduce to vulgar fractions $\cdot 01\dot{6}3\dot{4}$ and $9\cdot\dot{9}$.

(**503**) Add together $\frac{2}{3}$ of $1\frac{1}{2}$, $\frac{11}{12}$ of $\frac{9}{15}$, $\frac{7}{8}$ of $3\frac{2}{5}$.

(**504**) Divide $\frac{9}{11}$ of $\frac{19}{81}$ by $\frac{33}{153}$ of $\frac{114}{121}$.

(**505**) How many yards of canvas 18 inches wide would it take to cover a box 4 ft. long, 3 ft. high, and three-quarters of a yard wide?

(**506**) If £1025. are put out to simple interest for 13 years at $4\frac{1}{2}$ per cent., what will be the interest and amount?

(**507**) Find the square root of 488601 and of 84·82410.

(**508**) If 7 men can mow 6 acres in 12 hours, how many men will mow 15 acres in 14 hours?

(**509**) If $\frac{3}{8}$ of a yard of velvet cost £$\frac{2}{5}$., what will $\frac{5}{16}$ of a yard cost?

(**510**) The quick time or step in marching being two paces per second, or 120 per minute, at 28 inches each, at what rate per hour does a company march on a good road? and how long will they be in arriving at a garrison 20 miles distant, allowing a halt of 1 hour by the way for refreshment?

(**511**) Add together $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{11}{12}$.

(**512**) Find the G.C.M. of 2526 and 2947.

(**513**) Subtract ·002 from 11, and multiply ·01 by 1·01.

(**514**) Divide $2\frac{6}{7}$ of $1\frac{2}{5}$ by $3\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{1}{4}$.

(**515**) If ·5 of £. buys ·4 of a gallon, how much may be bought for 6*s.* 3*d.*?

(**516**) Divide 1·44 by 1·2, by 12, by ·012, and by 120.

(**517**) Find the simple interest on £375. at 4 per cent. for 11 years.

(**518**) Divide £1300. into three parts in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

(**519**) Find the square root of 449·44 and ·0001.

(**520**) What is the cost of carpeting a room 21 ft. by 30 ft. with carpet $\frac{1}{2}$ a yard wide at 4*s.* 6*d.* a yard?

(**521**) If 12 men do a piece of work in 15 days of 10 hours each, how many days of 9 hours will 10 men take to do it?

(**522**) Find the income produced by investing £4500. in the 3 per cents. at 90, and that produced by investing £2850. in the $3\frac{1}{2}$ per cents. at 95.

(**523**) A steam engine and a horse start together; the engine does the first mile in 5 minutes, the horse in 4; after this mile the pace of the engine is 30 miles per hour, that of the horse 20: which wins in a 3 mile race, and by how much?

(**524**) Divide (1) 1·69 by ·13; (2) 12·1 by ·011; (3) 001 by ·01.

(**525**) Reduce 12*s.* 6*d.* to the decimal of £1., and 7 lbs. to the decimal of 1 cwt.

(**526**) Divide $\frac{1}{4}$ of $\frac{1}{5}$ of $4\frac{2}{7}$ of $2\frac{1}{3}$ by $\frac{1}{9}$ of $4\frac{1}{5}$ of $3\frac{3}{4}$ of $1\frac{1}{7}$.

(**527**) Find the compound interest on £100. for 2 years at 10 per cent.

(**528**) If 24 men do a piece of work in 14 days of 9 hours, how many will do it in 12 days of 7 hours?

Extract the square root of 17161.

(**529**) Find the L.C.M. of 10, 14, 15, 21, 30, 42.

(**530**) What sum invested in the 3 per cents. at 92 will produce an income of £100. per annum?

(**531**) If cotton is bought at £2. 6*s.* 8*d.* per cwt., at what price per lb. must it be sold to bring in 5 per cent. profit?

(**532**) What will it cost to carpet a room 20 ft. 3 in. long by 16 ft. wide with carpet 2 ft. 3 in. wide at 3*s.* 4*d.* per yard?

(**533**) A and B together do a piece of work in 5 days; A alone would do it in 7 days; in how many days would B alone do it?

(**534**) Find the value of an acre of land at $8\frac{1}{2}$*d.* per square foot.

(**535**) Add together $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$, and express the sum in decimals.

(**536**) Simplify—

$$(1)\ \frac{1}{2+\dfrac{3}{4+\frac{5}{6}}}. \qquad (2)\ \frac{\frac{2}{9}\times\frac{3}{8}\div 2}{\frac{1}{6}\left\{\frac{4}{5}-\frac{1}{16}\right\}}.$$

(**537**) Reduce $\frac{4}{5}$ of 3*s.* 4*d.* to the fraction of a guinea, and 6·72 oz. to the decimal of a cwt.

(**538**) Divide 36·33 by 210, 35 by ·05, ·002 by 200.

(**539**) Find the square root of 2209 and of 22·1841.

(**540**) Find the simple interest on £2066. 15*s.* 10*d.* for 4 years at $2\frac{1}{2}$ per cent., and the compound interest on £9030. or 2 years at $3\frac{1}{3}$ per cent.

(**541**) Find the difference between the discount allowed at 5 per cent. on £810. paid 3 months before it is due, and the interest charged for the loan of the same sum for the same period at the same rate.

(**542**) A grocer mixes 13 lbs. of tea at 3*s.* 4*d.* with 3 lbs. at 4*s.* 10*d.* : at what price must he sell the mixture so as to gain a penny a pound?

(**543**) Find the value of £$3\frac{2}{5}$. + $7\frac{2}{3}$*s.* + $11\frac{3}{7}$ of a guinea, and of ·35 of a ton + ·39 of a cwt. + ·44 of a qr.

(**544**) Find the G.C.M. of 124321 and 197451.

(**545**) Reduce to its lowest terms—

$$\left\{1+\cfrac{1}{3+\cfrac{1}{3+\frac{1}{3}}}\right\}\div\left(\tfrac{1}{3}+\tfrac{1}{11}\text{ of }10\tfrac{2}{3}\right).$$

(**546**) Find the value of $\frac{1}{480}$ of a £. + $\frac{2}{15}$ of half-a-crown + $\frac{5}{42}$ of a guinea + $\frac{15}{2}$ of a penny.

(**547**) Give the rule (1) for placing the point in division of decimals; (2) for reducing a circulating decimal to a vulgar fraction.

(**548**) Reduce ·075 of a £. to the decimal of 4*s.* 6*d.*; and 2 sq. po. 9 yds. 4 ft. 72 in. to the decimal of 1 ac. 60 sq. yds.

(**549**) Find, by Practice, the value of 2 tons 7 cwt. 56 lbs. at £2. 10*s.* 6*d.* per cwt.

(**550**) A man buys eggs at $11\frac{1}{4}$*d.* per dozen, and sells them at 2*d.* each: what does he gain per cent.?

(**551**) Nine men were engaged to dig a trench 3 feet deep, $4\frac{1}{2}$ feet wide, and 120 yards long in a certain time; but before it was begun it was discovered that the trench must be 5 feet deep, 180 yards long, but only 4 feet wide: how many men must be employed in order that it may be completed in the required time?

(**552**) Find the simple interest on £387. 10*s.* for $5\frac{1}{4}$ years at $3\frac{1}{2}$ per cent.

(**553**) What sum, if laid out at $3\frac{1}{2}$ per cent. for 6 years, will produce £28. 17*s.* 6*d.* interest at simple interest?

(**554**) What will be the cost of papering a room 14 ft. 6 in. long, 10 ft. 6 in. broad, and 8 ft. high, with paper 2 feet wide at $7\frac{1}{2}$*d.* per yard?

(**555**) Find the L.C.M. of 3, 5, 7, 9, 15, 63.

(**556**) Divide $1-(\frac{1}{2}+\frac{1}{3}+\frac{1}{24})$ by $1-\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{20}$, and find the value of $\dfrac{\frac{1}{6}+\frac{1}{8}}{8\frac{1}{2}+\frac{3}{14}}$.

(557) Reduce 1 lb. Troy to the fraction of 1 lb. Avoirdupois.

(558) Multiply ·000725 by 31·25, and divide the product successively by 6·25, 625, and ·0625.

(559) Reduce to vulgar fractions ·0015625, ·05, $\cdot 03\dot{1}\dot{8}$.

(560) Find the value of 3·15625 of a £.

(561) Find the square root of 106929 and 4·1209.

(562) At the census of 1851 the population of Oxford was 27843; at the census of 1861 the population was 27560: what was the decrease per cent.?

(563) A can walk 5 miles while B is walking 4: supposing A to walk 6 hours a day, and B 7 hours a day, how many days will B take in walking a distance which A can accomplish in 14 days?

(564) Find the simple interest on £354. 3*s.* 4*d.* for $3\frac{1}{3}$ years at $2\frac{1}{4}$ per cent.

(565) A quadrangle is 50 feet long by 40 feet broad; it is crossed in each direction by a path 10 feet broad, and the remainder has to be turfed: how many strips of turf $1\frac{1}{2}$ foot long and 6 inches broad will be required for the purpose?

(566) Find the value of $\dfrac{\frac{7}{10}-\frac{2}{3}}{\frac{9}{12}+\frac{1}{5}} \div \dfrac{\frac{1}{3}}{9\frac{1}{2}}$, and of $\frac{1}{2}+\frac{2}{9}-\frac{4}{15}+\frac{5}{18}$.

(567) £4500. is divided between A, B, C, D; A receives $\frac{1}{2}$; B, C each $\frac{2}{5}$ of the remainder: how much is left for D?

(568) Reduce 1 lb. 12 oz. (Avoirdupois weight) to the fraction of 1 cwt. 2 qrs., and $\frac{5}{16}$ of 6*s.* 8*d.* to the fraction of a guinea.

(569) Express as vulgar fractions ·001375, $\cdot\dot{0}\dot{3}$, $7\cdot 193\dot{1}\dot{8}$, and divide ·175 by ·25, by 2·5, and by ·0025.

(570) State the tests of divisibility of numbers by 4, 9, and 11; and apply them to the number 71016.

(571) What is the value of a cargo of tallow, weighing 515 tons, at 51*s.* 3*d.* per cwt.?

(572) Five per cent. on a given sum amounts to £25. 13*s.* 4*d.*: find $4\frac{3}{4}$ and $4\frac{1}{2}$ per cent. on the same sum.

(573) Define Interest and Discount. What is the discount on £429. 5*s.* due 3 months hence at 4 per cent. per annum?

(574) Two bills for £456. 5*s.* and £274. 2*s.* 6*d.* are due on the 1st and 30th June respectively. What is their value on the 20th June, interest being reckoned at the rate of 5 per cent. per annum?

(575) Divide £3920. amongst 4 persons in the proportions of 2, 4, 6, 8.

(576) A speculator sells at a profit of 50 per cent., but his purchaser fails, and only pays 10*s.* in the £.: how much per cent. does the speculator gain or lose by his venture?

(577) A and B run a race. A starts at the rate of 400 yards a minute, but in every successive minute increases his pace by a yard a minute; B diminishes his pace by the same, and is overtaken by A in 4 minutes. What was B's pace at starting?

(578) What is the value of a cargo of sugar, weighing 756 tons, at 28*s.* 6*d.* per cwt.?

(579) Two bills for £273. 15*s.* and £456. 17*s.* 6*d.* are due on the 2nd and 22nd of July respectively: what is their value on the 12th July, interest being reckoned at the rate of 5 per cent. per annum?

(580) A speculator sells at a profit of 75 per cent., but his purchaser fails, and only pays 5*s.* in the £.: how much per cent. does the speculator gain or lose by his venture?

(581) The manufacturer will supply a certain article at 6*d.*: if a tradesman charges 8*d.*, what profit per cent. does he make?

(582) State the rule for finding the Greatest Common Measure of two numbers. Find the Greatest Common Measure of 2793 and 7980, and the Least Common Multiple of 16, 90, 91, 280, and 455.

(583) Simplify $\frac{\frac{16}{81}+\frac{4}{75}-\frac{81}{675}}{\frac{4}{9}-\frac{2}{5}+\frac{9}{25}}$, and find the value of

$$\frac{1\frac{1}{8}\div 1\frac{1}{5}}{\frac{2}{3}\text{ of }\frac{5}{9}\div 10\frac{1}{3}}\times\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{6\frac{15}{16}\text{ of }5\frac{1}{3}}\text{ of }£1.$$

(584) Divide 23·78 by 62·5, and 6400 by ·04096; and reduce 3 hrs. 42 min. 39 sec. to the decimal of a year.

(585) The length of a rectangular piece of ground is twice its breadth; its area is 30479·805 square feet; find its length and breadth.

(586) The rate of interest being 7 per cent., what is the discount on a sum of £1356. 13*s.* 4*d.* due 3 months hence; and what is the interest on the same sum for 9 months?

(587) How often will a tumbler containing $1\frac{4}{5}$ of a gill be filled out of a cask which originally contained 20 gallons, but had leaked 10 per cent.?

(588) If 100 lbs. of tea be bought at 4*s.* 4*d.* per lb. and sold at 5*s.*, and 100 lbs. of sugar be bought at 6*d.* a lb., and sold at

7d. a lb., how much short of 18 per cent. will be the profit realised on the whole transaction?

(**589**) Compare the simple and compound interest on £1000. for 3 years at 5 per cent.

(**590**) A newspaper contains 48 columns, averaging 100 lines in each, $8\frac{1}{2}$ words in each line, $4\frac{1}{4}$ letters in each word: what number of consonant letters will be required to print the paper if the vowels are 35 per cent.?

(**591**) If 81 gallons of water will fill a cistern 4 ft. 4 in. long, 2 ft. 8 in. broad, and 1 ft. $1\frac{1}{2}$ in. deep, how many cubic inches are contained in a pint?

(**592**) Simplify the expression—

$$\frac{8\frac{3}{5}-7\frac{3}{4}+5\frac{2}{3}-4\frac{1}{2}}{13-11\frac{9}{10}+10\frac{7}{9}-9\frac{17}{20}} \text{ of } \frac{2}{11} \text{ of } 365.$$

(**593**) Reduce $\frac{7}{410}$ to a recurring decimal. Reduce the recurring decimal $\cdot\dot{0}0054$ to a fraction. If a fraction is expressed by a decimal, in what cases will the decimal be non-recurrent? Can every recurring decimal be obtained from a vulgar fraction by the common process of division? If not, give an example to the contrary.

(**594**) Find the square roots of $\frac{15129}{182329}$, of $11\frac{37}{49}$, and of $76\frac{14}{15}$, to five places of decimals.

(**595**) State and prove the rule for finding the Greatest Common Measure of two numbers.

(**596**) If Australian wool is worth *2s. 3d.* per lb., and English wool only *1s. 8d.* per lb., how many fleeces of English wool ought to be set off against 1000 Australian, a fleece of the latter weighing only $\frac{11}{18}$ of a fleece of the former?

(**597**) A clerk's net salary, after paying *6d.* in the £. income-tax, is £195.; what would it have amounted to had there been no income-tax?

(**598**) If tea be bought at £·1. per lb. and sold at $\cdot\dot{1}$ of a guinea, what is the gain per cent.

(**599**) Divide 8008 guineas between A and B so that ·5 of A's money shall equal ·6 of B's.

(**600**) By selling tea at *2s. 8d.* per lb. a merchant gains $\frac{1}{8}$ of his outlay; he then raises the price to *3s. 1d.* per lb.: what does he gain per cent. on the original outlay?

(**601**) Find the price of 63 cwt. 3 qrs. 18 lbs. 8 ozs. at £8. *12s. 8d.* per cwt.

(**602**) How many hours a day must 171 men work to do in 91 days what it takes 143 men 133 days of 9 hours each to do?

(**603**) What is the present value of £1051. 5*s*. due a year hence at £5. 2*s*. 6*d*. per cent.?

(**604**) If the carriage of $5\frac{1}{4}$ cwt. for 114 miles costs 4*s*. 8*d*., how far ought $3\frac{1}{2}$ cwt. to be carried for the same money?

(**605**) Find the price of 104 ac. 3 rds. and 10 per. at £42. 15*s*. an acre.

(**606**) By the reduction of the income-tax from 7*d*. in the pound to 5*d*. a person saves £28. 2*s*. 6*d*. a year: what is his income?

(**607**) Reduce $5\frac{1}{2}$ gall. to the fraction of $1\frac{3}{8}$ pecks, and $4\frac{1}{2}$ ozs. to the decimal of 4 lbs. 1 oz. 4 drs.; also find the value of ·6875 of 3*s*. 4*d*.

(**608**) Find the dividend on £3469. 10*s*. at 15*s*. $6\frac{1}{2}$*d*. in the pound.

(**609**) What will be the expense of paving an area which measures 35 ft. 10 in. by 18 ft. 6 in. at 6*s*. 3*d*. per square yard?

(**610**) A bankrupt's estate pays 4*s*. $4\frac{1}{2}$*d*. in the pound: what loss will be sustained by a creditor whose claim is for £432. 12*s*.?

(**611**) How much coffee at 1*s*. $10\frac{1}{2}$*d*. per lb. must be given in exchange for 78 lbs. of tea at 5*s*. $1\frac{1}{2}$*d*.?

(**612**) If a person transfers £1234. from the $3\frac{1}{4}$ per cents. at $88\frac{7}{8}$ to the $3\frac{1}{2}$ per cents. at $95\frac{2}{3}$, what will be the difference in his income?

(**613**) If 8 men can build 2 roods of wall 1 brick in thickness in 3 days, how many men must be employed to build 5 roods, a brick and a half in thickness, in a week?

(**614**) If £20. in trade gain £16. in five months, what sum will gain £24. in 3 months at the same rate?

(**615**) Divide £45. between 3 persons, so that their shares shall be as the numbers 3, 5, 7.

(**616**) If 7 bush. 2 pks. be consumed by 10 horses in 7 days, how many horses will consume 3 qrs. 6 bush. at the same rate in 10 days?

(**617**) If 1000 sovereigns weigh 21 lbs. 5 oz. 16 dwts., what will be the weight of one sovereign?

(**618**) Prove that a square room, whose side is 17 ft. 6 in. long, will require 43 yds. 2 ft. 3 in. of carpet 2 ft. 4 in. wide.

(**619**) If $2\frac{3}{4} - 1\frac{5}{6}$ of an estate cost £440., what will $2\frac{1}{2}$ of $6\frac{5}{12}$ of it cost?

(**620**) If 152 sacks will hold 65 qrs. 2 bush. 2 pks., how many sacks will contain 18 qrs. 7 bush. 1 pk.?

(**621**) What length of wall paper 27 in. broad will be required for a room 18 ft. long 12 ft. broad and 11 ft. high?

(**622**) If a man walks 600 miles in 25 days, walking 8 hours a day, in how many days will he walk 330 miles, walking 10 hours a day?

(**623**) If it costs 16 guineas to supply 30 men, 48 women, and 60 boys with bread for a week, supposing a man to eat twice as much as a boy, and a woman to eat one-fourth less than a man, how much will it cost to supply 25 men, 60 women, and 72 boys for 10 days?

(**624**) If an income of £300. pays £11. 5*s.* for income-tax, how much will be paid for an income of £525. 10*s.*?

(**625**) The rental of a parish is £1850., and the rates are taken at 80 per cent. of the rental: what is the amount of a rate at 9*d.* in the pound?

(**626**) If £3. 10*s.* are the wages of 4 men for 6¼ days, what are the wages of 17 men for 4 weeks of 5½ days in each week?

(**627**) If 10 men and 15 boys can reap a field in 6 days, in how many days will 7 men and 12 boys reap it, supposing that 2 men can do as much as 3 boys?

(**628**) Three wheels making 60, 36, and 24 revolutions in a minute, start with certain points in their circumferences downwards: when will they first come all together again in the same position?

(**629**) If a certain MS. fill 6 printed sheets, each containing 32 pages, and each page 24 lines, with 9 words in each line, how many lines of 12 words each must there be in a page in order that another similar MS., twice as long, may fill 3 sheets, each containing 48 pages?

(**630**) If forty shillings weigh 7 oz. 5 dwts., and contain 10 dwts. 21 grs. of alloy, how much pure silver is there in one shilling?

(**631**) If 5½ per cent. would be gained by selling 121 yds. of silk for £26. 11*s.* 10¾*d.*, at what price per yard must it be sold to gain 12 per cent.?

(**632**) If the duty on goods weighing 2 cwt. 3 qrs. 5 lbs. amounts to £2. 18*s.* 8¼*d.*, what is the rate per cwt.?

(**633**) If 1 lb. of metal, containing copper and zinc in the ratio of ·84 to ·16, be mixed with 2 lbs., containing the same metals in the ratio of ·75 to ·25, find how much copper and zinc there is in the mixture.

(**634**) If 8 horses plough 11⅔ acres in 2 days, in how many days will 6 horses plough 17½ acres?

(**635**) If 17 yds. of silk cost £4. 8*s.* 6½*d.*, what will be the cost of 120 yds. at the same rate?

(**636**) For what sum must I sell a horse that cost me £75. 10*s.* so as to gain 5¼ per cent.?

(**637**) If 25 chests of tea weigh 1 ton 3 cwt. 1 qr. 21 lbs., how many chests of the same size will weigh 1 ton 11 cwt. 3 qrs. 14 lbs.?

(**638**) What is the cost of papering a room 6 yds. 1 ft. 1 in. long, 6 yds. 0 ft. 4 in. broad, 12 ft. high, with paper ⅙ of a yd. wide, at 4½*d.* per yard?

(**639**) If 7 bush. 2 pks. be consumed by 10 horses in 7 days, how many horses will consume 3 qrs. 6 bush. at the same rate in 10 days?

(**640**) If 44 labourers can do a piece of work in 15 days of 10 hours each, how many navvies must be employed to do ¾ more work in 7 days of 11 hours, supposing 3 navvies able to do the work of 5 labourers?

(**641**) How much money must be paid for £1500. stock of the 3 per cents. at 89⅞, allowing the broker ⅛ per cent. as commission; and what interest per cent. will be gained on the outlay?

(**642**) How long will 12 men take to do a piece of work which 8 men can do in 27 days?

(**643**) The annual premium on a fire insurance being £13. 5*s.* 3*d.*, the duty charged on it will be £19. 17*s.* 10½*d.*: how much per cent. of the premium is the duty?

(**644**) If the ton be diminished 10·714285% and the ounce increased 25%: how many ounces would there be in a ton?

(**645**) A draper bought 600 yards of silk at 3*s.* 4½*d.* per yard; and having sold 360 yards at 4*s.* 6*d.* per yard, and 81 yards at 3*s.* 9*d.* per yard, he was robbed of the rest: what was his whole gain or loss, and his gain or loss per cent. on the outlay?

(**646**) When eggs are 24 a shilling, how many must be given in payment of a debt of £1. 11*s.* 5½*d.*?

(**647**) A gentleman having three sons, aged 24, 16, and 8 years respectively, left his estate to be divided amongst them in shares proportioned to their ages. Eight years afterwards the second son died and left his portion to be divided between his brothers in shares inversely proportional to their ages. Find what fractional part of the father's estate each of the surviving sons then had.

(**648**) A surveyor's chain is 22 yards long, and is divided into 100 links: how many square links are there in a piece of land of $2\frac{1}{3}$ acres?

(**649**) A dairyman buys milk at $2\frac{1}{2}$*d.* per quart, dilutes it with water, and sells the mixture at 3*d.* per quart. His profits are 60 per cent. upon his outlay. How much water does he mix with each quart of milk?

(**650**) Supposing a labourer to receive 13*s.* 5*d.* per week of seven days, find what his wages will amount to from the 1st of April to Christmas Day of the same year, both days inclusive.

(**651**) A person holds £4450. in the Turkish Stocks. If he sells out at $52\frac{1}{8}$ and invests the proceeds in the Reduced Three per Cents. at $88\frac{7}{8}$, what will be his income, supposing the broker to have received $\frac{1}{8}$ per cent. commission on each transaction?

(**652**) Find the value of $\dfrac{(\frac{1}{2})^4 + (\frac{1}{3})^4 + (\frac{1}{6})^2}{(\frac{1}{2})^2 + (\frac{1}{3})^2 + \frac{1}{6}}$; and reduce £3. 12*s.* $2\frac{1}{4}$*d.* to the decimal of eleven guineas.

(**653**) Prove that the Least Common Multiple of two numbers divides every other common multiple of the two numbers. What is the least number which is divisible by every number up to 12 inclusive?

(**654**) Prove the rule for fixing the position of the decimal point when one decimal fraction is multiplied by another.

(**655**) Express as vulgar fractions in their lowest terms—(1) ·0625 × ·0032; (2) ·016 ÷ ·64; (3) $\cdot4\dot{5} - \cdot45$.

(**656**) From forty pounds Troy of metal containing eleven parts of pure gold and one part of alloy, eighteen hundred and sixty-nine sovereigns are coined. If the alloy be supposed worthless, find the value to the nearest farthing of one ounce Troy of pure gold.

(**657**) Extract the square root of $2\frac{33439}{50625}$.

(**658**) Find, by Practice, the price of 5 cwt. 3 qrs. 24 lbs. 12 oz. at £5. 2*s.* 8*d.* per cwt.

(**659**) If the carriage of 5 tons 5 cwt. 2 qrs. 1 lb. for 50 miles cost £27. 10*s.*, what will be charged for conveying $25\frac{1}{2}$ cwt. 117 miles?

(**660**) Reduce the following to vulgar fractions:—3·07, $1\cdot2\dot{8}\dot{3}$, $\cdot01\dot{2}\dot{3}\dot{6}$; and the following to decimals:—$\frac{39}{25}$, $\frac{191}{480}$, $\frac{16}{81}$.

(661) The true length of the year is 365·24224 days: if it be calculated at 365¼ days, how soon will the error amount to one whole day?

(662) Simplify $\dfrac{\frac{13}{42}\cdot\frac{1}{6}-\frac{11}{28}\cdot\frac{1}{9}}{\frac{16}{42}\cdot\frac{1}{6}-\frac{13}{28}\cdot\frac{1}{9}}$; and express $\dfrac{1}{10+\dfrac{1}{2+\frac{1}{30}}}$ under the form of a decimal.

(663) Find the value of 18 cwt. 1 qr. 21 lbs. at £27. 15*s.* per cwt.; and the amount of income duty payable on a salary of 300 guineas per annum when the tax is 5*d.* in the pound.

(664) Find the continued product of 50·75 by 3·045 by ·07105, and the quotient of ·0001776 by ·0784.

(665) A carpet contains 167·71545025 square feet: find its breadth (1) when it is as broad as it is long, (2) when it is 6¼ times longer than it is broad.

(666) Given the Least Common Multiple of the 1st and 2nd of three quantities, and also of the 1st and 3rd, how may the Least Common Multiple of all three be found from these data?

(667) Find the Least Common Multiple of 110, 1155, 1470.

(668) If I am entitled to $\frac{9}{10}$ of the interest on £750. from March 3 to July 27, at 5½ per cent. per annum, how much should I receive?

(669) A person owned $\frac{1}{5}$ of a coal-mine, and sold $\frac{2}{3}$ of $\frac{3}{8}$ of his share for £5125·25.: what was the value of the mine, and of his share in it?

(670) The population of a country amounts to 26584000, and increases at the rate of 10 per cent. in every 10 years: what will it amount to in 30 years?

(671) If I spend on an average £5. 18*s.* 8½*d.* per week, and save every year £216., what is my income?

(672) A mixture of 49 gallons of wine (at 15*s.*) and brandy (at 18*s.*) contains only one gallon of brandy: how much water should be added to it that it may contain in each 49 gallons ·2 of a gallon of brandy; and what should be the price per gallon of the whole mixture?

(673) In a borough returning two members, and having a population of 30000, of whom $\frac{2}{15}$ have votes, A polls 100 plumpers and 840 split votes, B 200 plumpers and 890 split votes, and C 110 plumpers and 730 split votes: what percentage of those who had votes gave them?

(674) A merchant buys some tea at 2*s.* 3*d.* per lb. and some at 3*s*. 6*d*.: in what proportion must he mix them, so that by selling the mixture at 4*s*. a lb. he may realise 20 per cent.?

(675) A tea-dealer buys a damaged cargo for £530., and after spending 7 per cent. on the cost price in preparing it for the market, he sells it at 120 per cent. profit; receiving for it the same gross sum as is received by another dealer who purchased a sound cargo and sells it for 32 per cent. profit. Find the value of the second cargo.

(676) Multiply together 3·1415927 and 1·3183099.

(677) Calculate to five places of decimals $\frac{1 \cdot 492641}{\cdot 71293}$.

(678) Reduce the recurring decimal $\cdot 2\dot{2}8571\dot{4}$ to a fraction in its lowest terms.

(679) Alcohol and water being mixed in the proportion of 5·295 parts by weight of the former to 6·027 of the latter, find what weight of each substance is contained in 1 pound of the mixture, and also the weight of a quantity of the mixture which contains 1 pound of water.

(680) Calculate $\sqrt{3 \cdot 14515936535}$ to seven decimals.

(681) If a man invests half of his fortune in land, a fifth in trade, a sixth in funds, and loses the remainder (namely, £2000.) in speculation, what sum had he to start with?

(682) £550. 10*s*. was borrowed January 1, 1861, at the rate of 5 per cent. per annum: what sum repaid the debt on the 20th of October in the same year?

(683) An Australian miner has 100 lbs. of gold dust. 8 per cent. of this is alloy beyond the standard, and coining it into sovereigns costs him 5 per cent.: what is its value, the sovereign weighing 122·274 grains?

(684) Reduce to their lowest denominations, as ordinary fractions, the product of the two fractional sums, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}$ and $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}$, and find the ratio of the former to the latter.

(685) Reduce the two sums of the preceding question with the aforesaid product and ratio to decimals, all correctly to five places.

(686) Extract the square root of 633679929 to the last integer figure by the ordinary process.

(687) Divide £25. 7*s*. 6*d*. between 4 men, 3 women, and 2 boys, giving each woman a fourth less, and each boy a half less than the share of each man.

(688) A grocer mixes four kinds of tea in the proportion of

1, 1·5, 2, 2·5 parts by weight of the several kinds respectively: required the number of pounds of each kind in one hundredweight of the mixture.

(**689**) Find the difference between the simple and compound interest on £116. 2*s.* 11*d.* for 3 years at 9 per cent. per annum.

(**690**) If the penny-a-mile fare be 40 per cent. below second-class fare, and the second-class fare 30 per cent. below the first, and if return tickets are issued at a fare and three-quarters, what ought to be the exact price of a return ticket first-class for 250 miles?

(**691**) Required the compound interest and amount of £457. 10*s.* for 3 years at 4 per cent. per annum.

(**692**) In a mile race, A starts at 270 paces of 48 inches per minute, and B at 300 paces of 44 inches per minute. After 4 minutes, B quickens his pace to 320. Which wins, by how much, and in what time?

(**693**) Find the length of the side of a square field containing 3952 ac. 3 ro. 18 per. $21\frac{1}{2}$ yds.

(**694**) From the sum of the fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$ subtract the sum of the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and exhibit the result in its lowest denomination.

(**695**) Reduce the two sums of the preceding question, and their difference, separately, to decimals; and verify by subtraction the accuracy of your results.

(**696**) Extract to the nearest integer, by the ordinary or by any other process, the square root of 12345678987654321.

(**697**) Calculate to the nearest farthing, by whatever process you consider the most convenient, the cost of 19 tons 19 cwt. 3 qrs. $27\frac{1}{2}$ lbs. of merchandise at £19. 19*s.* $11\frac{1}{2}$*d.* per ton.

(**698**) What is the simple interest on £345. 19*s.* 4*d.* for 2 years at 4 per cent. per annum, and on £546. 7*s.* 10*d.* for 48 days at 3 per cent. per annum?

(**699**) If teas at 5*s.* 6*d.*, 6*s.* 5*d.*, and 4*s.* 9*d.* a lb. be mixed in equal quantities, and the mixture be sold at £32. 18*s.* per cwt., what will be the gain or loss per cent.?

(**700**) What is the yearly interest on £27225. bought into the $3\frac{1}{4}$ per cents. at $97\frac{1}{2}$?

(**701**) In a common school for 3 hamlets, A, B, C, 60 per cent. of the scholars belong to A, 28·8 per cent. to B, and 11·1 (only 7 boys and 3 girls) to C: what is the number of children in the school; and what number belong to A, B, and C respectively?

(702) A offers for an estate £83000. now, and B £96000. at the end of 3 years: which is the better offer, and by how much, allowing 5 per cent. per annum compound interest?

(703) Two bottles of sherry and 6 pints of ale cost 12*s.* If the price of the sherry rose 10 per cent. and that of the ale 50 per cent., the cost would be 14*s.* What was the original price of a bottle of sherry and of a pint of ale?

(704) Divide $\frac{25}{21}+6\frac{3}{8}-7\frac{1}{28}$ by $5\frac{2}{3}-3\frac{1}{7}+\frac{17}{12}$, and multiply the result by $6\frac{5}{13}+9\frac{7}{25}-13\frac{1}{5}$.

(705) Find what fraction of a guinea is equal to the difference between $\frac{2}{7}$ of a crown and $\frac{13}{11}$ of a shilling.

(706) Calculate to five places of decimals the fraction—

$$\frac{3{\cdot}70271 \times {\cdot}64732}{{\cdot}043679}.$$

(707) Reduce the circulating decimal $1{\cdot}3\dot{2}37\dot{2}$ to a vulgar fraction in its lowest terms.

(708) Extract the square roots, to five places of decimals, of the numbers—3·9726523, ·39726523, and ·039726523.

(709) A contributes £$9350\frac{87}{192}$. to a fund of £30000., B contributes £$7409\frac{11}{96}$., C £$3284.\frac{33}{192}$., and D the remainder: what is the average sum contributed *by the last three*, and what annual percentage of profit does the largest contributor receive if he gets £497. 16*s.* $3\frac{1}{8}$*d.* a year?

(710) In what time will £229. 10*s.* amount to £258. 3*s.* 9*d.* at 5 per cent. per annum, simple interest?

(711) Which is the better stock for investment, the $3\frac{1}{4}$ per cents. at $92\frac{5}{8}$ or the $3\frac{1}{2}$ per cents. at par?

(712) A and B enter into business together; A gives $\frac{3}{7}$ of the capital. At the end of the first year they have made a profit of $3\frac{1}{2}$ per cent.; and at the end of the second a profit of 5 per cent.; at the end of the third year they are bankrupts, and can only pay 10*s.* in the pound. The remaining money is £16663. 10*s.* How much did each contribute?

(713) To the sum of the ordinary fractions $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$ add the sum of the ordinary fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and exhibit the result as a decimal.

(714) From the sum of the repeating decimals $\cdot 12\dot{3}\dot{4}$ and $\cdot\dot{1}23\dot{4}$ subtract twice the repeating decimal $\cdot 1\dot{2}3\dot{4}$, and exhibit the result in the form of a decimal of similar character.

(715) The first and last terms of an Arithmetical Progression are 1 and 6561, and the number of terms is 9: determine completely the Progression.

(**716**) The first and last terms of a Geometrical Progression are 1 and 6561, and the number of terms is 9: determine completely the Progression.

(**717**) Find the simple interest on £639. 8*s.* from May 10 to August 20, at $5\frac{1}{8}$ per cent. per annum.

(**718**) If a person invests in 4 per cent. stock at $77\frac{1}{2}$, at what rate per cent. will he receive interest for his money?

(**719**) 260 children were present for examination in a school, and of these $30\cdot\dot{7}6923\dot{0}$ per cent. were in Standards IV. to VI. 17 children failed in the first three standards, and the percentage of failures for the whole school was 8·3: how many children passed in Standards IV. to VI.?

(**720**) What capital would be required in order to gain £12. 1*s.* $0\frac{3}{4}$*d.* in 2 years 5 months at $3\frac{1}{2}$ per cent.?

(**721**) If the sum of £1200. be laid out at 10 per cent. compound interest, to what sum will it amount in a year and a half, (*a*) if the interest be paid yearly; (*b*) if it be paid half-yearly?

(**722**) If by selling an article for 8*s.* 3*d.* you lose $17\frac{1}{2}$ per cent., what sum should you have sold it for to have gained 40 per cent.?

(**723**) Add together $\frac{3}{11}$ of $\frac{7}{87}$ of $\frac{1}{84}$ of a pound, and $\frac{5}{33}$ of $\frac{57}{58}$ of $1\frac{3}{4}$ of a shilling, stating the result as a fraction of half-a-crown.

(**724**) Multiply 123·456 by 654·321 correctly to the nearest integer.

(**725**) Obtain the square root of 0·001111111 correct to seven places of decimals. Multiply the result by the square root of 7·29.

(**726**) Reduce $1\cdot45\dot{6}7\dot{8}$ to a vulgar fraction; proving the truth of any rule regarding recurring decimals which you may require to use.

(**727**) Having given that one foot is 0·3048 of a mètre, what is an acre in square mètres?

(**728**) If 13 cubic feet of stone weigh a ton, what will a cubic mètre weigh in kilograms? (A kilogram may be taken as $2\frac{1}{5}$ lbs.)

(**729**) A coal-pit has an output of 1000 tons a week, which is sold for £370. The price of large coal is 10*s.* a ton, of slack 4*s.* a ton: how much of the 1000 tons is slack? If the slack costs the coal-owner 6*s.* and the large coal 7*s.* a ton, what are his profits per week?

(**730**) Find the compound interest and amount of £49000. for 3¼ years at 7 per cent. per annum.

(**731**) A man buys a house for £450., keeps it empty for 12 months, and then sells it so as to gain 15 per cent. Would he have done better or worse if he had let it for the last 8 months out of the 12 at 8 guineas a month and sold it for cost price at the year's end?

(**732**) Out of an invading army of 62000 men 88·25 per cent. perished by fatigue and disease, 27·8 per cent. fell in action, and the remainder returned home safe: what percentage of the whole army escaped, and how many fell in action?

(**733**) The average price of edible birds' nests is estimated at £5. 2*s*. 6*d*. per lb., and the annual consumption at 242000 lbs. If the whole cost of collecting be 11 per cent. of the selling price, find the amount of the tradesmen's profit on the annual sale.

(**734**) Square root of 20 to 4 places of decimals.

(**735**) Find the compound interest on £3945·3125. for 4 years, at 4 per cent.

(**736**) A bankrupt can pay 6*s*. 8*d*. in the £.: if his assets were £500. more he could pay 7*s*. 4*d*. Find his debts and his assets.

(**737**) The manufacturer will supply a certain article at 1¾*d*.; if a tradesman charges 2*d*., what profit per cent. does he make?

(**738**) The price of diamonds per carat varies as their weight: if a diamond of 2 carats is worth £32., what is the value of a diamond of 3 carats?

(**739**) Wishing to pay 18 Kreutzer, I give 1 Thaler, and receive in change 22 Kreutzer, 10 Silber-Groschen, and half a Gulden. I know that 1 Thaler is 30 Silber-Groschen, and 1 Gulden is 60 Kreutzer. Find for me how many Gulden are worth 4 Thaler.

(**740**) A man walks a certain distance, and rides back in 3 hrs. 45 min.; he could ride both ways in 2½ hrs. How long would it take him to walk both ways?

(**741**) In a constituency in which each elector may vote for two candidates, half of the constituency vote for A, but divide their votes among B, C, D, E, in the proportions of 4, 3, 2, 1; of the remainder, half vote for B, and divide their votes among C, D, E, in the proportions of 3, 1, 1; two-thirds of the remainder vote for D and E, and 540 do not vote at all: find the order on the poll, and the whole number of electors.

(742) Find the compound interest on £5503·4375. for 4 years, at 4 per cent.

(743) A bankrupt can pay 5*s.* 9*d.* in the £.: if his assets were £500. more he could pay 6*s.* 6*d.* Find his debts and his assets.

(744) If 5 men can reap a field, whose length is 800 feet and breadth is 700 feet, in 3½ days, working 14 hours a day, in what time can 7 men reap a field whose length is 1800 feet and breadth 960 feet, working 12 hours a day?

(745) Find the difference between the simple and compound interest on £416. 13*s.* 4*d.* for 2 years at 5 per cent. per annum.

(746) Find the true discount on £280. due 2¾ years hence, at 3 per cent. per annum.

(747) A and B, working together, do a piece of work in 7 days. B alone can do it in 13 days. Suppose B works 3 days, how long will A take to finish the work?

(748) A landlord sends a chest of tea to be equally divided among his 48 cottage tenants; each receives 2 lbs. 7½ ozs.: what did the tea weigh, and what did it cost at 2*s.* 3*d.* a pound?

(749) Assuming the year to consist of 365·2422453 days, find the decimal which represents one day, to 10 decimal places.

(750) Find the amount of £1158. 17*s.* 6*d.* for 1 year 115 days at £2. 10*s.* per cent. simple interest.

(751) Find the true discount on £397. 6*s.* 8*d.* due 9 months hence at 4 per cent. per annum.

(752) A person sells out £5000. consols (3 per cent. stock) at 89½, and buys £2000. Russian 5 per cent. stock at 88¼, and with the rest of his money India 5 per cent. stock at 106: find the alteration in his income.

(753) If the rents of a parish be rated at £2514. 7*s.* 6*d.*, and a rate be levied to pay £84. 16*s.* 3*d.*, how much is this in the £., and how much will be paid by a farm rated at £115. 12*s.* 6*d.*?

(754) A chest of tea weighing 68 lbs. 4 ozs. net is given by a landlord to be equally divided among his cottage tenants: how many tenants has he, if each receives 1 lb. 5 ozs.? and if the tea costs him 2*s.* 2*d.* a lb., what is the price of the chest?

(755) 1 carat = 4 grains (Diamond weight) = 3⅕ grains (Troy): find how many ozs., &c. (Troy) are contained in the diamond whose weight is 6720 grains (Diamond).

(756) A bankrupt can pay his creditors 3*s.* 4½*d.* in the pound: how much will one receive whose debt is £882. 13*s.* 4*d.*?

(757) Express £893. 5 fl. 3 cents 2 mils. in £. and cents, and find 3½ per cent. on the amount. (£1. = 10 florins = 100 cents = 1000 mils.)

(758) Find, by Practice, the values of 1513 things at £5. 8*s*. 5*d*., and of 5 cwt. 3 qrs. 21 lbs., at £6. 10*s*. 8*d*. per cwt.

(759) Find the difference between the interest and discount on £1114. 11*s*. 8*d*. for 1¾ years, interest being reckoned at 4 per cent. per annum.

(760) Which is the better investment, 3½ per cent. stock at 98¼ or 3¾ per cents. at 105?

(761) Find the square root of 19740¼, and the cube root of 284890312.

(762) Divide 28561 into parts proportional to 1, 1, 4, 7; and also into parts proportional to $\frac{1}{12}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{2}$.

(763) 9 men were engaged to dig a trench 6 ft. deep, 4½ ft. wide, and 120 yds. long, in a certain time; but before it was begun it was discovered that the trench must be 5 ft. deep, 180 yds. long, but only 2 ft. wide: how many men must be employed that it may be finished in time?

(764) Find the discount on £1133. 8*s*. 7*d*. due 6 months hence, interest being reckoned at 6 per cent. per annum.

(765) £98. is divided amongst a certain number of persons, one-third of whom receive 15*s*. each, and the remainder 17*s*. each: what is the number of persons?

(766) The average price of edible birds' nests has been estimated at £5. 2*s*. 6*d*. per lb., and the annual consumption 242000 lbs. If a tradesman's profit be 89 per cent. on the selling price, what is the whole annual cost of collecting them?

(767) If a sequin be worth 9*s*. 4½*d*. and a carlino £5. 12*s*. 3¼*d*., how many sequins are equivalent to 900 carlini?

(768) Find the simple interest on £1563. 13*s*. 4*d*. for 3 years at 2½ per cent. per annum.

(769) What is the discount on £926. 11*s*. 5*d*. due 9 months hence, interest being reckoned at 4 per cent. per annum?

(770) £142. is divided amongst 160 persons, 80 of whom receive 4*s*. each more than each of the rest: how much does each receive?

(771) Which is the better investment, 3¼ per cent. stock at 97¼ or 3½ per cents. at 105?

(772) Find the square root of 72002$\frac{7}{9}$ and the cube root of 1334633301.

(773) Divide 46875 into parts proportional to 1, 4, 4, 6, and also into parts proportional to $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$.

(774) Two sets of men perform the same amount of work. Each man in the first set is stronger than one in the second in the ratio 7 to 6. The first set works 6 days a week for 10 weeks; and the second set works 5 days a week for 7 weeks. If there are 9 men in the first set, how many are there in the second?

(775) If a sequin be worth 9*s*. 5*d*., and a pistole 15*s*. $7\frac{3}{4}$*d*., how many pistoles are equivalent to 1502 sequins?

(776) Find the value of 212 tons 16 cwt. 1 qr. 5 lbs. at £28. per ton.

(777) Find, by Practice, the value of 1943 things at (1) £29. 19*s*., (2) 13*s*. 1*d*. each.

(778) What is the simple interest on £914. 16*s*. 8*d*. for $2\frac{1}{2}$ years at 3 per cent. per annum?

(779) Three merchants, A, B, and C, trading with a capital of £18000., find, after a certain time, their respective shares increased by £97. 0*s*. 2*d*., £36. 2*s*. 2*d*., £51. 11*s*. 8*d*.: how much had they severally subscribed to the original capital?

(780) How many sq. ft. and sq. in. remain out of 313 sq. ft. of carpeting after covering a room 16 ft. 9 in. long by 12 ft. 11 in. wide?

What is the price of the carpet required at 3*s*. 6*d*. per yard?

What is the cost of papering a room 14 ft. 6 in. long, 10 ft. 5 in. broad, 8 ft. high, with paper 2 ft. wide at $1\frac{1}{2}$*d*. per yard?

(781) If a piece of work can be finished in 45 days by 35 men, and if the men drop off by 7 at a time at the end of every 15 days, how long will it be before the work is finished?

(782) By selling a horse for £42. 10*s*. a person lost $7\frac{1}{2}$ per cent.: find the gain or loss per cent. by selling him for £48. 5*s*.

(783) If a metric system of area were adopted, so that 2 ac. 2 rds. 6 per. would be represented by 5·12, express the unit of measurement in square yards and decimal parts of a square yard.

(784) The price of £100. Bank Stock which pays $6\frac{1}{2}$ per cent. annual interest is $185\frac{6}{8}$; £6250. of this stock is transferred to the 3 per cent. Consols when the price of Consols is $92\frac{7}{8}$: find the loss in the annual interest, brokerage being neglected.

(785) Two circular gold plates, each 1 inch thick, of which the diameters are respectively 6 and 8 inches, are melted into one plate an inch thick: find its diameter, having given that the area of a circle is proportional to the square of its radius.

(786) I have a certain sum of money wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny I shall spend 5*d.* too much, if 50 a penny 10*d.* too little: how much have I to spend?

(787) What is a multiple? a common multiple? the Least Common Multiple? Find the L.C.M. of 13 × 17 × 19, 17 × 19 × 21, 19 × 21 × 13.

(788) What fraction is 4 lbs. Troy, 3 ozs. 10 dwts. 20 grs. of 6 lbs. 5 ozs. 6 dwts. 6 grs.? If 4 lbs. 3 ozs. 10 dwts. 20 grs. cost £5. 4*s.* 2*d.*, what will 6 lbs. 5 ozs. 2 dwts. 2 grs. cost?

(789) Find the cost of varnishing the floor of a room 14 ft. 4 in. broad and 15 ft. 6 in. long. at 6*d.* per square yard.

(790) Find, by Practice or otherwise, the cost of (i.) 3684 articles at £2. 6*s.* 9¼*d.* each; (ii.) 2 tons 7 cwt. 22 lbs. 5 ozs. at £16. per ton.

(791) Three and sixpence a week is charged for coals when they are 27*s.* a ton: how much is supposed to be consumed in a year?

(792) Find the difference between the discount and interest on £453. 14*s.* 5*d.* for 5 months at 6 per cent.

(793) The interest on £300. from June 12 to Sept. 30 is £2. 5*s.* 2⅓*d.*: at what rate is it calculated?

(794) A tradesman's prices are 25 per cent. above cost price; if he allows a customer 12 per cent. on his bill, what profit does he make?

(795) On a stream, B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 3 hrs. 45 min., from A to C in 2⅓ hrs.: how long would it take to go from C to A?

(796) I have to earn a certain sum of money by selling a certain number of nuts, and I find that if I sell at the rate of 40 a penny I shall earn 10*d.* too much, if 50 a penny 5*d.* too little: how many have I to sell?

(797) If 60 cubic inches of lead, together with 54 cubic inches of cork are equal in weight to 1538⅔ cubic inches of fir, and the weights of equal quantities of lead and fir are represented by the numbers 11·324 and ·45, what number represents the weight of an equal quantity of cork?

(798) The sidereal year being 365 days 6 hrs. 9 min. 9·6 sec., and the tropical year 365 days 5 hrs. 48 min. 49·7 sec., reduce their difference to the decimal of a sidereal year.

(**799**) Find (i.) in chains and links, (ii.) in yards and feet, the diameter of a circle which contains an acre; the area of a circle being the square of the radius multiplied by 3·1416.

(**800**) A watch gains 1 minute and 12 seconds per day. It is set right at noon on the 11th of November. What will be the true time when it points to noon on the following Christmas Day?

(**801**) A grocer buys coffee at the rate of £8. 10*s*. per cwt., and chicory at the rate of £2. 10*s*. per cwt., and mixes them in the proportion of 5 parts chicory to 7 coffee: at what rate must he sell the mixture so as to gain £$16\frac{2}{3}$. per cent. on his outlay?

(**802**) What is a measure? a common measure? the Greatest Common Measure? Find the G.C.M. of 13 × 17 × 19, 17 × 19 × 21, 19 × 21 × 13.

(**803**) What fraction is 8 lbs. Troy 1 oz. 19 dwts. 9 grs. of 13 lbs. 7 ozs. 5 dwts. 15 grs.? If 8 lbs. 1 oz. 19 dwts. 9 grs. cost £10. 6*s*. 6*d*., what will 13 lbs. 7 ozs. 5 dwts. 15 grs. cost?

(**804**) Find the cost of varnishing the floor of a room 14 ft. 6 in. broad and 15 ft. 4 in. long at 6*d*. per square yard.

(**805**) Find, by Practice, the cost of (i.) 1842 articles at £2. 6*s*. $9\frac{1}{4}$*d*. each, (ii.) 2 tons 7 cwt. 22 lbs. 5 ozs. at £32. per ton.

(**806**) A scuttle of coals is charged 6*d*. when coals are 27*s*. a ton: how much ought the scuttle to hold?

(**807**) Find the difference between the discount and interest on £313. 19*s*. for 8 months at 6 per cent.

(**808**) The interest on £300. from June 2 to September 20 is £2. 5*s*. $2\frac{1}{2}$*d*.: at what rate is it calculated?

(**809**) A tradesman's prices are 20 per cent. above cost price: if he allow a customer 10 per cent. on his bill, what profit does he make?

(**810**) On a stream, B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 5 hrs. 15 min., from A to C in 7 hrs.: how long would it take to go from C to A?

(**811**) What fraction of $\frac{8}{15}$ of a crown is $\frac{15}{8}$ of a guinea; and what proportion does their difference bear to their sum?

(**812**) Find the value of 33 po. $2\frac{3}{4}$ yds. at £653400. per acre.

(**813**) How many ounces of gold at £3·89375 per oz. are contained in a lump worth £123·9675?

(**814**) Find the discount on £196. 4*s*. 4½*d*. due 6 months hence at 8 per cent.

(**815**) Find the discount on £1000. due 2 years hence at 4 per cent. compound interest.

(**816**) How much in the £. is 12½ per cent.? A man receives only 15*s*. in the £. of what is due to him, and thereby loses £150. 10*s*. 6*d*.: what was due to him?

(**817**) An English mile is ·2136 of a German mile: what time will a man who walks a German mile in 2 hours take to walk an English mile?

(**818**) An extra-efficient earns 30*s*. for the corps. He has to advance 4*s*. for ammunition, which is returned to him upon application: if 25 per cent. of the men were to neglect to apply for it, what would be the average gain the corps would make for each extra-efficient?

(**819**) I have to be at a certain place at a certain time, and I find that if I walk at the rate of 4 miles per hour I shall be 10 minutes too late; if at the rate of 5 miles per hour I shall be 5 minutes too soon: how far have I to go?

(**820**) The price of diamonds per carat varies as their weight: if a diamond of 3 carats is worth £75., what is the value of a diamond of 2 carats?

(**821**) If 1 Gulden is $17\frac{1}{8}$ Silber-Groschen and 1 Thaler is 30 Silber-Groschen, and to pay a bill of 4·4 Gulden I give 2 Thalers 16 Silber-Groschen, have I any change to receive?—the smallest coin being $\frac{1}{100}$th of a Gulden.

(**822**) A man walks a certain distance and rides back in 5 hrs. 15 min. He could walk both ways in 7 hrs.: how long would it take him to ride both ways?

(**823**) In a constituency in which each elector may vote for two candidates, one-third of the constituency vote for A, but divide their votes among B, C, D, E, in the proportion of 4, 3, 2, 1; of the remainder, two-thirds vote for B, and divide their votes among C, D, E, in the proportions of 3, 1, 1; half of the remainder vote for D and E, and 540 do not vote at all: find the order on the poll, and the whole number of electors.

(**824**) What fraction of $\frac{12}{5}$ of a crown is $\frac{5}{12}$ of a guinea? and what proportion does their difference bear to their sum?

(**825**) Find the value of 16 po. 16½ yds., at £1306800. per acre.

(**826**) How many ounces of gold, at £3·89375. per ounce, are contained in a lump worth £113·6975.?

(**827**) (i.) Find the discount on £78. 9*s.* 9*d.*, due 8 months hence, at 6 per cent.

(ii.) Find the discount on £100., due 2 years hence, at 4 per cent. compound interest.

(**828**) How much per cent. is 2*s.* 4*d.* in the £.? A man receives only 5*s.* in the £. of what is due to him, and thereby loses £150. 10*s.* 6*d.*: what was due to him?

(**829**) An English mile is ·2136 of a German mile. What time will a man who walks 4 English miles an hour take to walk a German mile?

(**830**) An extra-efficient earns 10*s.* for the corps. He has to advance 4*s.* for ammunition, which is returned to him upon application: if 25 per cent. of the men were to neglect to apply for it, what would be the average gain the corps would make for each extra-efficient?

(**831**) I have to be at a certain place at a certain time, and I find that if I walk at the rate of 4 miles per hour I shall be 5 minutes too late; if at the rate of 5 miles per hour I shall be 10 minutes too soon: how far have I to go?

(**832**) Two persons, A and B, hold ground in common, at a rental of £54.: A puts in 23 horses for 27 days, and B 21 horses for 39 days. How much of the rent should be paid by each?

(**833**) What are the aliquot parts (greater than $\frac{1}{112}$) of 1 cwt.?
Find, by Practice, the value of 3119 things at £4. 7*s.* 7*d.* each; and of 9 tons 19 cwt. 3 qrs. 14 lbs. at £28. per ton.

(**834**) Find the square root of 121·2201; and the cube root of $423987\frac{457}{512}$. The latter root is a mixed number, whose fractional part is $\frac{1}{8}$.

(**835**) A debt of £18. 15*s.* 10*d.* is paid in crowns, shillings, and pennies, whose numbers are proportional to 3, 2, 1: find the number of each coin.

(**836**) Define Interest and Discount. Find the amount of £2160. 12*s.* 6*d.* in 1 year 73 days, at 5 per cent. interest.

(**837**) Find the discount on £4120. 8*s.* 7*d.* due 9 months hence, interest being reckoned at 4 per cent. per annum.
Does the creditor or debtor gain by computing interest instead of discount?

(**838**) When wheat is 15*s.* per bushel, 8 men can be fed for 12 days at a certain cost: for how many days can 6 men be fed for the same cost when wheat is 12*s.* per bushel?

(**839**) A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage and of the two circles of rails are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the four wheels in a complete circuit.

(**840**) Reduce ·145215 sq. miles to acres, roods, perches.

(**841**) On a scale 2·15 inches represent 19·8 furlongs: what length on it represents a mile?

(**842**) Find the product and quotient of 16777216 and 131072.

(**843**) Find the number of square yards in a room whose floor measures 6½ yards by 5½ yards.

What would be the cost of papering such a room if it were 10 feet high; the paper being 2 feet wide, and costing 2¾*d.* per yard?

(**844**) A Bernese gold Ducat and Pistole are worth 8*s.* 1½*d.* and 18*s.* 8*d.* respectively. How many Ducats are equivalent to 3510 Pistoles?

(**845**) Define a fraction, and multiply $10000\frac{4}{4976}$ by ·4789.

(**846**) State the rule for the division of decimals, and verify it by means of the quotients of ·0144÷1·2, and 1422·3÷·0011.

(**847**) What are the aliquot parts (greater than $\frac{1}{20}$) of £1.?

(**848**) Find, by Practice, the value of 1931 things at £5. 3*s.* 7*d.* each, and of 9 yds. 3 ft. 9 ins. at £2. 3*s.* 6*d.* per yard.

(**849**) A debt of £475. 3*s.* is paid in sovereigns, half-sovereigns, and shillings, whose numbers are proportional to 1, 2, 3. Find the number of each coin.

(**850**) Find the interest on £4321. 5*s.* for 2 years 146 days, at 2½ per cent. per annum.

(**851**) Find the present value of £1079. 2*s.* 5*d.*, due 1½ years hence, interest being reckoned at 5 per cent. per annum.

Is the present value increased or diminished by computing Interest instead of Discount?

(**852**) If 15 men are fed for 7 days at a certain cost when wheat is 12*s.* per bushel, what must be the price when 10 men can be fed for 8 days at the same cost?

(**853**) Show how many yards of carpet (2 ft. wide) it will take to furnish a room whose floor measures 8 yards by 7¼ yards.

If the room be 9½ ft. high, how many feet will it contain?

(**854**) If 15 men take 17 days to mow 300 acres of grass, how long will 27 men take to mow 167 acres?

(**855**) Find the amount of £550. 15*s.* for 2½ years, at 3½ per cent. per annum, compound interest.

(**856**) Find the true discount on £2500. due a year and a half hence, at 5 per cent.

(**857**) Assuming the length of the year at 365 days 6 hours, find the decimal which represents one hour to 10 decimal places.

(**858**) Extract the square roots of 1420913025 and 68·492176, describing the process.

(**859**) If £788. 2*s.* 6*d.* in 3 years 2 months amount to £887. 19*s.* 1*d.*, what is the rate per cent. of simple interest?

(**860**) Explain the meaning of discount. Find the true discount on £256. 7*s.* 6*d.* due 3 years hence, at 3 per cent.

(**861**) A person sells out £4000 Consols (3 per cent. stock) at $89\frac{1}{4}$, and with half the proceeds buys Dutch 4 per cents. at $87\frac{7}{16}$, and with the other half Dutch $2\frac{1}{2}$ per cents. at $54\frac{15}{16}$: what is the alteration in his income?

(**862**) The income of a parish is £6529. 10*s.* 6*d.*: how much in the £. will produce a rate of £300.?

(**863**) Extract the square roots of 780811249, of 360117·609604, and of ·00142884, describing the process.

(**864**) Find the amount of £1550. 15*s.* for $3\frac{1}{2}$ years, at $2\frac{1}{2}$ per cent. per annum, compound interest.

(**865**) Four merchants, A, B, C, D, trading with a capital of £23800., find after a certain time their respective shares increased by £26. 11*s.* 8*d.*, £37. 4*s.* 4*d.*, £53. 3*s.* 4*d.*, and £63. 16*s.*: how much had they severally subscribed to the original capital?

(**866**) A room, 16 ft. 8 in. long by 13 ft. 4 in. wide, is to be carpeted with carpet a yard wide, costing 6*s.* 9*d.* a yard: what is the area of the floor, and how much carpet will have to be bought, how much will be used, and how much will it cost?

(**867**) If the simple interest on £923. 18*s.* $1\frac{1}{2}$*d.* amounts to £17. 9*s.* $3\frac{3}{4}$*d.* in 138 days, what is the rate?

(**868**) An Austrian Souverain and gold Ducat are worth 13*s.* 11*d.* and 9*s.* 6*d.* respectively: how many Ducats are equivalent to 4560 Souverains?

(**869**) Simplify—

$$\frac{3\frac{2}{3}\times 9}{\frac{5}{6}\times\frac{7}{8}} \div \frac{\frac{1}{2}+\frac{1}{3}-\frac{9}{10}}{\frac{4}{9}-\frac{7}{18}}\text{; and } 1\div\left\{2+\left[1\div 2+[1\div(2+\tfrac{1}{2})]\right]\right\}.$$

(**870**) The sum of £328. 3*s.* $6\frac{1}{2}$*d.* is to be divided among four men, A, B, C, and D, in such proportion, that for every £3. given to A, B is to receive £5., C £8., and D £9.: what sum did each receive?

(**871**) What sum will amount to £425. 19*s*. 4$\frac{4}{5}$*d*. in ten years at 3½ per cent. simple interest; and in how many years more will it amount to £435. 11*s*. 7*d*.?

(**872**) Find the true discount on £125. due two years hence, at 4 per cent. per annum.

(**873**) Find the amount of £1000. in 6 years at 5 per cent. compound interest.

(**874**) A rectangular cistern, whose length is 13$\frac{3}{8}$ feet, and breadth 6 feet, contains 294¼ cubic feet of water: what is the depth of the water; and what is its weight when a cubic inch of water weighs 252·5 grains?

(**875**) A loaded waggon weighs 2 tons 2 cwt.; the waggon by itself weighs 18¾ cwt.; the load consists of 215 packages of equal weight. Find the weight of each.

(**876**) Find the cost of 3 cwt. 1 qr. 7 lbs. at 3*s*. 10¾*d*. a lb.

(**877**) The sum of £204. 6*s*. 3*d*. is to be divided between 6 men, 3 women, and 12 children. For every £1. that each woman receives, each man is to receive £2., and each child 10*s*.: how much will they severally receive?

(**878**) Find the amount of £215. 7*s*. 5¼*d*. in 3 years at 4½ per cent. simple interest.

Also find the true discount on £1000. due a year and a half hence, at 5 per cent.

(**879**) Find the amount, compound interest, of £1627. 12*s*. 1*d*. in 4 years, at 4 per cent.

(**880**) If 16 horses eat 96 bushels of corn in 42 days, in how many days will 7 horses eat 66 bushels?

(**881**) A wooden chest, 6 ft. 2 in. long, 3 ft. 4 in. high, and 2 ft. 9 in. wide, is made of 1½ in. board: find the quantity of wood used, and the cubical contents of the chest.

(**882**) Calculate the value of $\sqrt{(3+\sqrt{3})}$ to 6 places of decimals.

(**883**) If the light received by an object at different distances from a candle be inversely as the squares of the distances, and a mirror reflect $\frac{4}{9}$ of the light which falls on it, at what distance from the candle must an object be placed in order to receive the same amount of light directly which it would receive by reflection from the mirror so placed that the whole distance traversed by the light is 5 feet?

(**884**) From $\frac{10}{1144}$ of $\frac{18}{16}$ of $\frac{23}{240}$ of a mile subtract $\frac{55}{4}$ of $\frac{1}{24}$ of a foot, and express the result in mètres. (One mètre may be taken as 39$\frac{3}{8}$ inches.)

(**885**) Multiply 0·0316228 by itself, giving the result correctly to six significant figures.

(**886**) Obtain the square root of 0·03456789 correctly to 7 places of decimals.

(**887**) Find the product of $0{\cdot}\dot{5}3846\dot{1}$ and $0{\cdot}3\dot{2}8571\dot{4}$, reducing the result to a vulgar fraction, expressed in its simplest form.

(**888**) Given that a gallon of water weights 10 lbs., that a cubic foot of water weighs 1000 ounces, and that a litre is a cubic decimètre, find how many litres there are in a gallon.

(**889**) What is the average height of 107 recruits if 49 of them are 5 feet 6 inches high; 37, 5 feet 7½ inches; 13, 5 feet 8½ inches; 6, 5 feet 10 inches; and 2, 6 feet high? And what percentage of them would have been rejected if the standard had been 5 feet 8 inches?

(**890**) A clock goes ·04 per cent. too fast: how much will it gain in a day?

(**891**) Find the compound interest on £4000. for 2½ years, at 5 per cent. per annum.

(**892**) A certain pudding consists of 3 parts of raisins at 6*d.* a pound, 4 parts of suet at 8*d.* a pound, and 2 parts of flour at 3*d.* a pound, and it costs altogether 7*s.*: what was the cost of each of the ingredients?

(**893**) Find the yearly income from investing £3960. in railway stock standing at 103, and paying 5¼ per cent., brokerage being ⅛ per cent.

(**894**) A person invests £2645. in railway shares at 115, the annual dividend on each share being £5. He afterwards sells out at 125, and invests in the 3 per cent. consols at 93¾. Find the change in his income.

(**895**) A steam engine of 4¼-horse power, working 51 days at 6 hours a day, consumes 25 tons of coal: how much coal will be consumed by a 17-horse power engine, working at the same work for 3 days at 8½ hours a day?

(**896**) If of every £1000. spent on University buildings Trinity College had to pay £229., for how much of the £4014. 8*s.* 4*d.* spent upon the Literary Schools would the College be liable?

(**897**) On a certain line of railway the telegraph posts are placed 58 yards apart, and a passenger counts as he passes 37 in two minutes: at what rate is he travelling?

(**898**) There were new moons on May 9 at 6.16 A.M. and on June 7 at 9.55 P.M. Also the seventh from now, June 7, 1880, will be on Dec. 31 at 1.56 P.M. What was the exact interval between the last two? and is it greater or less than the average for the rest of the year?

(**899**) Which would yield the greater return for an investment, London and North Western Stock at 161½ or Metropolitan Stock at 123½, the former paying 6½ per cent. and the latter 5 per cent.?

(**900**) In the centre of a room 21 feet square there is a square of Turkey carpet; and the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively 16*s*. 6*d*. and 8*s*. 6*d*. per square yard: and the whole cost of the carpet and oil-cloth is £35. 4*s*. 6*d*. Find the width of the oil-cloth border.

(**901**) The difference between the incomes derived from investing a certain sum in 5 per cent. stock at 127 and 5½ per cent. stock at 135 is £4. 14*s*. Find the amount invested, and the income resulting from each investment.

(**902**) A man wishes to have his library arranged on shelves each containing the same number of volumes. He tries successively to put fifteen, sixteen and seventeen volumes on each shelf, and each time he has three volumes remaining. If the number of volumes in his library be the least number of volumes for which this would be the case, show that he cannot arrange it in the way that he desires, except upon shelves which each contain three volumes.

(**903**) Silver of the ordinary standard contains 92·5 per cent. of pure silver, and silver of the Britannia standard contains 11 ozs. 10 dwts. of pure silver to the pound Troy. Find the weight of pure silver in two bars of the ordinary and Britannia standards weighing respectively 55 lbs. 9 ozs. 8 dwts. 8 grs., and 91 lbs. 7 ozs. 1 dwt.

(**904**) A mètre is 39·3708 inches, and a hectare is 10000 square mètres. Express a hectare in acres correct to three decimal places.

(**905**) A. and B. ride a race of 50 miles on bicycles. The wheel of A.'s bicycle makes 3 revolutions in 2 seconds, and that of B. 960 revolutions in 10 minutes 15 seconds. A. accomplishes the distance in 3 hours 20 minutes, and wins by 5 minutes. Find the circumference of the wheel of each bicycle.

(**906**) A man has £12000 to invest. He invests ⅓ of this sum in 3 per cent. Consols at 96, $\frac{7}{24}$ in Railway debenture 4 per cent. stock at 105, and the remainder in French 3 per cent.

Rentes at 67½. Find his whole income, and the average rate of interest he makes on his investment.

Find also the increase in his capital, when each stock has risen 1 per cent.

(**907**) A proprietor of Railway stock received last year a dividend of 6½ per cent. on his stock, and paid an income-tax of 2*d.* in the pound. This year the dividends have fallen to 6 per cent., and the income-tax is increased a penny in the pound, and his net income is reduced by £125. Find the amount of stock which he holds.

(**908**) Add together $5{\cdot}6\dot{3}\dot{1}$, $\cdot04\dot{2}$, $\cdot\dot{9}7\dot{6}$ and $8{\cdot}349\dot{4}8766\dot{9}$.

(**909**) I was doubting whether to sell a horse at a price which would have ensured my losing 8⅓ per cent., when I got another bid of 218 guineas and thereby cleared 9 per cent.: what was the first bid made?

(**910**) A person had Railway preference stock bearing 5 per cent. interest which brought him in £800. a year. He sold out at 113 and reinvested in canal stock, which paid 3 per cent. He had to pay the broker ⅛ per cent. on each transaction. At what price did he reinvest so as to gain £40. per annum?

(**911**) The managers of a building company contracted to buy for £45000., through A., land which they might have bought direct from B. for £28000. The company expended £60000. in improving the property and had to borrow £20000. of this at 5 per cent. The dividend paid on the subscribed capital was 4 per cent. What was the loss in income to a holder of £8800. stock owing to this mismanagement?

(**912**) A butler stole wine from a butt of sherry which contained 25 per cent. of spirit; he replaced this by wine containing 9 per cent. only of spirit; the cask was then of 13 per cent. strength only: how much of the wine had he stolen?

(**913**) In what time will £1455. amount to £2328., if, in 3 years, £430. amount to £494·5.?

(**914**) If the wholesale dealer sell to a retailer at 10 per cent. profit, and the retailer sell to the consumer at 50 per cent. profit, what proportion of the price paid by the consumer is profit?

(**915**) A grocer buys coffee at the rate of £8. 10*s.* per cwt. and chicory at £2. 10*s.* per cwt., and mixes them in the proportion of 5 parts chicory to 7 coffee: at what rate must he sell the mixture so as to gain 16⅔ per cent. on his outlay?

(**916**) If I borrow money at 3 per cent. interest payable yearly, and lend it immediately at 5 per cent. payable half-yearly, receiving compound interest for the second half-year, and gain thereby at the end of the year £660., what was the sum of money which I borrowed?

(917) A milkman buys daily 36 gallons of milk at 8*d.* per gallon: how much water must he mix with it so that by selling the mixture at 9*d.* per gallon he may clear 25 per cent. profit?

(918) A. has stock in the 3 per cent. Consols which produces him £300. per annum. He sells out one-half at 92, and invests the proceeds in a railway company when a £50. share is worth £23. What dividend per cent. per annum ought the company to pay so that he may increase his income by £50.?

(919) A person takes £60. with him to France and changes it into French money at the rate of 25·125 francs per £1. He remains there 28 days, spending 37¾ francs per day. How much has he left?

(920) A closed vessel 3 feet in length, 2 feet in width, and 1 foot high is made of metal 1 inch thick, and weighs 21 lbs. 5 ozs. Find the weight of a solid block of the metal of the same dimensions.

(921) Three persons whose strides are respectively 2 ft. 6 in., 2 ft. 8 in., and 3 ft., walk a mile together: how often in this distance will they step together?

(922) The carpet for a room 28 feet long and 23 feet broad cost £5. 14*s.* 0½*d.*, and a carpet of the same kind for another room 26 feet long cost £5. 10*s.* 6*d.*: what was the breadth of the latter room?

(923) Find the side of a square containing 149296 sq. yds. 3 sq. ft. 52 sq. in.

(924) Find the simple interest on £415. 10*s.* for two years at 2½ per cent. per annum.

(925) The compound interest on a certain principal sum for two years at 5 per cent. exceeds the simple interest by 15*s.* 4*d.*: what was the sum?

(926) A gross profit of £19. 18*s.* 9*d.* is made by selling 11 cwt. 1 qr. 16 lbs. of tea at 2*s.* 9¾*d.* a lb.: what was the profit per cent.?

(927) Divide £10. 1*s.* 8*d.* among 4 men, 3 women, and 2 children, giving each man half as much again as each woman, and each woman twice as much as each child.

(928) If 3 men and 4 boys can do as much work as 2 men and 16 girls in the same given time, and 4 men and 2 boys as much as 12 boys and 12 girls, how should a man who receives

44 shillings for a piece of work reward a boy and a girl who have been helping him all the time?

(**929**) Extract the square root of 120409; also the cube root of $3\frac{1}{3}$ to two places of decimals.

The breadth of a room is twice its height and half its length, and its contents are 4096 cubic feet: find the dimensions of the room.

(**930**) Find the interest on £1875. at $3\frac{3}{4}$ per cent. for 16 months, and the discount on £6191. 10*s.* 1*d.* at $3\frac{3}{4}$ per cent. for 4 months.

(**931**) What is the amount, at compound interest, of £1864. at $4\frac{1}{2}$ per cent. in three years?

(**932**) If six grains of silver are worth five farthings, what should a crown weigh?

If the alloy in silver standard coin is $\frac{1}{21}$ of the mass, what would a crown be worth if it were pure silver?

(**933**) What will be the cost of purchasing £720. Russian 5 per cents. at $91\frac{1}{2}$? Also, if the stock be sold out again when the price has risen to $93\frac{7}{8}$, what profit is made by the double transaction?

N.B.—Usual brokerage charged at each transfer, $\frac{1}{8}$ per cent.

(**934**) Find to a penny the compound interest on £377·57. for 3 years at $3\frac{1}{2}$ per cent.

(**935**) A man buys a site of 3 ac. 3 rds. 15 po. for a house and garden at £440. an acre, and spends £3808. 15*s.* in building the house. He lets the premises for £357. 10*s.* a year: what rate per cent. does he get for his money?

(**936**) Find to four places of decimals the square roots of 8·1 and ·00081.

What is the cube of 50·09, and what is the cube root of 8000000?

(**937**) A person standing on a railway platform noticed that a train took 21 seconds to pass completely through the station, which was 88 yards long, and that it was 9 seconds in passing him. How long was the train, and at what rate per hour was it travelling?

(**938**) A person invests money in the 3 per cents. at 98, and after receiving three half-yearly dividends sells out at 99, paying $\frac{1}{8}$ per cent. on each of the two transactions. If the result of these transactions is to increase his capital by £236. 5*s.*, how much capital did he invest?

(**939**) Distinguish between 'commensurable' and 'incommensurable' numbers.

Show, without the use of algebraical symbols, that if two numbers are exactly divisible by a third, then the sum or difference of any two multiples of these numbers is also divisible by the third.

A hall is 58·99 feet long and 27·066 feet wide and it is to be completely paved with equal square tiles: what is the exact size of the largest tile that can be used?

(940) The present value of a certain debt due three years hence is £112. 10*s.* In two years' time, if it is not paid, its value will be £120. 2*s.* 6*d.* What is the debt?

(941) The breadth of a room is half as much again as its height: its length is twice its height; it costs £5. 5*s.* to paint its walls at 1¼*d.* per square foot: what are its dimensions?

(942) Three men can do as much work as 5 boys; the wages of 3 boys are equal to those of 2 men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350.: how long would it take if 20 boys and 20 men were employed, and how much would it cost?

(943) A. walks to a place at the rate of 4½ miles per hour: at 8 miles from his destination he meets B., and turns back with him (walking at B.'s rate) for a mile: if A. is half an hour late at his destination, what is B.'s rate, and at what rate should A. have walked after parting with B. so as to arrive at the proper time?

(944) If the difference between the interest and discount on a sum of money for 2 months at 4½ per cent. be 2*s.* 3*d.*, find the sum.

(945) If the price of the 3 per cent. stock be 96, a person can obtain an annual income of £1. more than he can if the price be 97. How much has he to invest?

(946) In an election A. receives promises from half the constituency, B. from $\frac{19}{40}$; 400 voters, however, had promised both candidates, of whom 300 vote eventually for B. and 100 for A. The electors who had not promised either do not vote. B. wins by 80 votes. What were the numbers for each?

(947) A box 1 ft. 8 in. long, 1 ft. 3 in. wide, and 1 ft. 4 in. high has to be lined inside with tinfoil: what length of tinfoil will be required if it is sold in sheets 5 in. wide?

(948) What sum of money will amount to £338. 8*s.* in 2½ years at $3\frac{1}{5}$ per cent.?

(949) In a coin weighing 4 ozs. and made of a metal formed by mixing 2 lbs. 6 ozs. of copper with 1 lb. 10 ozs. of tin, how much of each of these two metals is there?

(950) A cistern whose dimensions are 5 ft., 16 ft., and 4 ft. 6 in., is filled by a pipe in three-quarters of an hour: how long will it take three such pipes to empty another cistern whose dimensions are 9 ft., 19 ft. 6 in., and 5 ft. 4 in.?

(951) A farm of 430 acres is let at £1. 17*s*. 6*d*. per acre. It is subject to the following charges: Land Tax, 1 per cent.; Poor Rates, 7*d*. in the £.; other rates 11*d*. in the £.; Tithe 4*s*. 8*d*. per acre; Insurance, 4*s*. per £100. What is the net income derived from that farm?

(952) A man buys a horse for £156., and pays for it by a bill due 8 months afterwards. He sells it immediately for £180. cash. What does he gain per cent. on the transaction, reckoning the rate of interest at 4½ per cent.?

(953) If I transfer £3000. from 4 per cent. stocks at 90 to 3 per cents. at 72, what difference does it make in my income?

(954) Find the square root of 323 to four places of decimals, and the cube root of 40353607.

(955) If 2 cwt. 2 qrs. 20 lbs. of sugar at 6½*d*. per lb. is mixed with 4 cwt. 2 lbs. at £1. 17*s*. 4*d*. per cwt., and the mixture sold at 6*d*. per lb., what is the whole profit and the profit per cent.?

(956) The return of a business in which three persons have invested £150., £260., £400. respectively is £50. 12*s*. 6*d*.: what is the share of each?

(957) I invested £7740. in the 3 per cents. at 89⅞; a week later they rose to 96⅝; what would have been my loss of income had I waited till the end of the week, brokerage being ⅛ per cent.?

(958) A Turkey carpet 13 ft. 6 in. by 10 ft. 9 in. is laid on a floor 21 ft. 6 in. by 12 ft. 9 in.: find the quantity of felt required to cover the rest of the floor, and its cost at 3*s*. 9*d*. per square yard.

(959) Find the simple interest on £750. 10*s*. 7½*d*. for 64 days at 6¼ per cent. per annum.

(960) The receipts of a company are divided as follows: 45 per cent. for working expenses, 10 per cent. for reserve fund, a preference dividend of 4 per cent. on one-fourth of the capital, and the remainder, £54000., for dividend at 5 per cent. on the rest of the capital: find the capital and receipts.

(961) A merchant bought 1452 articles, ¼ of which he sells at a gain of 6 per cent., ⅓ at a gain of 8 per cent., and the remainder at a gain of 10 per cent.; if he had sold the whole at a gain of 12 per cent. he would have gained £22. 3*s*. 8*d*. more: find the cost price of each article, and prove your answer.

(**962**) Reduce £2. 17*s*. 5½*d*. to the decimal of £12.

·2640625 of a ton of sugar is sold for £7·39375.: find the price per lb.

(**963**) A rectangular field, twice as long as it is broad, contains 61 ac. 2 rd. 10 po. 29½ yds.: find the cost of fencing it at 1*s*. 3*d*. per yard of fencing.

Find the square root of 327·537604.

(**964**) Find the amount of £937. 10*s*. in 3 years, at compound interest, at 4 per cent. per annum.

The discount on an amount due a year and six months hence is £68. 10*s*. 3*d*.: what is the amount, simple interest being reckoned at 4½ per cent. per annum?

(**965**) A man derives an income of £220. 10*s*. from 4½ per cent. stock. He sells out at 112, and invests the proceeds in 4 per cent. stock, increasing his yearly income by £3. 10*s*. At what price does he buy the 4 per cent. stock?

(**966**) A cubic foot of water weighs 1000 ozs., and a gallon contains 277 cubic inches: find the weight of water in a cistern which contains 27000 gallons.

(**967**) A. begins a piece of work by himself; after 5 days B. comes to his help, and they together finish the work in 7 more days. A. could do the whole work by himself in 18 days. In how many days could B. do the work?

(**968**) Two clocks both indicate the right time at noon on July 3. One gains 60 seconds and the other loses 30 seconds in every 12 hours. When the first indicates noon on July 10, what time does the second indicate?

(**969**) A dealer buys a certain number of articles at the rate of 40 for a pound, and twice the same number at 50 for a pound. He sells at the rate of 48 for a pound. How much per cent. does he gain or lose by the transaction?

(**970**) Show that the interest on £16280. at 3½ per cent. for 9 months is equal to the discount on £3017. 7*s*. at 11 per cent. for 18 months.

(**971**) The population of a town increases every year at the rate of 2 per cent.: what was the number of inhabitants in 1874 if in 1877 it was 132651?

(**972**) Multiply the difference between $\cdot 31\dot{1}\dot{2}$ and $\cdot 311\dot{2}$ by 12·21, and exhibit the result in the form of a repeating decimal.

(973) The sum of £300. is invested in 3 per cent. stock at 96. What sum must be invested in 5 per cent. stock at 110 in order that the rate of interest on the whole investment may be 4½ per cent.?

(974) Reckoning simple interest, what sum of money lent out at 4 per cent. per annum will produce in three years the same amount of interest as £540. lent out at 5 per cent. will produce in two years?

(975) At what rate of interest will the present worth of £100. 10*s.* 3*d.*, payable two years hence, be £93. 10*s.*?

(976) A room is 18 ft. 3 in. long, 13 ft. 3 in. wide, and 10 ft. high. Find the difference of cost of papering the walls with English paper, which is 21 inches wide, and costs 6*d.* a yard, or with French paper, which is 18 inches wide, and costs 5½*d.* a yard, allowing 14 square yards for windows, &c.

(977) A garrison of 1500 men has just provisions enough to allow 26 ozs. of bread a day to each man for 38 days; but, an engagement coming on, the garrison is increased by 400 men: how many ounces of bread must be assigned to each so as to enable them to protract the siege for 27 days longer?

(978) For the ordinary parts of the Previous Examination 539 men put down their names; $\frac{5}{7}$ of the number entered for Part I., $\frac{8}{11}$ for Part II. How many men entered for both parts?

(979) How many yards of carpet $2{\cdot}1\dot{6}$ ft. wide will cover a room $25{\cdot}\dot{3}$ feet broad and $28{\cdot}\dot{8}$ feet long?

(980) If 75 men can perform a piece of work in 12 days of 10 hours each, how many men will perform a piece of work twice as large in a tenth part of the time if they work the same number of hours in the day, supposing that two of the second set can do as much work in an hour as three of the first set?

(981) Find the sum which must be invested in the 3 per cents. at 90, at simple interest payable half yearly, to amount in 23⅓ years to £2317. money, the price of the funds remaining unchanged. If the funds rose to 96, in how many years sooner could the required amount be realised?

(982) In a school the numbers for the week were:—Monday morning 67, Tuesday morning 60, Wednesday morning 65, Thursday morning 68, Friday morning 62, Monday afternoon ·5 more than the average of Monday and Tuesday mornings, Tuesday afternoon 59, Wednesday afternoon ·5 less than the average of Tuesday, Thursday the average of Monday morning and Tuesday afternoon, Friday afternoon 60. Find the average attendance for the week.

(983) Fifty cards, of which one is marked, are dealt round to form 5 heaps, and the heaps are then piled one on the other in such order that the heap containing the marked card is the middle one; the pack so made is again dealt round to five heaps, and the heaps piled as before; if now the pack be dealt to two heaps, show that the marked card will be the thirteenth in the heap which contains it.

(984) Multiply together ·001234 and ·0789: divide the product by 34·56, and extract the square root of the quotient, giving the result to three significant figures.

(985) Assume that 6 men can do as much work in an hour as 7 women, and 8 women as much as 11 boys, and that 5 men can do a certain piece of work in 10 hours. How long will it take 1 man, 2 women, and 3 boys together to do the same piece of work? Express the result decimally.

(986) A plot of land is sold at £1200. per acre. What is the price in francs per square mètre? (£1. = 25 francs; 1 mètre = 39$\frac{3}{8}$ inches.)

(987) A reduction of 30 per cent. in the price of eggs would enable a purchaser to obtain 54 more for a guinea: what may the present price be?

(988) Express $\sqrt{\frac{\cdot00456 \times \cdot987}{6\cdot54}}$ as an ordinary decimal fraction, correctly to three significant figures.

(989) Express $\sqrt{1\cdot\dot{2}\dot{7} \times 1\cdot\dot{5}7142\dot{8}}$ as a vulgar fraction in its lowest terms.

(990) What is one shilling and sixpence a gallon in francs per litre? Express the result decimally correct to four significant figures.

(A gallon of water weighs 10 pounds; a litre is a cubic decimètre; a gramme is the weight of a cubic centimètre of water; and you may assume that a kilogramme is 2$\frac{1}{5}$ lbs., and that £1. = 25 francs.)

(991) Assume that 4 English navvies can do as much work in a day as 5 French navvies, that 4 French navvies can do as much as 7 negroes. It is found that 13 English and 12 French navvies do a piece of work in 3 days. How long will it take 10 negroes? Express your result decimally to three significant figures.

(992) A milk-dealer buys pure milk at 11$\frac{1}{4}$*d.* per gallon. How much water must he add that he may sell at 5*d.* a quart, and obtain a gross profit of 100 per cent.?

(993) A reduction of 10 per cent. in the price of iron would enable a purchaser to obtain one hundredweight more for a sovereign: what may the present price be?

(**994**) Express $\sqrt{\frac{\cdot 0132 \times \cdot 543}{7\cdot 65 \times \cdot 0301}}$ as a decimal fraction, correctly to three significant figures.

(**995**) Reduce $[\cdot 0\dot{9}\dot{6} \times \cdot\dot{8}4615\dot{3}] \div [\cdot\dot{4}6153\dot{8} \times \cdot 8\dot{3}]$ to its simplest form.

(**996**) A reduction of 20 per cent. in the price of beef would enable a purchaser to obtain 6 lbs. more for a sovereign: what is the reduced price?

(**997**) Water runs into a cistern by one pipe and from it by two, in each pipe at a uniform rate when the pipe is open. The cistern holds 1000 gallons. If all three pipes be open at once, the cistern, supposed full, would empty itself in 200 minutes. If one escape pipe and the supply pipe only be open, it would, if empty, fill in 200 minutes; but if the other escape pipe and the supply pipe be open, in 100 minutes. How long would it take the two escape pipes to empty the cistern if the supply pipe were closed?

(**998**) A ton of stone measures 13 cubic feet: what does a cubic mètre weigh in kilogrammes? (1 mètre (linear) = $39\frac{3}{8}$ inches: 1 kilogramme = $2\frac{1}{5}$ pounds.)

(**999**) Express $\{(9\cdot 864 \times \cdot 01234) \div (\cdot 005678 \times \cdot 00008765)\}^{\frac{1}{3}}$ as a decimal to three places, and $\{(\cdot\dot{4}2857\dot{1} \times \cdot 7\dot{7}1428\dot{5}) \div (\cdot\dot{2}8571\dot{4} \times \cdot 0\dot{5}7142\dot{8})\}^{\frac{1}{2}}$ as a vulgar fraction.

(**1000**) If a farmer lays two tons of lime on an acre of land, how many grammes is that per square mètre?

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